1.3 Compound Interest: Future Value p. 20

Name ___________________

Date ____________________

Goal: Determine the future value of an investment that earns compound interest.

1. **compounded annually**: When compound interest is determined or paid yearly.

2. **compounding period**: The time over which interest is determined; interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily.

3. **Rule of 72**: A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years. The Rule of 72 is most accurate when the interest is compounded annually.

LEARN ABOUT the math

Yvonne earned $4300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% **compounded annually** . She decided to invest in a CSB, instead of keeping the money in a savings account, because the CSB will earn more interest.

What is the future value of Yvonne's investment after 10 years?

Example 1: Using reasoning to develop the compound interest formula (p.20)

Reflecting

A. The compound interest earned (I) on an investment at the end of any compounding period is the difference between the value of the investment at that time (A) and the original principal (P):

$$I = A - P$$

How can this relationship be represented symbolically using the variables I, A, P, i, and n?

B. For Yvonne's investment, the number of compounding periods in the term was the same as the number of years. Suppose that the interest had been compounded semi-annually. How many compounding periods would there have been at maturity? Explain.

Example 2: Determining the future value of an investment with semi-annual compounding (p. 22)

Matt has invested a $23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

b) Compare the principal and the future values at 5 years and 10 years. What do you notice?

c) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

Example 3: Determining the future value of investments with monthly compounding (p. 24)

Both Joli, age 50, and her daughter Lena, age 18, plan to invest \$1500 in an account with an annual interest rate of 9%, compounded monthly.

a) If both women hold their investments until age 65, what will be the difference in the future values of their investments?

b) Lena's older step-brother Cody, age 34, also plans to invest \$1500 at 9%, compounded monthly. Determine the future value of his investment at age 65.

Example #4: Comparing interest on investments with different compounding periods (p.25)

Céline wants to invest $3000 so that she can buy a new car in the next 5 years. Céline has the following investment options:

A. 4.8% compounded annually
B. 4.8% compounded semi-annually
C. 4.8% compounded monthly
D. 4.8% compounded weekly
E. 4.8% compounded daily

Use the TVM solver on the TI-83/TI-83 Plus/TI-84 to compare the interest earned by each of these options from terms of 1 to 5 years.

Example #5: Estimating doubling times for investments (p.27)

Both Berta and Kris invested $5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

b) Verify your estimates by determining the doubling time for each CSB.

c) Estimate the future value of an investment of $5000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

In Summary

Key Ideas

- The future value of an investment that earns compound interest can be determined using the compound interest formula

$$A = P(1 + i)^n$$

where A is the future value, P is the principal, i is the interest rate per compounding period (expressed as a decimal), and n is the number of compounding periods.
- The more frequent the compounding and the longer the term, the greater the impact of the compounding on the principal and the greater the future value will be.

Need to Know

- When using the compound interest formula, use an exact value for i. For example, for an annual interest rate of 5% compounded monthly, substitute $\frac{0.05}{12}$ for i instead of the rounded value 0.004 16....
- Four common compounding frequencies are given in the table below. The table shows how the interest rate per compounding period (i) and the number of compounding periods (n) are determined.

Compounding Frequency	Times per Year	Interest Rate per Compounding Period (i)	Number of Compounding Periods (n)
annually	1	i = annual interest rate	n = number of years
semi-annually	2	$i = \frac{\text{annual interest rate}}{2}$	n = (number of years)(2)
quarterly	4	$i = \frac{\text{annual interest rate}}{4}$	n = (number of years)(4)
monthly	12	$i = \frac{\text{annual interest rate}}{12}$	n = (number of years)(12)

- The total compound interest earned on an investment (I) after any compounding period can be determined using the formula

$$I = A - P \quad \text{or} \quad I = P[(1 + i)^n - 1]$$

- The Rule of 72 is a simple strategy for estimating doubling time. It is most accurate when the interest is compounded annually. For example, \$1000 invested at 3% interest, compounded annually, will double in value in about $\frac{72}{3}$ or 24 years; \$1000 invested at 6% will double in about $\frac{72}{6}$ or 12 years.

HW: 1.3 pp. 30-32 #1, 3, 4, 5, 7, 11 & 13