F Math 12

# *1.1 Simple Interest p.6*

Name ______________

Date ______________

**Goal:** Solve problems that involve simple interest.

1. **term**: The contracted duration of an investment or loan.
2. **interest ($i$)**: The amount of money earned on an investment or paid on a loan.
3. **fixed interest rate**: An interest rate that is guaranteed not to change during the term of an investment or loan.
4. **principal ($P$)**: The original amount of money invested or loaned.
5. **simple interest**: The amount of interest earned on an investment or paid ona loan based on the original amount (the principal) and the simple interest rate.
6. **maturity**: The contracted end date of an investment or loan, at the end of the term.
7. **future value ($A$)**: The amount that an investment will be worth after a specified period of time.
8. **rate of return**: The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

To determine simple interest only: $\boldsymbol{i = Prt}$ where:

$i =$ interest earned  $P =$ Principal (initial amount)  $r =$ rate of interest  $t =$ time in years

To determine future value: $\boldsymbol{A = P(1 + rt)}$ where: $A =$ future value

$A = P + Prt$

Annual = 1/year  Semi-Annually = 2/year  Monthly = 12/years

Weekly= 52/years  Daily= 365/years

***INVESTIGATE*** the math

Sera is 20 years old and needs money to pay for college. When she was born, her grandparents bought her a $500 Canada Savings Bond (CSB) with a **term** of 10 years. They chose a CSB as an investment because they liked the security of loaning money to the government. The **interest** earned was determined using a **fixed interest rate** of 6% per year on the original investment and was paid at the end of each year until Sera's 10th birthday.

How can you determine the current value of Sera's CSB?

A. How much interest was earned on the **principal** by the end of the first year?

$i = Prt$
$= (500)(0.06)(1)$
$= \$30$

B. Determine the **simple interest** earned each year, the accumulated interest, and the value of the investment for the first 4 years. Organize your calculations in a table.

| Year | Value of Investment @ Start of Year ($) | Simple Interest Earned Each Year ($) | Accum. Interest ($) | Value of Investment @ End of Year ($) |
|---|---|---|---|---|
| 0 | | | 0 | 500 |
| 1 | 500 | $30 | $30 | $530 |
| 2 | $530 | $30 | $60 | $560 |
| 3 | $560 | $30 | $90 | $590 |
| 4 | $590 | $30 | $120 | $620 |

C. Is the simple interest earned each year constant or variable? Explain.

doesn't change ∴ constant

D. Describe the relationship between the number of years, the interest earned each year, and the accumulated interest.

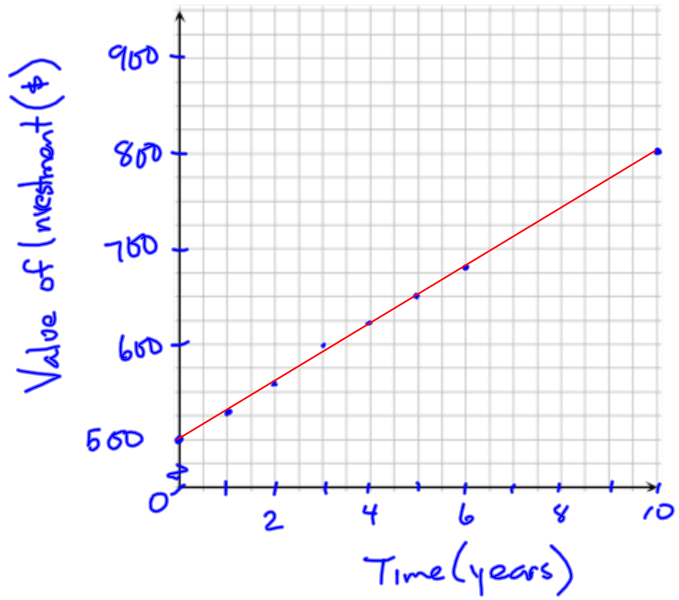
years × interest earned = accumulated interest

E. Use the relationship for part D to predict the value of the investment after 10 years.

$10 \times \$30 = \$300$ ; $\$500 + \$300 = \$800$

F. Graph the growth of the investment until its maturity at 10 years using "Time (years)" as the domain and "Value of the investment ($)" as the range. Is your prediction in part E supported by your graph?

x ← (domain)   → y (range)




$y = mx + b$

$y = 30x + 500$

(a)   /a

↑ means year from latin → annum

**Example 1**: Solving a simple interest problem (p.8)

Marty invested in a $2500 guaranteed investment certificate (GIC) at 2.5% simple interest, paid annually, with a term of 10 years.

a) How much interest will accumulate over the term of Marty's investment? Use the formula $i = Prt$, where $I$ = interest, $P$ = the principal, $r$ = rate as a decimal and $t$ = time

$$i = Prt$$
$$= (2500)(0.025)(10)$$
$$= \$625$$

b) What is the **future value** of his investment at maturity?

$$\text{future value} = P + i$$
$$= \$2500 + \$625$$
$$= \$3125$$

c) Use Marty's investment to write an algebraic expression for the future value of an investment

$$A = P + i$$
$$A = P + Prt$$
$$A = P(1 + rt)$$

**Example 2**: Representing the growth of a simple interest investment (p. 9)

Sunni invested $15 000 in a savings account. Sunni earned a simple interest rate of 8%, paid semi-annually (twice a year) on her investment. She intends to hold the investment for 4.5 years, when she will withdraw all the money to buy a car. Determine the value of the investment at each half year until she withdraws the money.

