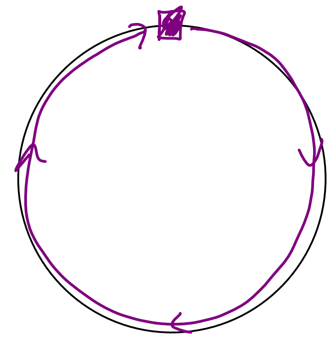
Circular Motion and Gravitation Notes

1 – Centripetal Acceleration and Force

This unit we will investigate the special case of kinematics and dynamics of objects in **uniform circular motion.** constant speed

First let's consider a mass on a string being twirled in a horizontal circle at a constant speed.
Let's determine the speed of the object.
Remember that speed is defined as: $V = \frac{d}{t}$

We define the period of motion (T) as the time it takes to complete one rotation.
How far does it travel in one rotation? circumference



We can find the circumference of the circular path (distance traveled) by:

$$c = 2\pi r$$

Therefore the speed of an object in uniform circular motion is:

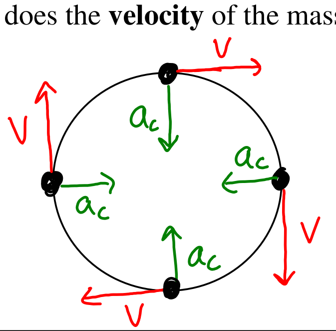
$$\boxed{V = \frac{2\pi r}{T \leftarrow \text{period}}}$$

Ok so we've figured out its speed, but is the mass accelerating?

Remember that the mass is traveling at a constant speed. However, acceleration is defined as:

$$\vec{a} = \frac{\Delta \vec{V}}{t}$$

So how does the **velocity** of the mass **change** with respect to **time**?



Notice that the direction of the velocity at any time is …
tangent to the circle

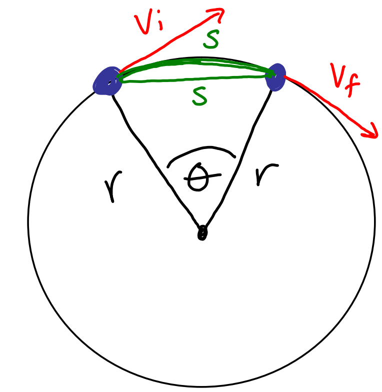
So even though it may be traveling at a constant speed anything traveling in a circular path is accelerating because the direction of its velocity is always changing.

The acceleration of an object in uniform circular motion is:

$$\boxed{a_c = \frac{V^2}{r} = \frac{4\pi^2 r}{T^2}}$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

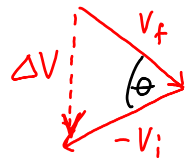
Let’s do a quick derivation of this formula:



$V_i = V_f$

$a = \frac{\Delta \vec{V}}{t}$

$\Delta \vec{V} = \vec{V}_f - \vec{V}_i$



It is worth noting from the above derivation that the direction of the change in velocity is always.... towards the centre of the circle

Therefore the acceleration of an object in circular motion is always towards the… towards the centre also!

This is the definition of **centripetal**, which means **center-seeking**.

$$\frac{\Delta V}{V} = \frac{s}{r}$$

$$\Delta V = \frac{V \cdot s}{r}$$

V

$$a = \frac{\Delta V}{t} = \frac{V \cdot \boxed{s}}{r \cdot \boxed{t}} = \frac{V \cdot V}{r} \qquad \boxed{\therefore a_c = \frac{V^2}{r}}$$

Whenever an object is accelerated there must be a…

net force

This force is known as **centripetal force**, F_c. This is not a new force, it is simply the **net force** that accelerates an object towards the center of its circular path.

Examples:

1) A mass is twirled in a circle at the end of a string, the centripetal force is provided by tension

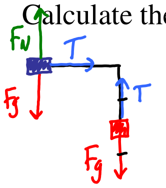
2) When a car rounds a corner on a highway, the centripetal force is provided by friction

3) When the Moon orbits the Earth, the centripetal force is provided by gravity

Newton’s Second Law we can help us to determine a formula for centripetal force:

$$\boxed{F_c = ma_c = \frac{mV^2}{r} = m\frac{4\pi^2 r}{T^2}}$$

Example:
A 0.50 kg mass sits on a frictionless table and is attached to hanging weight. The 0.50 kg mass is whirled in a circle of radius 0.20 m at 2.3 m/s. Calculate the centripetal force acting on the mass.



