Definitions

In physics we have two types of measurable quantities: **vectors** and **scalars**.

Scalars: have *magnitude* (magnitude means size) only

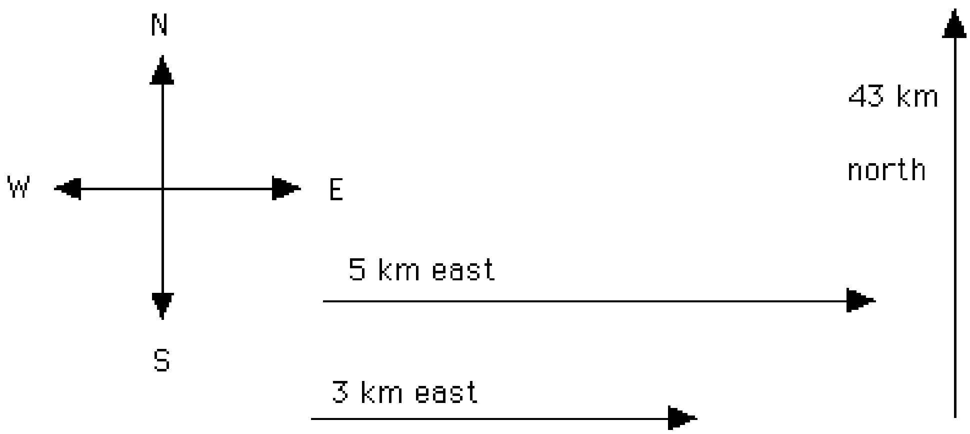
- Examples of scalar quantities include *time*, *mass*, *volume*, *area*, *distance*, *speed*, *work*, *energy*, *power*.
- Note (1): *distance* is the length of a path travelled; for example, running north for 300m, then turning around and running south for 200 m means you ran a total distance of 500 m.
- Note (2): *speed* is the rate of *distance* travelled, or distance travelled divided by a *time* interval. Speed is a scalar quantity because both distance and time are also scalar quantities.

Vectors - have both magnitude and direction

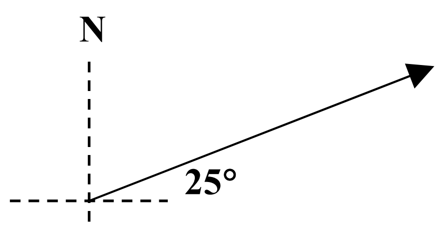
- Examples of vector quantities include *displacement*, *velocity*, *acceleration*, *force*, *momentum*, *impulse*, *field strength*.
- Note (3): *displacement* is the change in position of a moving object, or the difference between starting and finishing points. For example, running north for 300m, then turning around and running south for 200 m means your total displacement is 300 m (N) – 200 m (S) = 100 m due north from your starting point.
- Note 4: *velocity* is the rate of change in position, or net *displacement* divided by a *time* interval. Velocity is a vector quantity because displacement is a vector quantity.
- Note 5: *acceleration* is the rate of change in *velocity*, or change in velocity divided by *time*. This makes acceleration a vector quantity – e.g.; acceleration due to gravity is -9.8 m/s^2, or 9.8 m/s^2 down.

In this unit, we will be focusing on vector analysis. To show a vector, draw an arrow in the direction of the vector and scale the length to reflect the vector's magnitude.

