4.4 Permutations When Objects Are Identical p. 260

Name ________________

Date ________________

Goal: Determine the number of permutations when some objects are identical.

INVESTIGATE the Math

1. The permutations of the 4 different letters A, B, E, and F are:

ABEF	ABFE	AEBF	AFBE	AEFB	AFEB
BAEF	BAFE	EABF	FABE	EAFB	FAEB
BEAF	BFAE	EFAB	FEAB	EBAF	FBAE
BEFA	BFEA	EFBA	FEBA	EBFA	FBEA

How many permutations are there?

2. a) ***What happens if two of the letters are the same?*** Investigate this by converting each F to an E in the list below. Then count the number of permutations of the letters A, B, E, and E.

ABEF	ABFE	AEBF	AFBE	AEFB	AFEB
BAEF	BAFE	EABF	FABE	EAFB	FAEB
BEAF	BFAE	EFAB	FEAB	EBAF	FBAE
BEFA	BFEA	EFBA	FEBA	EBFA	FBEA

There are _______ permutations of the letters A, B, E, and E.

b) How does this number compare with step 1?

3. a) ***What happens if three of the letters are the same?*** Investigate this by converting each F and E to a B. Then count the number of permutations of the letters A, B, B, and B.

ABEF	ABFE	AEBF	AFBE	AEFB	AFEB
BAEF	BAFE	EABF	FABE	EAFB	FAEB
BEAF	BFAE	EFAB	FEAB	EBAF	FBAE
BEFA	BFEA	EFBA	FEBA	EBFA	FBEA

There are _______ permutations of the letters A, B, B, and B.

b) How does this number compare with step 1?

4. Generalize the pattern from the investigation to determine the number of permutations of:

a. A, B, C, D, D

b. A, B, D, D, D

c. A, D, D, D, D

d. A, B, B, C, C

e. A, A, A, B, B

Generalization

The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on, is:

Example 1: Determine the number of permutations of all the letters in the following the words.

a. STATISTICIAN

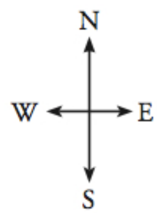
b. CANADA

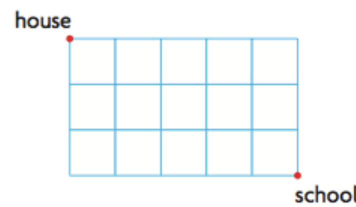
Example 2: Solving a conditional permutation problem involving identical objects (p. 263)

How many ways can the letters of the word CANADA be arranged, if the first letter must be N and the last letter must be C?

Example 3: Solving a permutation problem involving routes (p. 264)

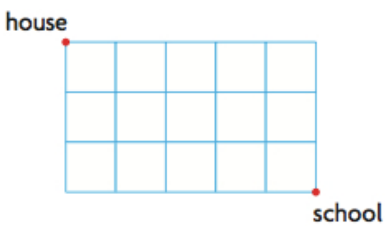
Julie's home is three blocks north and five blocks west of her school. How many routes can Julie take from home to school if she always travels either south or east?

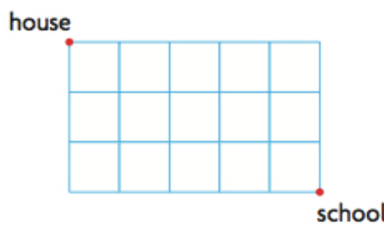


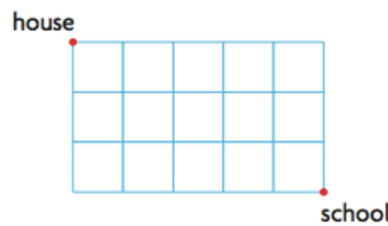


Method 1: Using permutations

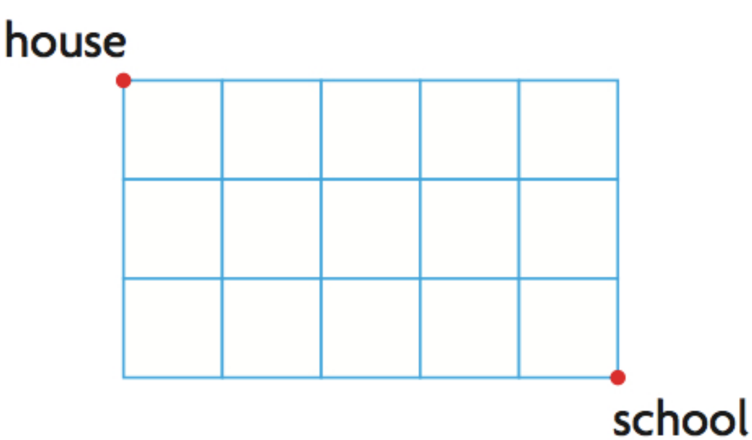
Possible Routes:







Method 2: Use a diagram



In Summary

Key Ideas

- There are fewer permutations when some of the objects in a set are identical compared to when all the objects in a set are different. This is because some of the arrangements are identical.
- The number of permutations of n objects, where a are identical, another b are identical, another c are identical, and so on, is

$$P = \frac{n!}{a!b!c!\ldots}$$

For example, in the set of four objects a, a, b, and b, the number of different permutations, P, is

$$P = \frac{4!}{2! \cdot 2!}$$

$$P = 6$$

The six different arrangements are *aabb*, *bbaa*, *abab*, *baba*, *abba*, and *baab*.

Need to Know

- Dividing $n!$ by $a!$, $b!$, $c!$, and so on deals with the effect of repetition caused by objects in the set that are identical. It eliminates arrangements that are the same and that would otherwise be counted multiple times.

HW: 4.4 p. 266-269 # 5, 6, 7, 9, 11 & 15