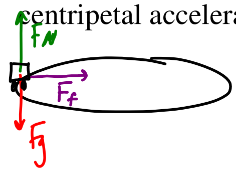
$$F_c = \frac{mv^2}{r} = \frac{(0.50\,kg)(2.3\,m/s)^2}{0.20\,m}$$
$$= 13.225\,N$$
$$= \boxed{13.2\,N}$$

Calculate the mass of the hanging weight.

$$F_g = T = F_c = mg \qquad m = \frac{F_c}{g} = \frac{13.2\,N}{9.8\,m/s^2}$$
$$= \boxed{1.35\,kg}$$

Example:
A car traveling at 14 m/s goes around an unbanked curve in the road that has a radius of 96 m. What is its centripetal acceleration?

flat

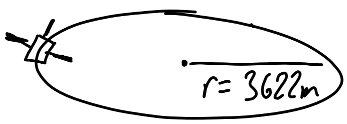


$$a_c = \frac{V^2}{r} = \frac{(14\,m/s)^2}{(96\,m)} = \boxed{2.04\,m/s^2}$$

What is the minimum coefficient of friction between the road and the car's tires?

$$F_c = \boxed{F_f = ma_c}$$
$$F_f = \mu F_N = \mu F_g = \mu mg$$
$$\mu \cancel{m} g = \cancel{m} a_c$$
$$\mu = \frac{a_c}{g} = \frac{2.04\,m/s^2}{9.8\,m/s^2} = \boxed{0.21}$$

Example:
A plane makes a complete circle with a radius of 3622 m in 2.10 min. What is the speed of the plane?



$$V = \frac{2\pi r}{T} = \frac{2\pi(3622\,m)}{126\,s} = \boxed{180\,m/s}$$

$$T = 2.10\,min \times \frac{60\,s}{1\,min} = 126\,s$$

One last note on a little thing called centri**fugal** force. While centripetal means center- seeking
centrifugal means center- fleeing.

An **inertial frame of reference** is a one where Newton's Law's are true.
In an inertial frame of reference, centrifugal force is actually an *apparent* force - it does not exist. It is simply the apparent force that causes a revolving or rotating object to move in a straight line.

However, Newton's First Law tells us that we do not need a force to keep an object moving in a straight line, you only need a force to *deflect* an object from moving in a straight line.

Example:
When riding in the backseat of a car that is turning a corner, you slide across the seat, seeming to accelerate outwards, away from the center of the turning circle.
Explain why the force in this case is actually working towards the center of the turn and not outwards.

- By Newton's 1st Law you want to continue moving in a straight line.
- It is this inertia that you feel that is mistaken for a force pushing outward.
- In reality the car (seatbelt, friction, the door) pulls you in to the center of the circle that it is traveling in.

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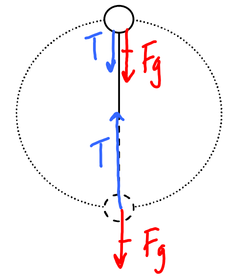
2 – More Centripetal Problems

We have already seen the forces acting on a mass moving in a horizontal circle, now let's see how this differs from a mass moving in a vertical circle.

Draw the forces acting on a mass on a string being spun in a vertical circle at the top and bottom of its path.



As with any object moving in a circle there is a net force acting on it, towards the center of circle.

This net force is a centripetal force.

Notice that at the top of its arc the centripetal force (or net force) is:

$$F_c = T + F_g$$

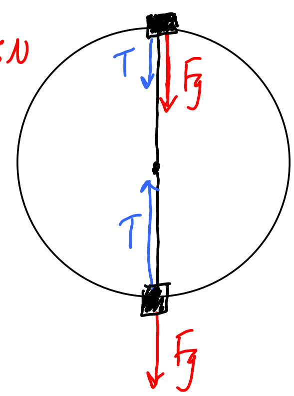
Also at the bottom of the arc the centripetal force is:

$$F_c = T - F_g$$

Example:
A 1.7 kg object is swung from the end of a 0.60 m string in a vertical circle. If the time of one revolution is 1.1 s, what is the tension in the string:

$F_g = mg = (1.7)(9.8) = 16.66\,N$

$F_c = m\frac{4\pi^2 r}{T^2} = \frac{(1.7)4\pi^2(0.60)}{1.1^2} = 33.3\,N$



a) at the top?