$P =$ $15 000

$r =$ 0.08

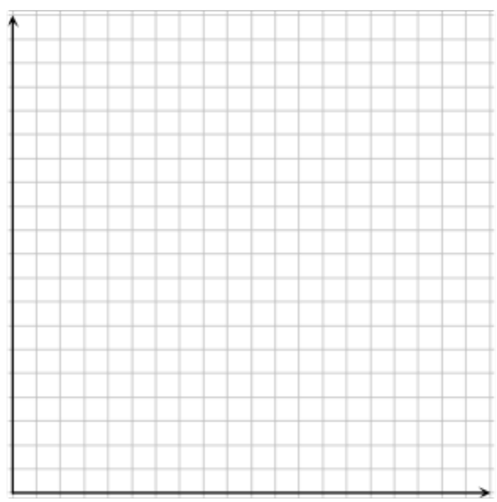
$t =$ 9 (semi annually for 4.5 years → 2 x 4.5)

$i = Prt$

$= \$15\,000(0.08)(0.5)$

$= \$600$

| year | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 |
|---|---|---|---|---|---|---|---|---|---|---|
| Value of Invest. | 15 000 | 15 600 | 16 200 | 16 800 | 17 400 | 18 000 | 18 600 | 19 200 | 19 800 | 20 400 |



skipped

**Example 3:** Determining the duration of a simple interest investment (p. 10)

Ingrid invested her summer earnings of $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

a) How long will it take for the future value of the investment to grow to $8000?

$A = 8000$
$P = 5000$
$r = 0.08$
$t = ?$

$A = P + Prt$

$8000 = 5000 + 5000(0.08)t$

$\frac{3000}{400} = \frac{400t}{400}$

$7.5 = t$

It will take 8 years (need to round up to the nearest interest payment period)

b) What is Ingrid's **rate of return** ?

BEDMAS!

$A = P(1 + rt)$

$= 5000(1 + 0.08)(8))$

$= 5000(1 + 0.64)$

$= 5000(1.64)$

$= 8200$

Rate of Return

$RR = \frac{i}{P}$

$= \frac{\$3200}{\$5000}$

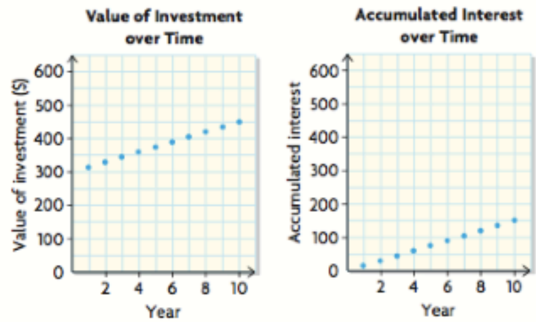
$= 0.64 \rightarrow 64\%$

The rate of return over 8 years is 64%

## In Summary

### Key Ideas

- Simple interest is determined only on the principal of an investment.
- The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest earned over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. For example, the following graphs show principal of \$300 invested at 5% interest, paid annually, over a term of 10 years.




### Need to Know

- The amount of simple interest earned on an investment can be determined using the formula

$$I = Prt$$

  where $I$ is the interest, $P$ is the principal, $r$ is the annual interest rate expressed as a decimal, and $t$ is the time in years.
- The future value or amount, $A$, of an investment that earns simple interest can be determined using the formula

$$A = P + Prt$$
$$\text{or } A = P(1 + rt)$$

  where $P$ is the principal, $r$ is the interest rate expressed as a decimal, and $t$ is the time in years.
- Unless otherwise stated, an interest rate is assumed to be annual, or per annum.
- Even though interest rates are usually annual, interest can be paid out at different intervals, such as annually, semi-annually, monthly, weekly, and daily.

HW: 1.1 pp. 14-17 #1, 4, 6, 8, 9 & 13

F Math 12

## *1.2 Exploring Compound Interest p.8*

Name _______________

Date _______________

**Goal:** Compare simple interest with compound interest.

1. **compound interest**: The interest that is earned or paid on both the principal and the accumulated interest.

***Explore*** the math

Guaranteed investment certificates (GICs) can earn either simple or compound interest. If a GIC earns simple interest annually, the same amount of interest is earned every year. If a GIC earns **compound interest** annually, the interest at the end of the first year is earned on the principal, but the interest at the end of the second year is earned on the principal plus the interest from the first year. Each year after that, the interest is earned on the principal plus all the accumulated interest from the previous years.

Both Ewan and Rena received a $1000 prize in a story-writing contest.

- Ewan bought a $1000 simple interest GIC with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Rena bought a $1000 compound interest GIC with her prize money. It also has a 5-year term and earns 3.6% paid annually.

How do the future values of Ewan's and Rena's investments compare at maturity?

A. With a partner, compare your answers and the strategies you used to determine the difference between the two investments at maturity.