Here are some examples of displacement vectors, using a scale of approximately 1 cm = 1 km:



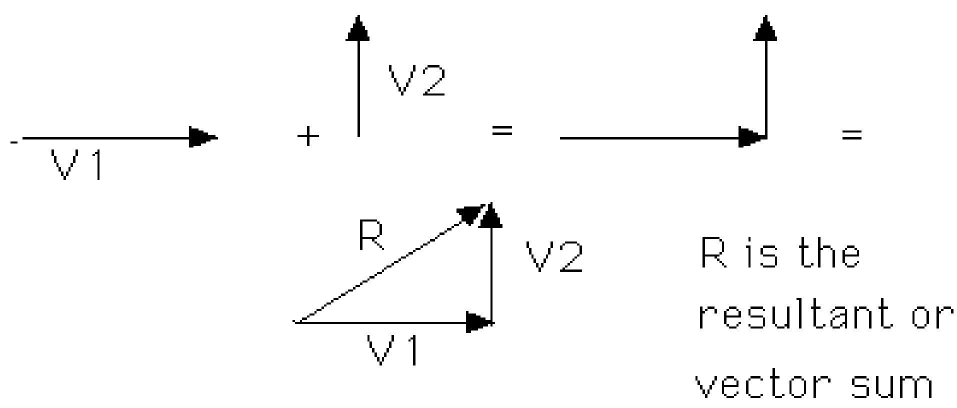
To show a vector that is not N, S, E or W:



This vector direction can be described in two ways:
a) **25°** north of due east, or **25° N of E**
b) **65°** east of due north, or **65° E of N**

Vector Addition

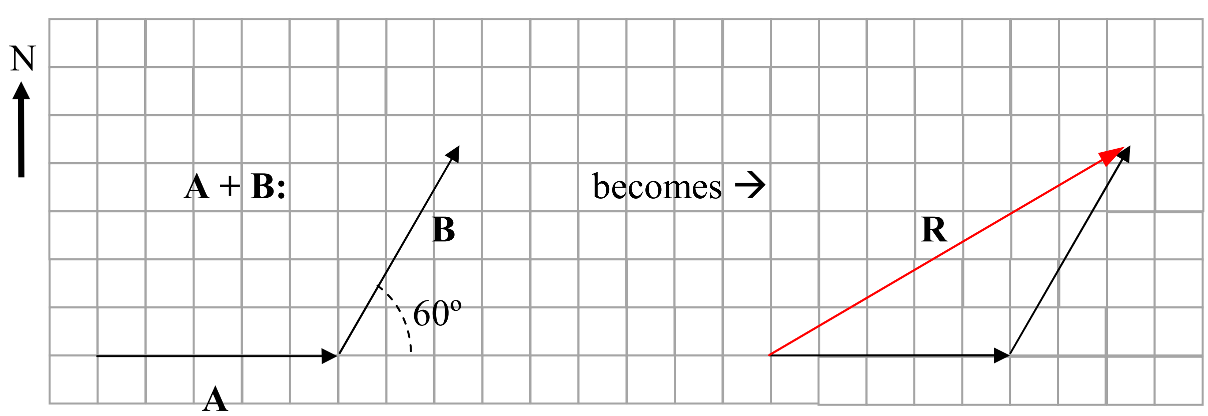
Make a diagram in which the vectors are placed one after another, the tail of the second vector on the head of the first. This is the tip-to-tail method.



Example 1.

A man walks 250 meters due east (vector A), then 250 meters 60º north of east (vector B). Using scale diagrams on graph paper, determine the magnitude and direction of the resultant displacement.

- Adopt an appropriate scale (e.g. 1 square side = 50 m)
- Note that the correct length of vector B must be measured first, then drawn on the grid.
- Vector-add **A + B** by drawing each vector tip-to-tail. Then draw the resultant vector from the start-position to the finish-position.



- Now measure length **R**. This should be done by measuring the number of grid lengths, multiplied by 50 m. Measured correctly, **R** should equal approximately 433 m.

- Finally, use a protractor to find the direction of **R**. This angle should be measured relative to either a horizontal (east/west) or a vertical (north/south) grid line.
- The answer: **R = 433 m at 30° N of E**

Example 2.
Add the displacements $D_1 + D_2 + D_3 = R$ where $D_1 = 6$ km north, $D_2 = 3$ km east, and $D_3 = 4$ km (45° S of E).