$F_c = T + F_g$

$T = F_c - F_g = 33.3 - 16.66 = \boxed{17\,N}$

b) at the bottom?

$F_c = T - F_g$

$T = F_c + F_g = 33.3 + 16.66 = \boxed{50\,N}$

Now suppose the mass is spun with just enough speed to keep it moving in a circular path. What is the tension in the string at the top?

$T = 0$

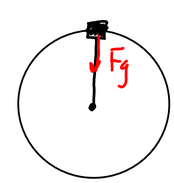
We say that the mass at the peak of the arc is **weightless**, because the net force working on it is only gravity. This is the same as an object in total free fall.

Example:
An object is swung in a vertical circle with a radius of 0.75 m. What is the minimum speed of the object at the top of the motion for the object to remain in circular motion?

$F_c = F_g$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$v = \sqrt{gr} = \sqrt{(9.8)(0.75)} = \boxed{2.7\,m/s}$



Notice that if the velocity of the object… constant

it depends only on… radius and period $\boxed{v = \frac{2\pi r}{T}}$

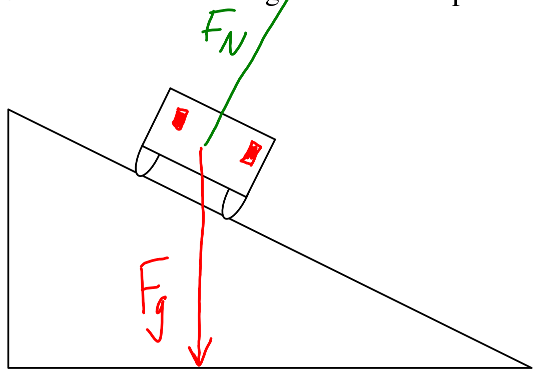
For the special case of finding the minimum speed of an object at the top of its circular arc we can use the equation:

$$\boxed{F_c = F_g \quad \therefore} \qquad v = \sqrt{gr}$$

Banked Curves (and other 2-D Problems:

When cars travel at high speeds on highways, they do not rely solely on friction to keep the cars from sliding off the road. A greater centripetal force can exist if the turn is banked.

Consider a car traveling at a constant speed around a frictionless banked corner.



On a frictionless corner only F_g and F_N act on the car.

Note that in this case FN is larger because it both:

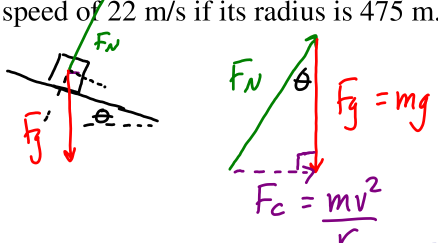
(1) Matches F_g

(2) accelerates inwards

The sum of F_N and F_g must equal F_c

Example

Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22 m/s if its radius is 475 m.

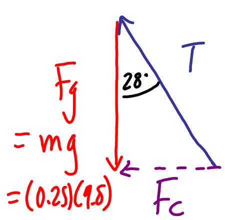


$$\tan\theta = \frac{F_c}{F_g} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g} = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left[\frac{22^2}{(475)(9.8)}\right] = 6^\circ$$

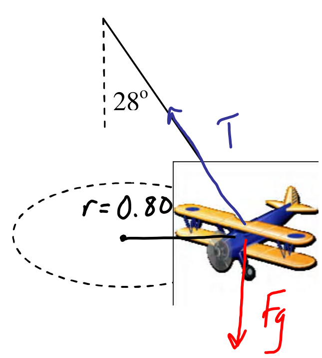
Example

A 0.25 kg toy plane is attached to a string so that it flies in a horizontal circle with a radius of 0.80 m.. The string makes a 28° angle to the vertical. What is its period of rotation?



$$\tan 28° = \frac{F_c}{F_g}$$

$$F_c = F_g \tan 28° = 1.30N$$



$$F_c = \frac{m4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{m4\pi^2 r}{F_c}} = \sqrt{\frac{(0.25)4\pi^2(0.80)}{1.30}} = \boxed{2.5s}$$

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3 – Gravitation

Newton discovered that gravity attracts any two objects depending on their masses and their distance apart.