- Use $A = P(1 + rt)$ to determine the future value of Ewan's investment

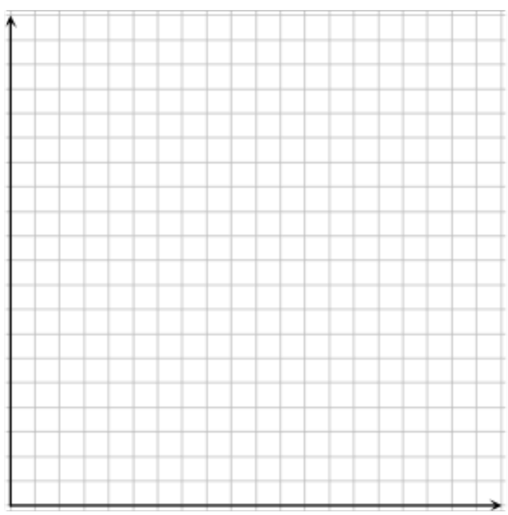
$$A = 1000(1 + (0.036)(5))$$
$$= \$1180$$

- Use the following table to determine the future value of Rena's investment

| Year | Value of Investment @ Start of Year ($) | Interest Earned Each Year ($) | Accum. Interest ($) | Value of Investment @ End of Year ($) |
|---|---|---|---|---|
| 0 | | | | 1000 |
| 1 | 1000 | $i = Prt$ \$36<br>$i = (1000)(0.036)(1)$ | 36 | 1036 |
| 2 | 1036 | 37.30 | 73.30 | 1073.30 |
| 3 | 1073.30 | 38.64 | 111.94 | 1111.94 |
| 4 | 1111.94 | 40.03 | 151.97 | 1151.97 |
| 5 | 1151.97 | 41.47 | 193.44 | 1193.44 |

B. Graph both investments on the same coordinate grid. How are the shapes of the graphs different? Explain why.

omit!



C. $\overbrace{\text{How much would Ewan need to invest}}^{P}$ at 3.6% simple interest to earn the same as Rena in 5 years?

$$A = P + Prt \rightarrow A = P(1 + rt)$$

$$1193.44 = P(1 + (0.036)(5))$$

$$\frac{1193.44}{1.18} = \frac{P(1.18)}{1.18}$$

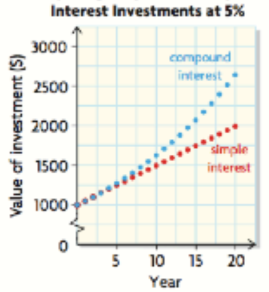
$$P = \$1011.39$$

## In Summary

### Key Ideas

- Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest. Previously earned interest is reinvested over the course of the investment.
- If the same principal is invested in a compound interest account and a simple interest account, with the same interest rate for the same term, the compound interest investment will grow faster (non-linear) than the simple interest investment (linear). For example, the graphs show principal of $1000 invested over 20 years at 5% simple interest (red graph) and 5% compound interest (blue graph), both paid annually.

**Comparing Simple and Compound Interest Investments at 5%**




### Need to Know

- Financial institutions pay compound interest on investments at regular equal intervals. If interest is paid annually, it is calculated at the end of the first year on the principal and then added to the principal. At the end of the second year, the interest is calculated on the balance at the end of the first year (principal plus interest earned from the previous year). This pattern continues every year until the end of the investment term.

HW: 1.2 p. 19 #1-3

# 1.3 Compound Interest: Future Value p. 20

Name ______________

Date ______________

**Goal:** Determine the future value of an investment that earns compound interest.

1. **compounded annually**: When compound interest is determined or paid yearly.

2. **compounding period:** The time over which interest is determined; interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily.

3. **Rule of 72**: A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years. The Rule of 72 is most accurate when the interest is compounded annually.

***LEARN ABOUT*** the math

Yvonne earned $4300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% **compounded annually** . She decided to invest in a CSB, instead of keeping the money in a savings account, because the CSB will earn more interest.

What is the future value of Yvonne's investment after 10 years?

**Example 1**: Using reasoning to develop the compound interest formula (p.20)

$$A = P + Prt$$
$$= P(1 + rt)$$

$$A_1 = 4300(1 + 0.038(1))$$
$$= 4300(1.038)$$
$$= \$4463.40$$

$$A_2 = 4463.40(1.038)$$
$$= 4633.00$$

$A_3 = 4633.00(1.038)$
$= \$4809.06$

$A_3 = 4300 \times 1.038 \times 1.038 \times 1.038$
$= 4300(1.038)^3$
$= \$4809.06$

$A_{10} = 4300(1.038)^{10}$
$= \$6243.70$

$\boxed{A = P(1+i)^n}$

A = future value
P = pricipal
* i = interest per compounding period *
n = number of compounding periods

**Reflecting**

A. The compound interest earned ($I$) on an investment at the end of any compounding period is the difference between the value of the investment at that time ($A$) and the original principal ($P$):

$$I = A - P$$

How can this relationship be represented symbolically using the variables $I$, $A$, $P$, $i$, and $n$?

$I = P(1+i)^n - P$
$I = P[(1+i)^n - 1]$

2 per year ←

B. For Yvonne's investment, the number of compounding periods in the term was the same as the number of years. Suppose that the interest had been compounded semi-annually. How many compounding periods would there have been at maturity? Explain.