(see Vectors Ex 2 for answer)

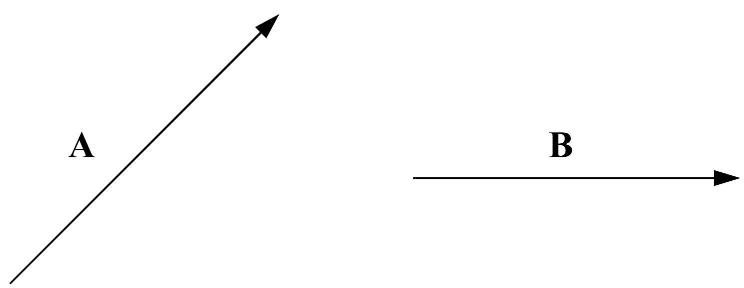
Vector Subtraction

In algebra, subtracting a number is the same as adding the opposite of that number. For example:

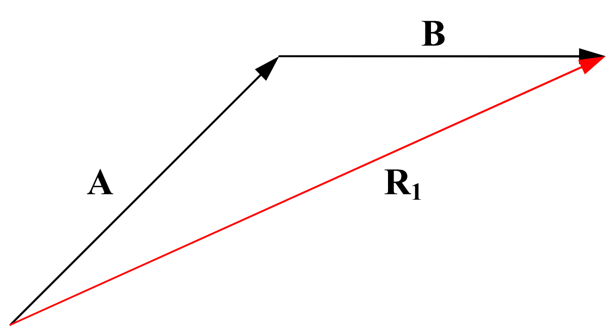
$$16 - 11 = 16 + (-11) = 5$$

Think about vector opposites. Given a vector **B** that points due east, **-B** is just the other direction from **B**, but with the same *magnitude* (the term 'magnitude' means size):

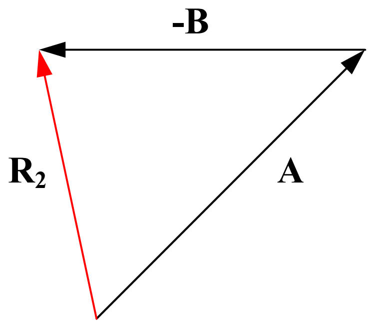
Now consider two vectors, **A** and **B**, as follows:



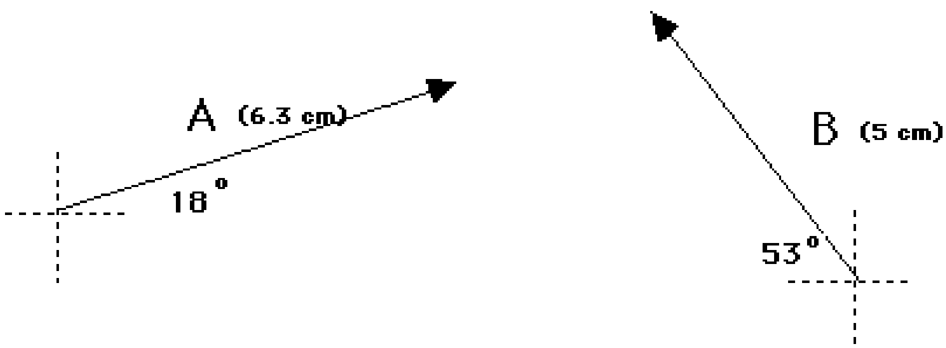
Adding **A** + **B** produces resultant $\mathbf{R_1}$:



Subtracting **A - B** is the same as adding **A + (-B)**; this produces resultant $\mathbf{R_2}$:



Example 3.
Given the vectors shown:



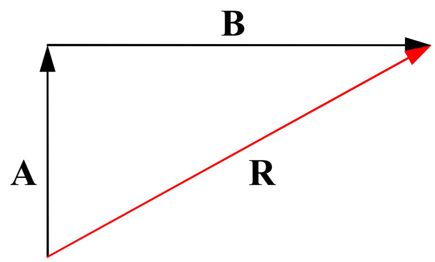
Draw diagrams and find R for:
(a) A + B = R
(b) A - B = R
(c) B - A = R