F_g is proportional to the two masses

$$F_g \propto m_1 m_2$$

F_g is inversely proportional to the square of the distance between their centers of mass

$$F_g \propto \frac{1}{r^2}$$

or

$$\boxed{F_g = \frac{G m_1 m_2}{r^2}}$$

Where: $G = 6.67\times10^{-11}\ N m^2/kg^2$

m_1, m_2 = masses

r = distance between centers of mass

Example

Calculate the force of gravity between two 75 kg students if their centers of mass are 0.95 m apart.

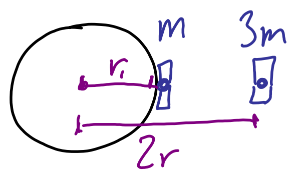
$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67\times10^{-11})(75)(75)}{(0.95)^2}$$

$$= 4.2\times10^{-7} N$$

Common Question Alert!!!

You will often see problems that ask something like this…

A satellite weighs 9000 N on Earth's surface. How much does it weigh if its mass is tripled and its orbital radius is doubled?



$$F_{g1} = \boxed{\frac{G m_1 m_2}{r_1^2}} = 9000 N$$

$$F_{g2} = \frac{G m_1 (3 m_2)}{(2r)^2}$$

$$F_{g2} = \frac{3}{4}\boxed{\frac{G m_1 m_2}{r^2}} = \frac{3}{4}(9000)$$

$$= \boxed{6750 N}$$

While we're at it, let's make sure we clear up another common misconception: mass vs. weight.

Mass:

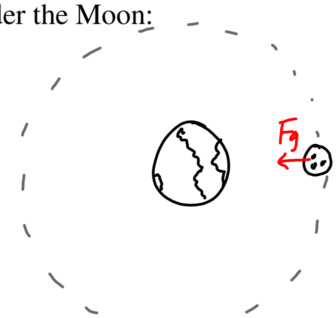
- amount of matter
- constant everywhere

Weight:

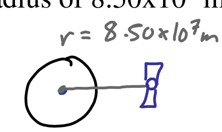
- gravitational attraction (F_g)
- changes depending on location

Satellites in Orbit

A satellite of the Earth, such as the moon, is constantly falling. But it does not fall towards the Earth, rather it falls *around* the Earth. Just as if you were in an elevator that was falling towards the Earth you would feel weightless if you were on an artificial satellite falling around the Earth. Consider the Moon:



Example: A 4500 kg Earth satellite has an orbital radius of 8.50×10^7 m. At what speed does it travel?



$$F_c = F_g$$

$$\frac{\cancel{m_2} v^2}{\cancel{r}} = \frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}}$$

$$v = \sqrt{\frac{G m_1}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{8.50\times10^7}}$$

$$= 2200 m/s$$

Circular Motion and Gravitation Notes

4 – Gravitational Fields

Scientists had difficulty explaining how two objects that are not in contact can exert a force on one another. In order to help conceptualize how this can occur, we had invented the idea of *FIELDS*.

A field is defined as…

an area of influence.

To help imagine how these fields work, consider a **campfire.** It seems as though the fire is emitting a *heat field*.

As you approach the fire the …

the field strength increases.

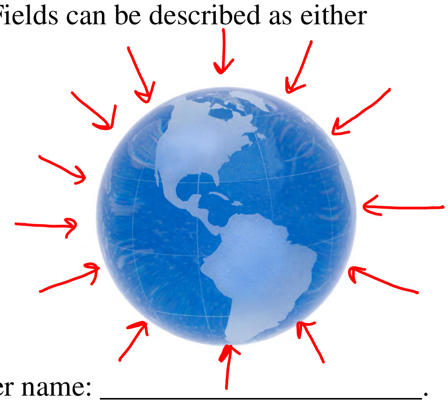
As you increase the size of the fire the …

the field strength increases.

Just like this so-called heat field, gravitational fields surround any mass. Fields can be described as either **vector** or **scalar**.

While heat is measured by temperature (a scalar) its field is also scalar. Gravitational fields are **force fields** and as such are vector.

Vector fields, like vector quantities, are represented by arrows. In this case, the density of the arrows represents the magnitude of the field strength…



We are already quite familiar with gravitational field strength by its other name: acceleration due to gravity.

Recall that: $F_g = mg$

Therefore

$$\boxed{g = \frac{F_g}{m}}$$

Where g = acceleration due to gravity
= gravitational field strength
= 9.80 m/s^2 near Earth's surface

This formula works fine if we stay put on Earth, but it falls way short once we leave Terra Firma because…

g varies with distance.