$2 \times 10 = 20$ compounding periods

$P = 4300$
$i = \frac{0.038}{2}$ *
$n = 2 \times 10 = 20$

$A = P(1+i)^n$
$A_{10} = 4300\left(1 + \frac{0.038}{2}\right)^{20}$
$= 4300(1.019)^{20}$
$= \$62465.45$

* semi annual compounding period

$\rightarrow$ 2 times a year

**Example 2**: Determining the future value of an investment with semi-annual compounding (p. 22)

Matt has invested a $23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years.

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

$$A = P(1+i)^n$$

$$A_5 = 23\,000\left(1 + \frac{0.136}{2}\right)^{5\times 2}$$
$$= \$44\,405.87$$

$$A_{10} = 23\,000\left(1 + \frac{0.136}{2}\right)^{10\times 2}$$
$$= 23\,000(1.068)^{20}$$
$$= \$85\,733.96$$

b) Compare the principal and the future values at 5 years and 10 years. What do you notice?

@ 5 years, almost doubled
@ 10 years, almost quadrupled

c) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

$$I = Prt$$

$$I_5 = (23000)(0.136)(5)$$
$$= \$15\,640$$

$$I_{10} = (23000)(0.136)(10)$$
$$= \$31\,280$$
$$(I_5 \times 2)$$

simple interest only pays interest on the principal so future value will be less than compound interest

**Example 3:** Determining the future value of investments with monthly compounding (p. 24)

Both Joli, age 50, and her daughter Lena, age 18, plan to invest \$1500 in an account with an annual interest rate of 9%, compounded monthly.

* N is the # of compounding periods

a) If both women hold their investments until age 65, what will be the difference in the future values of their investments?

TVM

* N: 15 × 12
I%: 9
PV: −1500
FV: \$ 5757.06
P/Y: 12
C/Y: 12

always the same # when investing

Lena

N: (65−18) × 12
I%: 9
PV: −1500
FV: \$101 461.71
P/Y: 12
C/Y: 12

Difference: \$101 461.71 − 5757.06 = \$95 704.65

b) Lena's older step-brother Cody, age 34, also plans to invest \$1500 at 9%, compounded monthly. Determine the future value of his investment at age 65.

$$A = P(1+i)^n$$
$$= 1500\left(1 + \frac{0.09}{12}\right)^{(65-34)\times 12}$$
$$= 1500(1.0075)^{372}$$
$$= \$\,24\,168.61$$

N: 372
I%: 9
PV: −1500
FV: 24 168.61
P/Y: 12
C/Y: 12

**Example #4:** Comparing interest on investments with different compounding periods (p.25)

Céline wants to invest $3000 so that she can buy a new car in the next 5 years. Céline has the following investment options:

**A.** 4.8% compounded annually
**B.** 4.8% compounded semi-annually
**C.** 4.8% compounded monthly
**D.** 4.8% compounded weekly
**E.** 4.8% compounded daily

Use the TVM solver on the TI-83/TI-83 Plus/TI-84 to compare the interest earned by each of these options from terms of 1 to 5 years.

if the principal and interest rate do not change, then more compounding periods in a given amount of time result in more interest earned.

eg. for a 3 year term:

| A | B | C | D | E |
|---|---|---|---|---|
| N: 3×1 | N: 3×2 | N: 3×12 | N: 3×52 | N: 3×365 |
| I%: 4.8 | I%: 4.8 | I%: 4.8 | I%: 4.8 | I%: 4.8 |
| PV: −3000 | PV: −3000 | PV: −3000 | PV: −3000 | PV: −3000 |
| FV: 3453.07 | FV: 3458.76 | FV: 3463.66 | FV: 3464.42 | FV: 3464.62 |
| P/Y: 1 | P/Y: 2 | P/Y: 12 | P/Y: 52 | P/Y: 365 |
| C/Y: 1 | C/Y: 2 | C/Y: 12 | C/Y: 52 | C/Y: 365 |

**Example #5:** Estimating doubling times for investments (p.27)

Both Berta and Kris invested $5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

Berta: $\frac{72}{8}$

~9 years to double

Kris: $\frac{72}{9}$

~8 years to double

b) Verify your estimates by determining the doubling time for each CSB.

Berta →
N: 9.01
I%: 8
PV: -5000
FV: 10 000
P/Y: 1
C/Y: 1

Kris →
N: 8.04
I%: 9
PV: -5000
FV: 10 000
P/Y: 1
C/Y: 1

c) Estimate the future value of an investment of $5000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

9 years → $10 000
18 years → $20 000
27 years → $40 000

Rule of 72: $\frac{72}{8}$

9 years to double

N: 9
I%: 8
PV: -5000
FV: 9995.02
P/Y: 1
C/Y: 1

N: 18
I%: 8
PV: -5000
FV: 19 980.10
P/Y: 1
C/Y: 1

N: 27
I%: 8
PV: -5000
FV: 39 940.31
P/Y: 1
C/Y: 1

estimates are very close!

