F Math 12

5.5 Conditional Probability p. 328

Name _____

Date _____

Goal: Understand and solve problems that involve dependent events.

1. **dependent events**: Events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn).
2. **conditional probability**: The probability of an event occurring given that another event has already occurred.

INVESTIGATE

Situation #1: Drawing two balls from the pot, ***without*** replacement.

A ball is randomly selected from the pot and **is not** replaced. Then a second ball is drawn.

Define the flowing events:

A: The first ball is white
B: The second ball is white

Find: P(A) =

P(B given that the first ball drawn is white) =

P(A and B) =

Events A and B are ***dependent*** because

Situation #2: Drawing two balls from the pot, ***with*** replacement.

A ball is randomly selected from the pot and **is** replaced. Then a second ball is drawn.

Define the flowing events:

A: The first ball is white
B: The second ball is white

Find: P(A) =

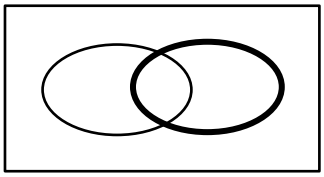
P(B given that the first ball drawn is white) =

P(A and B) =

Events A and B are ***independent*** because

Probabilities of Events A and B

General Case:



$P(A \cap \boldsymbol{B}) =$

"probability of A and B"

Equivalently:

$P(\boldsymbol{B}|A) =$

"probability of B given A"

Note: Events A and B are independent if $P(B|A) = P(B)$

For independent events: $P(A \cap B) =$

Example 1: Two cards are drawn without replacement from a shuffled deck of 52 cards.

Define the following events:

A: The first card is a face card.
B: The second card is a face card.

Determine:

a) $P(A \cap B)$

b) $P(A' \cap B)$

Example 2: According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

Example 3: Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probability that

a) both cards are hearts?

b) neither card is a heart?

c) exactly one of the two cards is a heart?

d) both cards are aces?

Example 4: A company has two factories that make computer chips. Suppose 70% of the chips come from Factory 1 and 30% come from Factory 2. In Factory 1, 25% of the chips are defective; in Factory 2, 10% of the chips are defective.

a) Suppose it is not known from which factory a chip came.
What is the probability that the chip is defective?

b) Suppose a defective chip is discovered.
What is the probability that the chip came from Factory 1?

In Summary

Key Ideas

- If the probability of one event depends on the probability of another event, then these events are called **dependent events**. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events.
- If event B depends on event A occurring, then the **conditional probability** that event B will occur, given that event A has occurred, can be represented as follows:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Need to Know

- If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

- A tree diagram is often useful for modelling problems that involve dependent events.
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events.

HW: 5.5 p. 350-353 #2-5 & 7