However, we can derive a more useful formula:

$$F_g = \cancel{m_2} g = \frac{G m_1 \cancel{m_2}}{r^2}$$

$$\boxed{g = \frac{G m_1}{r^2}}$$

Example:
What is the gravitational field strength on the surface of the Moon?

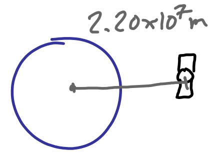
$m_{moon} = 7.35\text{x}10^{22}$ kg
$r_{moon} = 1.74\text{x}10^{6}$ m

$$g = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2}$$

$$= 1.62 \text{ m/s}^2$$

(or N/kg)

Example:
A satellite orbits the Earth at a radius of $2.20\text{x}10^{7}$ m. What is its orbital period?



$$F_c = F_g$$

$$\cancel{m}a_c = \cancel{m}g$$

$$a_c = g$$

$$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_1}} = \sqrt{\frac{4\pi^2(2.20\times10^7)^3}{(6.67\times10^{-11})(5.98\times10^{24})}}$$

$$= \boxed{32500\text{s}}$$

Geosynchronous Orbit

The orbital speed of a satellite will depend on the strength of gravitational field at the orbital radius.

Consider the following situations. Which identical satellite will ~~be travelling~~ faster in each case? Why?

a) Satellite A orbits the Earth at twice the orbital radius of Satellite B.

b/c it is closer ∴ F_g is greater ∴ must move faster to stay in orbit.

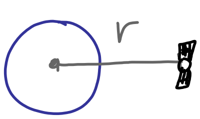
b) Satellite A orbits the Sun at the same orbital radius that Satellite B orbits the Earth.

b/c the Sun is larger ∴ same as above

The orbital period of the satellite depends only on the mass of the planet and the orbital radius of the satellite. It stands to reason therefore that at a certain orbital distance the orbital period will match the rotational period of the planet. Such a satellite is said to be in geosynchronous (or geostationary) orbit.

Example:
Find the orbital radius of a satellite that is geostationary above Earth's equator.

$$T_{Earth} = 24\text{hr} \times \frac{60\text{min}}{1\text{hr}} \times \frac{60\text{s}}{1\text{min}} = 86400\text{s}$$



$$a_c = g$$

$$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2}$$

$$r = \sqrt[3]{\frac{Gm_1 T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(86400)^2}{4\pi^2}}$$

$$= \boxed{4.2\times10^7\text{ m}}$$

What is the speed of this satellite?

$$V = \frac{2\pi r}{T} = \frac{2\pi(4.2\times10^7)}{86400}$$

$$= \boxed{3070\text{ m/s}}$$

Circular Motion and Gravitation Notes

5 – Potential Energy, Satellites and Escape Velocity

Gravitational Potential Energy

We have already discussed gravitational potential energy.

$E_p =$ mgh

However, as we have seen, g is not a constant but rather depends on mass of planet and distance.

Let's Derive!!!

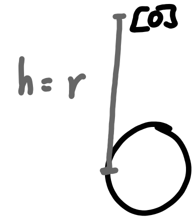
$g = \dfrac{Gm_1}{r^2}$ and $E_p = m_2gh$

$E_p = mgh$

$= m_2\left(\dfrac{Gm_1}{r^{\cancel{2}}}\right)\cancel{h}$

$= \dfrac{Gm_1m_2}{r}$

$$\boxed{E_p = -\frac{Gm_1m_2}{r}}$$



If you look at your formula sheet you will notice that this equation has a negative sign. What's the deal?

Whenever we talk about gravitational potential energy, we have to use a reference point. At this reference point we assign a gravitational potential energy of zero.

When determining the potential energy on a mass provided by the gravitational force generated by a second mass, we assign the ZERO reference point when the distance between the objects is infinite

This means whenever the objects get closer together the potential energy between them gets less. Compared to infinity the potential energy of the object will always be negative.

Let's sketch a graph showing the relationship between gravitational potential energy of one object relative to another and the distances between their centers.