## In Summary

### Key Ideas

- The future value of an investment that earns compound interest can be determined using the compound interest formula

$$A = P(1 + i)^n$$

where $A$ is the future value, $P$ is the principal, $i$ is the interest rate per compounding period (expressed as a decimal), and $n$ is the number of compounding periods.
- The more frequent the compounding and the longer the term, the greater the impact of the compounding on the principal and the greater the future value will be.

### Need to Know

- When using the compound interest formula, use an exact value for $i$. For example, for an annual interest rate of 5% compounded monthly, substitute $\frac{0.05}{12}$ for $i$ instead of the rounded value 0.004 16....
- Four common compounding frequencies are given in the table below. The table shows how the interest rate per compounding period ($i$) and the number of compounding periods ($n$) are determined.

| Compounding Frequency | Times per Year | Interest Rate per Compounding Period ($i$) | Number of Compounding Periods ($n$) |
|---|---|---|---|
| annually | 1 | $i$ = annual interest rate | $n$ = number of years |
| semi-annually | 2 | $i = \frac{\text{annual interest rate}}{2}$ | $n$ = (number of years)(2) |
| quarterly | 4 | $i = \frac{\text{annual interest rate}}{4}$ | $n$ = (number of years)(4) |
| monthly | 12 | $i = \frac{\text{annual interest rate}}{12}$ | $n$ = (number of years)(12) |

- The total compound interest earned on an investment ($I$) after any compounding period can be determined using the formula

$$I = A - P \quad \text{or} \quad I = P[(1 + i)^n - 1]$$

- The Rule of 72 is a simple strategy for estimating doubling time. It is most accurate when the interest is compounded annually. For example, \$1000 invested at 3% interest, compounded annually, will double in value in about $\frac{72}{3}$ or 24 years; \$1000 invested at 6% will double in about $\frac{72}{6}$ or 12 years.

HW: 1.3 pp. 30-32 #1, 3, 4, 5, 7, 11 & 13

F Math 12

# 1.4 Compound Interest: Present Value p. 34

Name ____________

Date ____________

**Goal:** Determine the principal or present value of an investment, given its future value and compound interest rate.

1. **present value:** The amount that must be invested now to result in a specific future value in a certain time at a given interest rate.

***INVESTIGATE*** the math

In 5 years, after graduating from college, Cal wants to spend a year travelling in Canada's three territories. He plans to start in Yukon and then travel east to the Northwest Territories and Nunavut. Cal has determined that he will need at least $15 000 for his trip. To reach this goal, he wants to invest money now. He has chosen a GIC at 7%, compounded annually.

How much does Cal need to invest now so that he will have $15 000 in 5 years?


$A = \$15\,000$

$P = ?$

$i = 0.07$

$n = 5$

$$A = P(1+i)^n$$
$$15\,000 = P(1+0.07)^5$$
$$\frac{15\,000}{(1.07)^5} = \frac{P(1.07)^5}{(1.07)^5}$$
$$\$10\,694.79 = P$$

N : 5
I% : 7
PV :
FV : 15 000
same $\begin{cases} P/Y : 1 \\ C/Y : 1 \end{cases}$

**Example 1:** Determining the present value of investments earning compound interest (p.35)

Ginny is 18 years old. She has inherited some money from a relative. Ginny wants to invest some of the money so that she can buy a home in Milk River, Alberta, when she turns 30. She estimates that she will need about $170 000 to buy a home.

a) How much does she have to invest now, at 6.5% compounded annually?
b) What is the ratio of future value to present value for Ginny's investment?
c) How would the ratio change if the interest rate decreased to 6% but was compounded semi-annually?

a) $A = \$170\,000$
$P = ?$
$i = 0.065$
$n = 12$ $(30-18)$

$$\frac{A}{(1+i)^n} = \frac{P\cancel{(1+i)^n}}{\cancel{(1+i)^n}}$$

$$P = \frac{A}{(1+i)^n}$$
$$= \frac{170\,000}{(1+0.065)^{12}}$$
$$= \$79\,846.09$$

b) $$\frac{A}{P} = \frac{170\,000}{79\,846.09}$$
$$= 2.14$$

c) $\frac{A}{P} = \frac{\cancel{P}(1+i)^n}{\cancel{P}}$

$i = 0.06 \div 2$
$= 0.03$
interest rate per compounding period

$\boxed{\frac{A}{P} = (1+i)^n}$

$n = 12 \times 2$
$= 24$
compounding periods

$\frac{A}{P} = (1+0.03)^{24}$
$= 2.03$

**Example 2:** Determining the present value of an investment that is compounded quarterly
(p. 37)

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that $40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.
4 times a year

a) How much money should they have invested?
b) How much interest will they earn over the term of their investment?

a) $P = ?$
$A = \$48\,000$
$i = 9.6\% \div 4 = 2.4\% \rightarrow 0.024$
$n = 3 \times 4$
$= 12$

$P = \frac{A}{(1+i)^n}$
$= \frac{40\,000}{(1+.024)^{12}}$
$= \frac{40\,000}{1.024^{12}}$
$\$30\,092.66$

b) $I = A - P$
$= 48000 - 30\,092.66$
$= \$9907.34$

**Example 3:** Determining an unknown interest rate and unknown term (p. 38)

Laura has invested $15 500 in a Registered Education Savings Plan (RESP). She wants her investment to grow to at least $50 000 by the time her newborn enters university, in 18 years.