(see Vectors Ex 3 for answer)

Adding and Subtracting Perpendicular Vectors using Trigonometry

Given these vectors: **A = 15 m/s (N)**, **B = 25 m/s (E)**, find the magnitude and direction of the following:

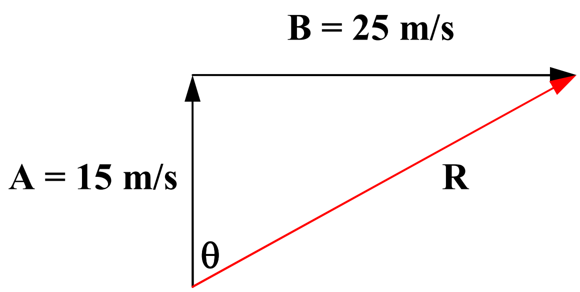
a) A + B = R:



First, find the magnitude of the resultant using pythagoras: $\mathbf{a^2 + b^2 = R^2}$

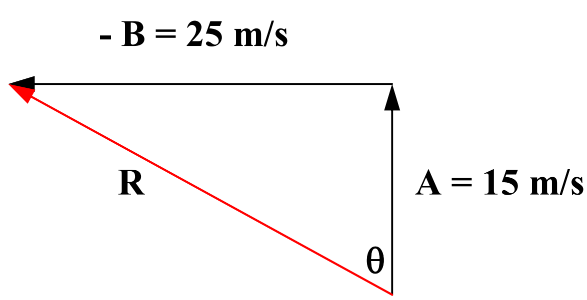
→ $\mathbf{15^2 + 25^2 = R^2}$ → **R = 29 m/s**

Now, use inverse (arc) tangent to find the angle:



$\tan\theta = \frac{25}{15}$ --> $\theta = 59°$ --> **R = 29 m/s (59° E of N)**

b) A - B = R



→ Here, **R = 29 m/s (59° W of N)** → same magnitude, but different direction!

Vector Analysis: Relative Velocity

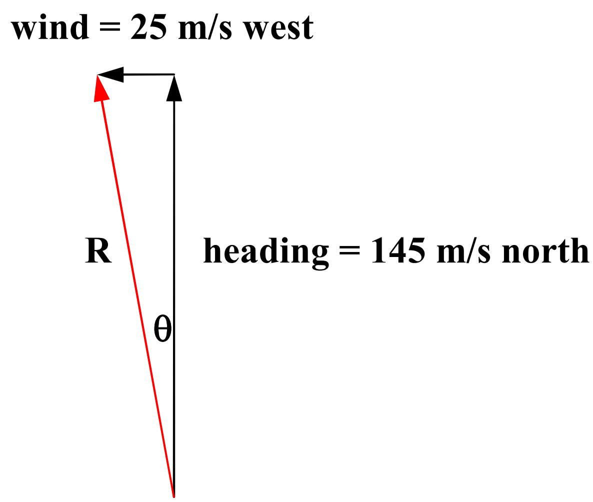
Suppose you are standing beside a highway watching car A move north at 20 m/s and car B moving south at 25 m/s. These are the observations you make as a stationary observer. However, the reality is quite different to each of the drivers in the two cars.

According to the driver in car A (i.e. relative to her), you are moving south at 20 m/s towards the car (the driver in car A thinks of herself as stationary and that you are moving). Car A also thinks that car B is moving at 45 m/s south! Similarly, the driver in Car B thinks that car A is moving at 45 m/s north.

These ideas are used to solve navigation problems for heading, displacement and time of travel.

Consider the following problem: A plane heads due north at 145 m/s. What is the plane's velocity relative to the ground if there is a 25 m/s wind blowing due west?