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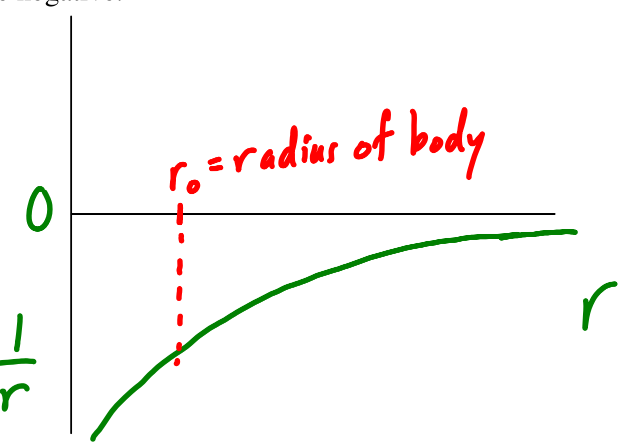
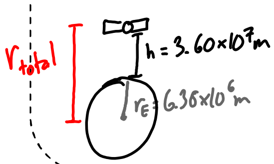


$E_p = -\dfrac{Gm_1m_2}{r}$ $\quad E_p \propto -\dfrac{1}{r}$

Example:

A 2500 kg satellite is in orbit 3.60x10^7 m above the Earth's surface. What is the gravitational potential energy of the satellite due to the gravitational force due to the Earth?



$E_p = -\dfrac{Gm_1m_2}{r} = \dfrac{-(6.67\times10^{-11})(5.98\times10^{24})(2500)}{4.238\times10^7}$

$= \boxed{-2.35\times10^{10}\ J}$

$r_{total} = 4.238\times10^7$ m

Note: The potential energy of this satellite relative to some infinite position is.. negative.

It would need more energy to break free of Earth's pull.

Example cont:
What is the **total** energy of the satellite in the last question?

$E_p = -2.35 \times 10^{10}\ J$

$E_T = E_p + E_K$

$a_c = g$

$\frac{v^2}{\cancel{r}} = \frac{Gm_1}{r^{\cancel{2}}}$

$v = \sqrt{\frac{Gm_1}{r}}$

$= \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{4.238\times10^7}}$

$= 3068\ m/s$

$E_K = \frac{1}{2}mv^2$

$= \frac{1}{2}(2500)(3068)^2$

$= 1.176 \times 10^{10}\ J$

$E_T = E_K + E_p$

$= 1.176\times10^{10} + (-2.35\times10^{10})$

$= -1.174 \times 10^{10}\ J$

$= \boxed{-1.17 \times 10^{10}\ J}$

The funny thing about satellites…

$a_c = g \qquad \frac{v^2}{\cancel{r}} = \frac{Gm_1}{r^{\cancel{2}}}$

$v^2 = \frac{Gm_1}{r}$

$E_K = \frac{1}{2}m_2v^2$

$= \frac{1}{2}\left(\frac{Gm_1m_2}{r}\right)$

$E_p = \frac{-Gm_1m_2}{r}$

$-E_p = \frac{Gm_1m_2}{r}$

$\therefore E_K = \frac{1}{2}(-E_p)$

$E_T = E_K + E_p$

$= -\frac{1}{2}E_p + E_p$

$\therefore \boxed{E_T = \frac{1}{2}E_p}$

Change in Potential Energy

A change in potential energy can be found by using:

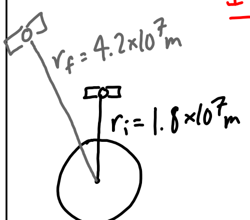
$\Delta E_p = E_{pf} - E_{pi}$

$$= -\frac{Gm_1m_2}{r_f} + \left(+\frac{Gm_1m_2}{r_i}\right) = \frac{Gm_1m_2}{r_i} - \frac{Gm_1m_2}{r_f} = Gm_1m_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

Example:
How much work is required to move a 4500 kg Earth satellite from an orbital radius of 1.8x10^7 m to a radius of 4.2x10^7 m?

Ignore E_K

$W = \Delta E_p$

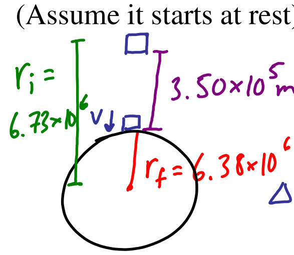


$\Delta E_p = Gm_1m_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$

$= (6.67\times10^{-11})(5.98\times10^{24})(4500)\left(\frac{1}{1.8\times10^7} - \frac{1}{4.2\times10^7}\right)$

$= \boxed{5.7\times10^{10}\ J}$

Example:
The International Space Station drops a 250 kg waste shuttle from an altitude of 3.50x10^5 m. At what speed would it impact Earth if there were no air friction? (Assume it starts at rest)