a) What interest rate, compounded annually, will result in a future value of $50 000? Round your answer to two decimal places.
b) Suppose that Laura wants her $15 500 to grow to at least $60 000 at the interest rate from part a). How long will this take?

a) $P = \$15\,500$
$A = \$50\,000$
$i = ?$
$n = 18$

$$\frac{A}{P} = \frac{\cancel{P}(1+i)^n}{\cancel{P}}$$

$$\frac{A}{P} = (1+i)^n$$

$$\sqrt[18]{\frac{50\,000}{15\,500}} = \sqrt[18]{(1+i)^{18}}$$

$$1.0672 = 1+i$$

$$0.0672 = i$$

$$6.72 = i$$

TVM

b) N : ?
I% : 6.72
PV : -15 500
PMT : 0
FV : 60 000
same $\begin{cases} \text{P/Y : 1} \\ \text{C/Y : 1} \end{cases}$

21 years

## In Summary

### Key Idea

- The present value of an investment that earns compound interest can be determined using the formula

$$P = \frac{A}{(1 + i)^n}$$

  where $P$ is the present value (or principal), $A$ is the amount (or future value), $i$ is the interest rate per compounding period (expressed as a decimal), and $n$ is the number of compounding periods.

### Need to Know

- Any equivalent form of the compound interest formula may be used to solve a compound interest problem.

  $A = P(1 + i)^n \qquad P = \dfrac{A}{(1 + i)^n} \qquad \dfrac{A}{P} = (1 + i)^n$

- To compare investments, usually with the same term or principal, the ratio of the future value to the present value can be determined using the form: $\dfrac{A}{P} = (1 + i)^n$
- Using a formula, using the financial application on a graphing calculator, and using spreadsheet software are all valid strategies for solving a compound interest problem.

HW: 1.4 pp. 40-42 #3, 5, 6, 7, 9, 10 & 14

F Math 12 **_1.5 Investments Involving Regular Payments p. 46_**

Name ______________

Date ______________

| |
|---|
| **Goal:** Determine the future value of an investment that earns compound interest involving regular payments. |

**THESE TYPES OF PROBLEMS CAN ONLY BE SOLVED USING THE TVM SOLVER OR A SPREADSHEET OR BY DOING A LOT OF REPETATIVE CALCULATIONS BY HAND!**

***INVESTIGATE*** the math

Pokiak is now 18 years old, and he needs money for his post-secondary education. On his 14th birthday, his family deposited $1000 into a Registered Education Savings Plan (RESP) at 3% interest, compounded annually. Since then, Pokiak has deposited $1000 of his own money, earned by working part-time, into the account each year.

How much money is in Pokiak's RESP account, and how much interest has it earned altogether?

N = 4

I% = 3%

PV = -1000

PMT = -1000

FV = ?

P/Y = 1

C/Y = 1

A = $5309.14

I = A − all deposited monies

= 5309.14 − 5000

= $309.14

Age → 14 $\underline{\$1k}$ 15 $\underline{\$1k}$ 16 $\underline{\$1K}$ 17 $\underline{\$1K}$ 18

$1k

**Example 1**: Determining the future value of an investment involving regular deposits (p.47)

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

N = 5×2

I% = 3.8

PV = 0

PMT = −500

FV = ?

P/Y = 2

C/Y = 2

A = $5449.90

How much did she deposit?

$500 × 10 = $5000

I = A − deposits

= $5449.90 − $5000.00

= $449.90

**Example 2**: Comparing a regular payment investment with a single payment investment (p. 49)

Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

a) What was their future value?
b) What principal amount did Blake invest 5 years ago?
c) Who earned more interest? Why?

Adam

N = 5

I% = 5

PV = 0

PMT = −200

FV = ?

P/Y = 1

C/Y = 1

a) Adam's future value (A) = $1105.13
∴ Blake's A = $1105.13

b) N = 5
I% = 5
PV = ?
PMT = 0
FV = $1105.13
P/Y = 1
C/Y = 1

figure out Blake's principal investment

principal = $865.90

c) Blake earned more interest.

$1105.13 − $865.90 = $239.23

A − P

Adam's interest: $1105.13 − $1000
$105.13

difference: 239.23 − 105.13
$134.10

Blake earned $134.10 more interest because he deposited a larger principal rather than yearly deposits

**Example 3:** Determining the interest rate of a regular payment investment (p. 51) quarterly $\$750 \times 4 \times 3$

Jeremiah deposits $750 into an investment account at the end of every 3 months. Interest is compounded quarterly, the term is 3 years, and the future value is $10 059.07. What annual rate of interest does Jeremiah's investment earn?

N = $3 \times 4$

I% = ?

PV = 0

PMT = $-750$

FV = 10 059.07

P/Y = 4

C/Y = 4

$\left.\begin{matrix}4\\4\end{matrix}\right\}$ always the same

I% = 8% /a

can't solve by hand!

$$A = P(1+i)^n$$

$$\frac{10059.07}{9000} = \frac{9000(1+i)^{12}}{9000}$$

$$\sqrt[12]{1.11767\ldots} = \sqrt[12]{(1+i)^{12}}$$

$$1.00931\ldots = 1+i$$

$$0.00931\ldots = i$$

$$r = 4i$$

$$=$$

**Example 4:** Determining the regular payment amount of an investment (p.52)

Celia wants to have $300 000 in 20 years so that she can retire. Celia has found a trust account that earns a fixed rate of 10.8%, compounded annually.

a) What regular payments must Celia make at the end of each year to meet her goal of $300 000?
b) How much interest will she earn over the 20 years?