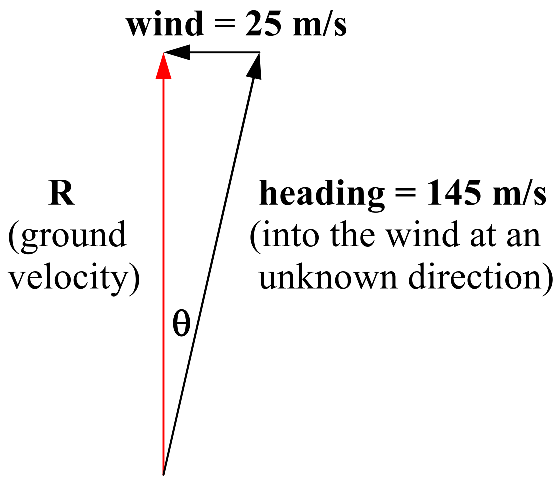
- The term "relative to the ground" describes how the plane appears to move according to an observer standing on the ground.
- To such an observer, the plane is being moved by:
  a) its engine, at 145 m/s due north (the heading);
  b) the wind, at 25 m/s due west.
- Therefore, the plane's velocity relative to the ground is simply the *resultant* of vector-adding engine heading and wind velocity. It represents the actual path of the aircraft over the ground.



- Using Pythagoras and simple trig, the plane's velocity relative to the ground is R = 147 m/s at 9.7° W of N.

Navigation involves directing a boat or plane in the correct direction to deal with wind or water currents. If the pilot in this same plane wanted to go straight north, what direction would she aim the plane (What is the heading?) Plane velocity is still 145 m/s and the wind is still 25 m/s to the west.

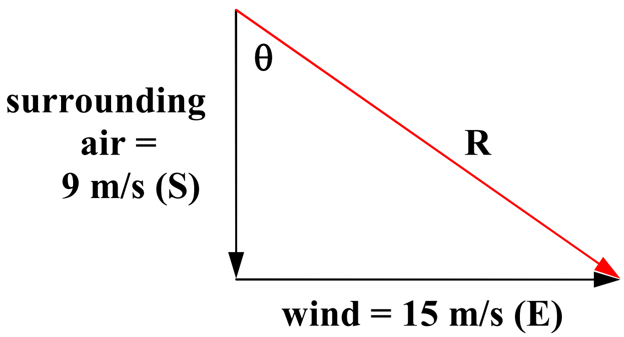
- Remember, **wind + heading = ground velocity (the resultant)**.
- In this case, we know the magnitude, but not the direction, of the heading.
- The vector-addition diagram looks like this:



- As you can see, vector-adding creates a resultant that points due north (where the pilot wants to head), and is <u>not</u> the hypotenuse.
- Using inverse-sine, the proper heading for the plane is 9.9° E of N. Note that the resultant speed in this case can also be determined, and is 143 m/s. This velocity is less than the engine's capable velocity, caused by the plane pointing into the wind.

Finally, think about this relative velocity problem: There is a 15.0 m/s wind blowing due east and you start riding your bike north at 9.0 m/s. What is the velocity of the wind in your face?

- This time, YOU are the observer. Relative to you, the bike is stationary.
- Even though you are travelling north, you observe that the ground and surrounding air is travelling *south*, at 9.0 m/s.
- You also observe the wind moving east at 15.0 m/s.
- Therefore, the apparent wind velocity that hits your face is simply the addition of these two vectors:



Remember, the 9 m/s points <u>south</u> because it represents the wind caused by your motion.

- The apparent velocity of the wind in your face is 17.5 m/s at 59° E of S.

Example 4.
A plane with an air speed of 105 m/s heads west when a 25 m/s north wind is blowing. What is the velocity of the plane relative to the ground?

(see Vectors Ex 4 for answer)

Example 5.
A plane is capable of 120 m/s is still air. Where must the pilot head the plane in order to end up going due north when there is a 35m/s west wind?

