## A Review of Static Electricity

Forces related to electrical charges can be explained using the model of the atom developed by Neils Bohr in the early part of the 20th century.

Consider the following model of a Boron atom:


## Relevant Points:

- There is a huge force of attraction between electrons and protons, but electrons repel electrons with an equally large force, as do protons with protons.
- The force of attraction between the protons in the nucleus and the fast moving electrons provides the centripetal force required to hold them in orbit around the nucleus. This electrical force is billions of times stronger than the external gravitational force exerted by Earth!
- Electrons have been given the name negative and protons positive.
- Atoms have the same number of protons as electrons, and are electrically neutral.
- Atoms can only lose or gain electrons to become positively or negatively charged; a charged atom is called an ion.
- The innermost electrons are bound tightly to the nucleus, but the outermost electrons are bound relatively loosely and can be dislodged. The force required to do this depends on the substances present.
- Electrons can be transferred from one material to another, resulting in objects becoming positively and/or negatively charged. Only electrons can travel in solid objects.
- Electrons cannot be created or destroyed (called the Law of Conservation of Charge).
- Insulators are poor conductors of electricity. The outer orbit electrons are anchored fairly strongly to the nucleus of the atoms contained within these materials, leaving very few free electrons to move through.
- Conductors allow electrons to flow easily through them. Within the atoms of these materials, the outer orbits are so loosely held to the nucleus that no outer electron sems to belong to any particular atom. These electrons are free to roam from one atom to another as well as throughout the materials.
- Semi-conductors allow limited movement of a few free electrons, so some electron flow occurs. In such materials, a charge can be built up, but will disappear fairly quickly.

For any given solid object, its charge is always indicated from the point of view of a lack or an excess of electrons of a body. This is because only electrons have the ability to move within solids (protons and neutrons are locked in place as the nuclei of all atoms contained within the solid).
> Neutral: Every proton in the material has a balancing electron leaving no excess or lack of electrons.

$>$ Positive: a deficiency of electrons. Foe example, when an acetate rod is rubbed with silk, electrons are displaced from the acetate to the silk. Acetate now has a lack of electrons and is positive.

> Negative: an excess of electrons. For example, when an ebonite or vinyl rod is rubbed with wool, electrons are removed from the wool and go to the rod, creating an excess of electrons in the rod to become negative.


Because electrons are attracted to protons, objects tend to be neutral, with no charge. However, a charge can be placed on an object in a number of ways:

## A. Charging by Conduction

When two objects are in contact with each other, electrons can move directly from one material to another. For example:

- If a charged conductor touches a neutral conductor, electrons will immediately move from one to another. Electrons flow from the negative body to the positive body, or from the more negative to the less negative body, or from the less positive to the more positive body.
- If a charged conductor touches a neutral conductor, electrons will immediately move from one to another. Electrons flow from the negative body to the positive body, or from the more negative to the less negative body, or from the less positive to the more positive body.
- If two insulators are rubbed together, the increased contact will cause electrons to "rub" off the material that holds electrons more loosely. These electrons are transferred to the material that holds its outer electrons more tightly, so that the latter material becomes negatively charged. Some examples:
- rubbing ebonite or vinyl with wool will make both ebonite and vinyl negatively charged and the wool positively charged
- rubbing acetate or glass with silk or rayon makes the former two materials positive and the latter two negatively charged
An insulator can also charge a conductor in this way, so long as the electrons have no means to escape the conductor. Aircraft, for example, are often charged on the metallic surface by air rubbing the plane's aluminum skin when in flight.


## B. Charging by Induction

Because electrons in conductors can move easily and because electrons repel other electrons but attract protons, a charge brought near to a conductor, but not touching it, will cause the electrons in the conductor to move away from an external negative charge or towards a positive external charge. Note that when electrons move inside a material in this manner, so that one side becomes negative and the other side positive, we say that charge has been separated. This is what happens in a chemical cell (mistakenly called a battery).

Examine the three diagrams below. Note that in $\mathbf{B}$ and $\mathbf{C}$, the presence of a charged rod causes a temporary induced charge. In particular, even though the ball remains neutrally charged (i.e. no net gain or loss of electrons), the near end of the ball is always charged oppositely to the charged rod, resulting in attraction. It is this induced charge which causes charged objects to attract neutral objects.


An electroscope is a simple device that detects static charges. Importantly, an electroscope can show the effects of charging by both conduction and induction.

Examine the pith ball electroscope below. When a charged rod is brought near the neutral pith ball, the ball attracts, touches and then repels.


As already shown, any charge attracts a neutral object. Consider the diagrams below:
$>$ In this case a positive charged acetate rod attracts the pith ball.

> The pith ball touches the acetate rod...

$>$...and the now positively charged ball repels the positive acetate rod.


This phenomenon demonstrates that like charges repel, unlike charges attract, and either charge attracts neutral.