N = 20

I% = 10.8

PV = 0

PMT = ?

FV = 300 000

P/Y = 1

C/Y = 1

a) Payment -> $4781.09

b) $I = A - P$

$= 300\,000 - (4781.09)(20)$

$= \$204\,378.20$

**Example 5:** Determining the term of a regular payment investment (p.53)

semi-annually

On Luis's 20th birthday, he started making regular \$1000 payments into an investment account at the end of every 6 months. He wants to save for a down payment on a home. His investment earns 3.5%, compounded semi-annually. At what age will he have $\boxed{\text{more than}}$ \$18 000?

N = ? *

I% = 3.5

PV = 0

PMT = -1000

FV = 18 000

P/Y = 2

C/Y = 2

N= 15.78 ÷ 2 = 7.89 -> 8 years

* remember N is the number of compounding periods. To get an answer in years, you must divide by the compounding periods per year!

**September 20, 2012**

## In Summary

### Key Idea

- The present value of an investment that earns compound interest can be determined using the formula

$$P = \frac{A}{(1 + i)^n}$$

where $P$ is the present value (or principal), $A$ is the amount (or future value), $i$ is the interest rate per compounding period (expressed as a decimal), and $n$ is the number of compounding periods.

### Need to Know

- Any equivalent form of the compound interest formula may be used to solve a compound interest problem.

$A = P(1 + i)^n \qquad P = \frac{A}{(1 + i)^n} \qquad \frac{A}{P} = (1 + i)^n$

FV to P ratio, bigger is better!

- To compare investments, usually with the same term or principal, the ratio of the future value to the present value can be determined using the form: $\frac{A}{P} = (1 + i)^n$
- Using a formula, using the financial application on a graphing calculator, and using spreadsheet software are all valid strategies for solving a compound interest problem.

HW: 1.5 pp. 55-57 #1, 5, 7, 8, 9, 10, 13 & 17

F Math 12 **_1.6 Solving Investment Portfolio Problems p. 58_**

Name ________________

Date ________________

| |
|---|
| **Goal:** Analyze, compare, and design investment portfolios that meet specific financial goals. |

1. **portfolio**: one or more investments held by an individual investor or by a financial organization.

**Example 1:** Determining the future value and doubling time of an investment portfolio (p.59)

Phyllis started to build an investment portfolio for her retirement.

- She purchased a $500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. } you must solve FV by hand !!
- Three years ago, she also purchased a $4 000 GIC that earned 6%, compounded monthly. TVM Solver

a. What was the value of Phyllis' portfolio 10 years after she started to invest?

b. Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?

solve the GIC first b/c it's easier

GIC

N = 3 x 12

I% = 6%

PV = -4 000

PMT = 0

FV = ?

P/Y = 12

C/Y = 12

FV=$4786.72

F Math 12 **_1.6 Solving Investment Portfolio Problems p. 58_**

Name ______________

Date ______________

| |
|---|
| **Goal:** Analyze, compare, and design investment portfolios that meet specific financial goals. |

1. **portfolio**: one or more investments held by an individual investor or by a financial organization.

**Example 1:** Determining the future value and doubling time of an investment portfolio (p.59)

Phyllis started to build an investment portfolio for her retirement.

- She purchased a \$500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. } you must solve FV by hand !!
- Three years ago, she also purchased a \$4 000 GIC that earned 6%, compounded monthly. TVM Solver

a. What was the value of Phyllis' portfolio 10 years after she started to invest?

b. Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?

solve the GIC first b/c it's easier

GIC

N = 3 x 12

I% = 6%

PV = -4 000

PMT = 0

FV = ?

P/Y = 12

C/Y = 12

FV=\$4786.72

F Math 12 **_1.6 Solving Investment Portfolio Problems p. 58_**

Name ______________

Date ______________

| |
|---|
| **Goal:** Analyze, compare, and design investment portfolios that meet specific financial goals. |

1. **portfolio**: one or more investments held by an individual investor or by a financial organization.

**Example 1:** Determining the future value and doubling time of an investment portfolio (p.59)

Phyllis started to build an investment portfolio for her retirement.

- She purchased a $500 Canada Savings Bond (CSB) at the end of each year for 10 years. The first five CSBs earned a fixed rate of 4.2%, compounded annually. The next five CSBs earned a fixed rate of 4.6% compounded annually. } you must solve FV by hand !!
- Three years ago, she also purchased a $4 000 GIC that earned 6%, compounded monthly. TVM Solver

a. What was the value of Phyllis' portfolio 10 years after she started to invest?

b. Phyllis found a savings account that earned 4.9%, compounded semi-annually. She redeemed her portfolio and invested all the money in the savings account. About how long will it take her to double her money?

solve the GIC first b/c it's easier

GIC

N = 3 x 12

I% = 6%

PV = -4 000

PMT = 0

FV = ?