$\Delta E_K = -\Delta E_p$

$\Delta E_p = Gm_1m_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$

$\Delta E_p = 6.67\times10^{-11}(5.98\times10^{24})(250)\left(\frac{1}{6.73\times10^6} - \frac{1}{6.38\times10^6}\right)$

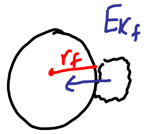
$= -8.128\times10^8\ J$

$\Delta E_K = -\Delta E_p = 8.128\times10^8\ J$

$\Delta E_K = E_{Kf} - \cancel{E_{Ki}}^{\,0}$

$E_{Kf} = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2E_K}{m}} = \boxed{2550\ m/s}$

Example: A 2.35x10^{16} kg asteroid falls towards the Earth from a really, really, REALLY far way away. How much energy is released when it impacts with the Earth?



$E_{Kf} = ?$

$r = \infty$

$v = 0$

$E_{pi} = 0$

$E_{Ki} = 0$

$$\boxed{= 1.5\times10^{24}\ J}$$

$$\Delta E_K = -\Delta E_p$$

$$E_{Kf} - \cancel{E_{Ki}} = -(E_{pf} - \cancel{E_{pi}})$$

$$E_{Kf} = -E_{pf}$$

$$= -\left(-\frac{Gm_1m_2}{r_f}\right) = \frac{Gm_1m_2}{r} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(2.35\times10^{16})}{6.38\times10^{6}}$$

Launching (Escape Velocity)

What goes up must come down, unless we throw it really, REALLY hard.

Escape velocity is the minimum speed an object requires in order to break free from Earth's pull and achieve orbit. It should stand to reason that if an object is going to be completely freed from the Earth gravitational pull that we need to supply it with enough ________________________________ to match its ______________________________________ at infinite.

In terms of equations this means that:

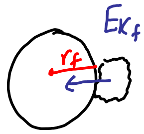
=

Example:
At what speed do you need to throw a 1.0 kg rock in order for it to leave the Earth's gravitational pull?

Does the mass of the rock matter?

Example: A 2.35x10^{16} kg asteroid falls towards the Earth from a really, really, REALLY far way away. How much energy is released when it impacts with the Earth?

$E_{Kf} = ?$



$r = \infty$
$v = 0$

$E_{Pi} = 0$

$E_{Ki} = 0$

$$\boxed{= 1.5 \times 10^{24}\ J}$$

$$\Delta E_K = -\Delta E_P$$

$$E_{Kf} - \cancel{E_{Ki}} = -(E_{Pf} - \cancel{E_{Pi}})$$

$$E_{Kf} = -E_{Pf}$$

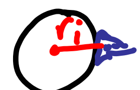
$$= -\left(-\frac{Gm_1m_2}{r_f}\right) = \frac{Gm_1m_2}{r} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(2.35\times10^{16})}{6.38\times10^{6}}$$

Launching (Escape Velocity)

What goes up must come down, unless we throw it really, REALLY hard.

Escape velocity is the minimum speed an object requires in order to break free from Earth's pull ~~and achieve orbit~~. It should stand to reason that if an object is going to be completely freed from the Earth gravitational pull that we need to supply it with enough Kinetic energy to match its potential energy at infinite.

$E_{Ki} = ?$ $E_{Kf} = 0$



$E_{Pi} = ?$ $E_{Pf} = 0$

In terms of equations this means that:

$$\Delta E_P = -\Delta E_K$$

=

$$\cancel{E_{Pf}} - E_{Pi} = -(\cancel{E_{Kf}} - E_{Ki})$$

$$-E_{Pi} = E_{Ki}$$

$$-\left(-\frac{Gm_1\cancel{m_2}}{r}\right) = \frac{1}{2}\cancel{m_2}v^2$$

$$\frac{Gm_1}{r} = \frac{1}{2}v^2$$

$$\boxed{V_{Escape} = \sqrt{\frac{2Gm_1}{r_i}}}$$

Example:
At what speed do you need to throw a 1.0 kg rock in order for it to leave the Earth's gravitational pull?

$$V_{escape} = \sqrt{\frac{2Gm_1}{r}} = \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.38\times10^{6}}}$$

$$= \underline{1.1\times10^{4}\ m/s}$$

Does the mass of the rock matter?

No, not for velocity but yes for Ek.