Introduction Understanding Radicals

a) Label each of the following parts of a radical.

$\sqrt[3]{8}$

b) What is the index of $\sqrt{5}$?

c) What is the difference between an entire radical and a mixed radical?

d) Is it possible to write a radical without using the radical symbol $\sqrt{\ }$?

Example 1 Convert each entire radical to a mixed radical.

a) $\sqrt{20}$

Prime Factorization Method	Perfect Square Method

b) $\sqrt{32}$

Prime Factorization Method	Perfect Square Method

c) $\sqrt[3]{16}$

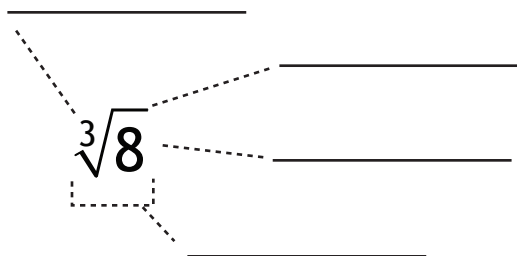| Prime Factorization Method	Perfect Cube Method

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Example 2 Convert each entire radical to a mixed radical using the method of your choice.

a) $\sqrt{24}$

b) $\sqrt{72}$

c) $\sqrt{49}$

d) $\sqrt[3]{81}$

e) $\sqrt[3]{64}$

f) $\sqrt[4]{48}$

Example 3 Convert each mixed radical to an entire radical.

a) $3\sqrt{3}$

Reverse Factorization Method	Perfect Square Method

b) $6\sqrt{2}$

Reverse Factorization Method	Perfect Square Method

c) $2\sqrt[3]{5}$

Reverse Factorization Method	Perfect Cube Method

Example 4 Convert each mixed radical to an entire radical using the method of your choice.
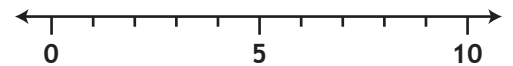
a) $4\sqrt{2}$
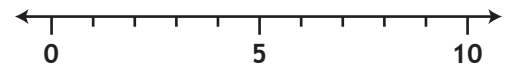
b) $5\sqrt{3}$

c) $3\sqrt[3]{3}$

d) $2\sqrt[4]{3}$

Example 5 Estimate each radical and order them on a number line.

a) $\sqrt{42}$ $\sqrt{20}$ $\sqrt{8}$ $\sqrt{14}$

b) $\sqrt[3]{92}$ $\sqrt[3]{169}$ $\sqrt[3]{54}$ $\sqrt[3]{35}$

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Example 6   Simplify each expression without using a calculator.

a) $\dfrac{2\sqrt{12}}{4}$

b) $\dfrac{3\sqrt[3]{27}}{36}$

c) $\dfrac{3}{4}\sqrt{32}$

d) $\sqrt{\dfrac{49}{81}}$

e) $\dfrac{3\sqrt[3]{72}}{\sqrt{64}}$

Example 7 Write each power as a radical.

a) $3^{\frac{1}{2}}$

b) $(-4)^{\frac{1}{3}}$

c) $2^{\frac{4}{3}}$

d) $(-7)^{\frac{2}{5}}$

e) $\left(\dfrac{2}{3}\right)^{\frac{3}{2}}$

f) $16^{0.25}$

Example 8 Write each radical as a power.

a) $\sqrt{5}$

b) $\sqrt[4]{9}$

c) $\sqrt[3]{2^2}$

d) $\left(\sqrt[5]{-3}\right)^4$

e) $\left(\sqrt[3]{\dfrac{5}{7}}\right)^2$

f) $\sqrt{\left(\dfrac{3}{4}\right)^2}$