(see Vectors Ex 5 for answer)

Example 6.
A boat is capable of 12 m/s in still water. If a river flows at 7.0 m/s due east and is 500 m wide:

(a) What is the velocity of the boat relative to the shore if the boat heads south, perpendicular to the current?
(b) How long would it take to cross the river?
(c) Where would the boat have to aim in order to end up directly across from its starting point?

(see Vectors Ex 6 for answer)

Components of Vectors

Vectors that are perpendicular to each other are independent of each other, and their effects can be considered separately.

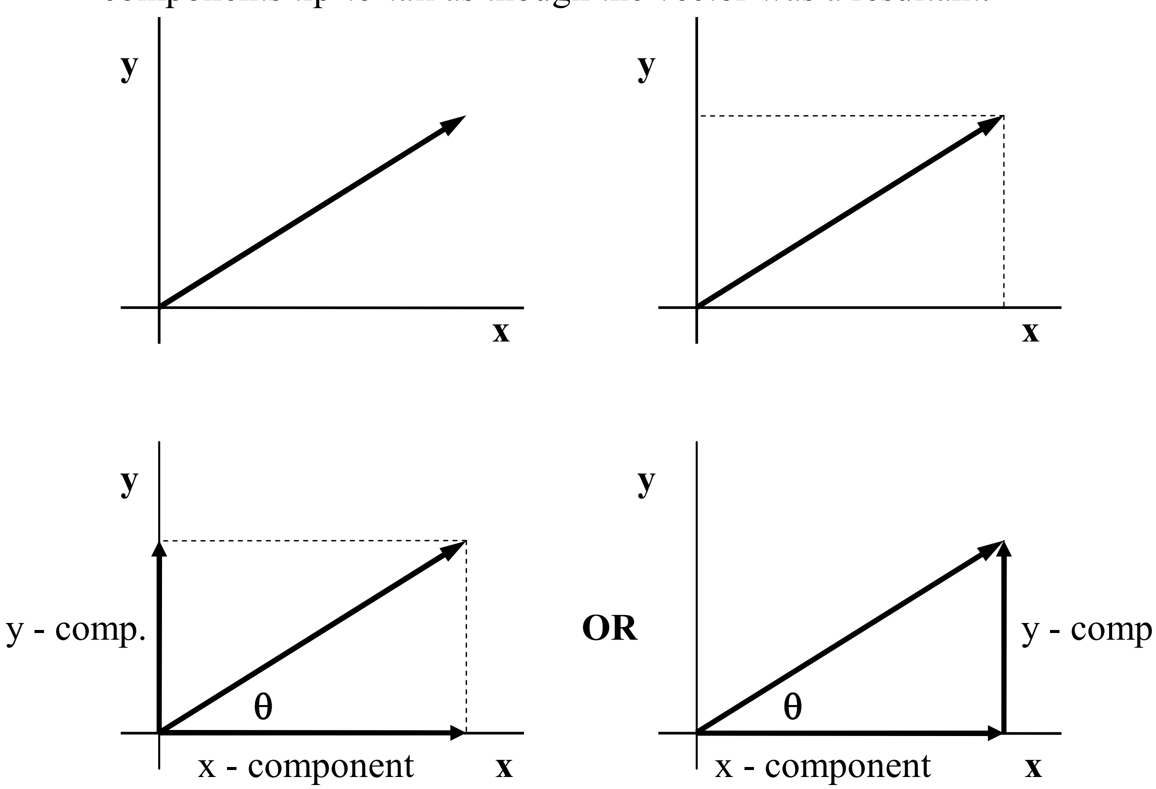
If a vector can be taken apart so that we have only its effects on the X and Y axes, then one-dimensional equations can be used on each axis.

Taking apart a vector like this is called <u>resolving</u> it into perpendicular components. In general, such components are either:

a) horizontal and vertical, or
b) parallel and perpendicular to an inclined surface.