P/Y = 12

C/Y = 12

FV=$4786.72

Need to calculate the value of the CSB by hand as the interest rate changes part way through the investment.

$A = P(1+i)^n$

| Year | *P* ($) | *i* | *n* | *A* ($) |
|---|---|---|---|---|
| 1 | 500 | 0.042 | 9 | $A = 500(1+0.042)^9 = \$724.07$ |
| 2 | 500 | 0.042 | 8 | $A = 500(1.042)^8 = \$694.88$ |
| 3 | 500 | 0.042 | 7 | $666.87 |
| 4 | 500 | 0.042 | 6 | $639.99 |
| 5 | 500 | 0.042 | 5 | $614.20 |
| 6 | 500 | 0.046 | 4 | $598.54 |
| 7 | 500 | 0.046 | 3 | $572.22 |
| 8 | 500 | 0.046 | 2 | $547.06 |
| 9 | 500 | 0.046 | 1 | $523.00 |
| 10 | 500 | 0.046 | 0 | $500 |
| | | | **Total:** | $6080.81 |

Portfolio value: $4786.72 + $6080.81

= $10 867.53

b) Rule of 72: $\frac{72}{4.9} = 14.69$ years

Her new investment will double in about 14.7 years.

**Example 3**: Comparing the rates of return of two investment portfolios (p.62)

Jason and Malique are each hoping to buy a house in 10 years. They want their money to grow so they can make a substantial down payment.

- A 10-year $2 000 GIC that earns 4.2%, compounded semi-annually
- A savings account that earns 1.8%, compounded weekly, where he saves $55 every week
- A 5-year $4 000 bond that earns 3.9%, compounded quarterly, which he will reinvest in another bond at an interest rate of 4.1%

- A tax-free savings account (TFSA) that earns 2.2%, compounded monthly, and has a current balance of $5 600
- The purchase, at the end of each year, of a 10-year $500 CSB that earns 3.6%, compounded annually
- A savings account that earns 1.6%, compounded monthly, where she saves $200 every month

In 10 years, whose portfolio will have the greater rate of return on investment?

| | J GIC | J SA | J $B_5$ | J $B_{10}$ |
|---|---|---|---|---|
| N = | 10×2 | 52×10 | 4×5 | 4×5 |
| I% = | 4.2 | 1.8 | 3.9 | 4.1 |
| PV = | −2 000 | 0 | −4 000 | −4856.65 |
| PMT = | 0 | −55 | 0 | 0 |
| FV = | ? | ? | ? | ? |
| P/Y = | 2 | 52 | 4 | 4 |
| C/Y = | 2 | 52 | 4 | 4 |
| FV = | $3030.71 | $31 329.72 | $4856.65 | $5955.45 |

Total FV = $40 315.88

$$RR = \frac{\text{interest earned}}{\text{investment}} \rightarrow \frac{A-P}{P}$$

$$= \frac{40\,315.88 - (2\,000 + 55 \times 52 \times 10 + 4000)}{34\,600}$$

$$= \frac{5715.88}{34\,600}$$

$$= 0.165$$

$$= 16.5\%$$

# Malique

- A tax-free savings account (TFSA) that earns 2.2%, compounded monthly, and has a current balance of $5 600
- The purchase, at the end of each year, of a 10-year $500 CSB that earns 3.6%, compounded annually
- A savings account that earns 1.6%, compounded monthly, where she saves $200 every month

**TFSA**

N : 12 x 10
I% : 2.2
PV : -5 600
PMT : 0
FV : $6976.62
P/Y : 12
C/Y : 12

**CSB**

N : 10
I% : 3.6
PV : 0
PMT : -500
FV : $5892.88
P/Y : 1
C/Y : 1

**SA**

N : 12 x 10
I% : 1.6
PV : 0
PMT : -200
FV : $26007.87
P/Y : 12
C/Y : 12

$$RR = \frac{\text{interest earned}}{\text{investment}}$$

$$= \frac{A - P}{P}$$

$$= \frac{(6972.62 + 5892.88 + 26007.87) - (5600 + 5000 + 2400)}{34\,600}$$

$$= \frac{38\,843.37 - 34\,600}{34\,600}$$

$$= 0.123$$

12.3%

Jason has the better rate of return, about 16%

**September 21, 2012**

## In Summary

### Key Ideas

- Rate of return is a useful measure for comparing investment portfolios.
- An investment portfolio can be built from different types of investments, such as single payment investments (for example, CSBs and GICs) and investments involving regular payments. Some of these investments, such as CSBs, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time but offer lower interest rates. Investments that involve greater principal amounts invested or greater regular payment amounts when contracted tend to offer higher interest rates.
- The factors that contribute to a larger return on an investment are time, interest rate, and compounding frequency. The longer that a sum of money is able to earn interest at a higher rate compounded more often, the more interest will be earned. For investments involving regular payments, the payment frequency is also a factor.

### Need to Know

- Financial applications on calculators or spreadsheets and online financial tools at banking websites are valuable tools for analyzing and comparing investment portfolios.

HW: 1.6 pp. 64-67 #3, 5, 6, 7, 8 & 10