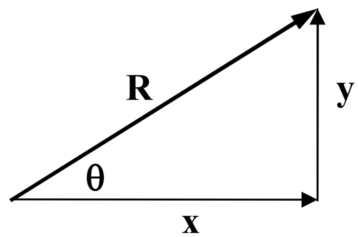
To resolve a vector into perpendicular components:

1. Place the vector on an X-Y grid.
2. Drop perpendicular lines to each axis.
3. Draw your components on each axis at the perp. cuts **<u>or</u>** draw the components tip-to-tail as though the vector was a resultant.



Once a vector is resolved into its components this way, trigonometry can be used to find the magnitude of the components.

Consider the following diagram:



Vector **R** can represent any vector quantity (e.g. force **F**, velocity **v**, etc.). Therefore, **x** and **y** represent the components of the vector quantity.

F --> $\mathbf{F_x}$, $\mathbf{F_y}$ components

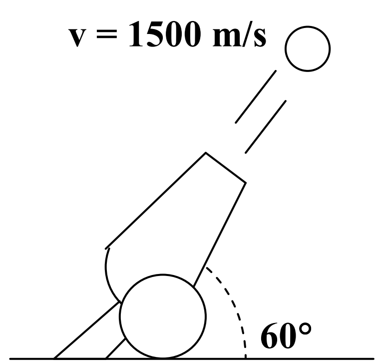
v --> $\mathbf{v_x}$, $\mathbf{v_y}$ components

In the diagram above,

$\sin\theta = \frac{y}{R}$ therefore $y = R\sin\theta$

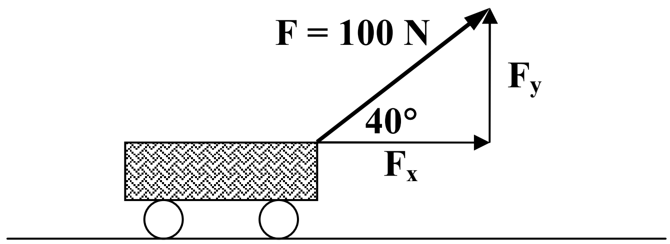
$\cos\theta = \frac{x}{R}$ therefore $x = R\cos\theta$

Example 7.
A cannon is shot at a muzzle velocity of 1500m/s at an angle of 60º to the horizontal. What are the vertical and horizontal components of the velocity?



(see Vectors Ex 7 for answer)

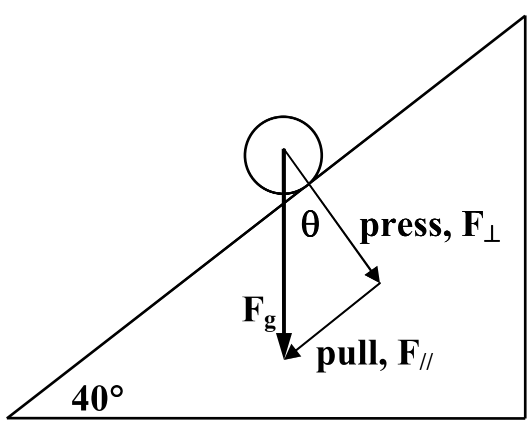
Example 8.
A boy pulls a wagon with a force of 100 N at 40 degrees to the horizontal. Find the pulling force (F_x) and the lifting force (F_y). (64 N, 77 N)



(see Vectors Ex 8 for answer)

Note that components are not always drawn as horizontal or vertical components. Sometimes, it's easier to show components that are *parallel* and *perpendicular* to surface where the object is perched.

Think about this problem: a ball rests on a slope. What component of it's weight F_g presses it onto the surface, and what component acts as a pulling force down the plane? The ball weighs 600 N and the slope is 40° to the horizontal. (460 N, 386 N)



As can be seen, the pull and press are perpendicular components, and can be solved using the same methods as in previous examples.

Our superior knowledge of geometry allows us to see that by similar triangles, **$\theta = 40°$**; from this, the problem can be solved:

→ **press, $F_\perp = F_g \cos 40 = 460$ N**

→ **pull, $F_{//} = F_g \sin 40 = 386$ N**

As will be seen later, this way of creating components will be very useful in solving force-related problems for objects on a slope.

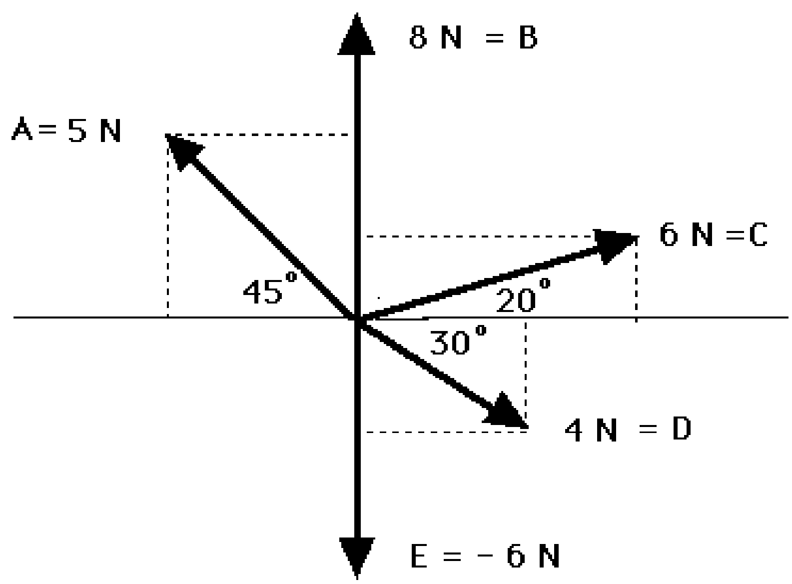
Finally, components can be used to add vectors when a large number of vectors (at least 4) are presented. Some or all of these steps can be used:

1. Draw all vectors on a set of X and Y axes
2. Draw in the X and Y components on the axes
3. Make a table adding the X and the Y components separately
4. Use vector addition to get the resultant of the X component sum and the Y component sum.

Now look at the following problem:

Add 8N (N)+ 5 N (45ºN of W) + 4N (30º S of E) + 6N (20º N of E) + 6 N (S)

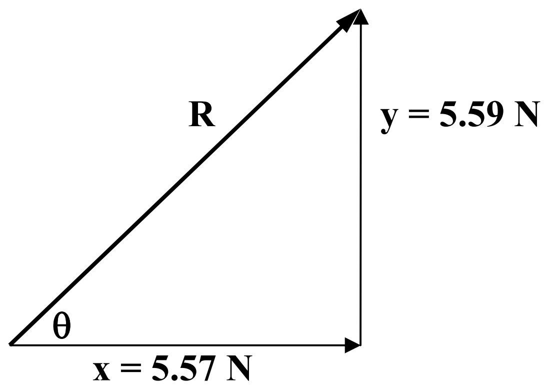
- Start with the drawing:



- Next, make a table of components and add them up; in this case, positive x-values are to the right, while positive y-values are 'up'

sum of X components	**sum of Y components**
A_x = - 5 cos 45º	**A_y = 5 sin 45º**
B_x = 0	**B_y = 8**
C_x = 6 cos 20º	**C_y = 6 sin 20º**
D_x = 4 cos 30º	**D_y = -4 sin 30º**
E_x = 0	**E_y = -6**
+ 5.57 N (to the right)	**+ 5.59 N** ('up')

- Now find the magnitude of the resultant of these two values:



→ $\mathbf{R} = \sqrt{5.59^2 + 5.57^2} = \mathbf{7.89\ N}$

- Finally, determine the direction:

$$\theta = \tan^{-1}\frac{5.59}{5.57} = 44.9^{o}$$

- The answer: **R = 7.89 N at 45° N of E**