What follows is a drawing of a leaf electroscope being charged:
$>$ In diagram A, the electroscope is 'neutral'; the number of positive and negative charges is equal, and the negative charges are equally distributed among the positive charges.
$>$ In diagram $\mathbf{B}$, a negatively charged rod is brought towards the electroscope top. While the electroscope as a whole stays 'neutral', each electroscope leaf has a negative charge induced in it because of the movement of electrons away from the rod. The similar nature of their charges causes the leaves to begin to spread apart by induction.
$>$ In diagram $\mathbf{C}$, the rod touches the electroscope top, allowing some of its electrons to enter the electroscope, giving it a net negative charge. The electroscope leaves continue to be spread apart, this time by conduction.


Finally, consider grounding. A ground is any conductor with a large enough mass to be able to easily give up or receive electrons without that action much affecting the overall neutral charge of the ground. Any object with a huge reservoir of charged particles compared with the charged object in question can act as a ground (e.g. you). As you might expect, the Earth is the largest and most effective ground.

## Electrostatic Forces

The space around any charged object or particle that can cause a force to act on another charged object or particle is called its electric field. This field will cause a force of attraction or repulsion to act on another charged object.

Any point charge will exert an electrostatic force that depends on three factors:

- the size of its charge, in Coulombs (C).
- the size of charge of another object that is being attracted or repelled by the first object.
- the distance between the two charges; the further the distance, the weaker the force of attraction or repulsion.

From this, the size of the force between point charges is expressed as follows, and is known as Coulomb's Law:

$$
F=k \frac{q \mathbf{Q}}{\mathbf{R}^{2}}
$$

The units in this equation:
$>\mathbf{R}$ is the distance between the two centers of charge
$>\mathbf{Q}$ and $\mathbf{q}$ are the magnitude of each charge, measured in Coulombs
$>\mathbf{k}$ is the constant of proportionality; i.e.; $k=9.0 \times 10^{9} \mathbf{N}-\mathbf{m}^{2} / \mathbf{C}^{\mathbf{2}}$
Note that the smallest unit of charge is called the elementary charge, represented by the charge of either an electron or a proton. Charge is measured in Coulombs, where one Coulomb $=6.24 \times 10^{18}$ electrons. Therefore the coulomb charge on one electron or proton is the reciprocal of this number, or

$$
1 e^{-} \text {or } 1 p^{+}= \pm 1.6 \times 10^{-19} \mathrm{C} \quad \text { (sign depends on charge) }
$$

Note that because the coulomb is such a large unit, charges are often measured in microcoulombs or $\mu \mathrm{C}$.

Example 1. Find the electrostatic force between a $+3.0 \mu \mathrm{C}$ charge and a +8.0 $\mu \mathrm{C}$ charge, 0.25 m apart.

Example 2. A -6.0 $\mu \mathrm{C}$ charge and a $-4.0 \mu \mathrm{C}$ charge repel each other with a force of 7.0 N . How far apart are these point charges? ( 18 cm )
(see Electrostatics Ex 2 for answer)

We can use free-body diagrams to examine the relationship between two or more electrostatic forces. That is, if two or more electric forces act on one body the resultant net force is the vector sum of the electrostatic forces.

Example 3. What is the force on the $\mathbf{3 . 0} \mu \mathrm{C}$ charge if the charges are positioned along one line as follows.


To solve, follow these steps:
$>$ First, draw a free-body diagram of particle B, showing the direction of each vector acting on this particle.
$>$ Second, find the magnitude of each electrostatic force $\left(\mathbf{F}_{\mathbf{A B}}\right.$ and $\left.\mathbf{F}_{\mathbf{B C}}\right)$ that acts on the charge at $\mathbf{B}$. Don't include the sign of the charge in your calculations.
$>$ Finally, vector-add the two forces to find the resultant net force on $\mathbf{B}$.
(see Electrostatics Ex 3 for answer)

On the following page is a similar problem, but slightly more difficult. You will need to build a forces triangle and use some trigonometry skills in your last steps to solve.

Example 4. Three charges are laid out as in the following diagram. Find the resultant force due to the other two charges on charge $B$.
$A \bigodot 50 \mu \mathrm{C}$


$80 \mu \mathrm{C}$ $50 \mu \mathrm{C}$
(see Electrostatics Ex 4 for answer)

We can also use free-body diagrams to examine the relationship between electrostatic and other forces.

Example 5. Examine the electroscope arrangement below, where two pith balls have identical charges and are repelling each other.


If each string has a length $I=0.20 \mathrm{~m}$, the distance of separation between the charged pith balls is $r=0.05 \mathrm{~m}$, and the mass of the balls is 0.010 kg each, find the magnitude of the charge on each pith ball.

Some helpful hints:
$>$ first, find angle $\phi$, equal to $\frac{\theta}{2}$ :


$$
\sin \phi=\frac{2.5}{20} \phi=7.2^{\circ}
$$

$>$ Next, sketch a f.b. diagram of one of the pith balls and create a forces triangle.
$>$ Finally,from the diagram drawn, equate the electric and gravitational force; this should produce an equation from which charge $\mathbf{q}$ can be solved (remember that both charges are equal in this question).

## (see Electrostatics Ex 5 for answer)

Finally, just like with gravity, the electric force between two charges will change if either of the values for charge changes, or if the distance between the particles changes. And, similar to gravity,

$$
\mathbf{F} \propto q \text { and } \quad F \propto \frac{1}{\mathbf{r}^{2}}
$$

For example:
$>$ if either charge $\mathbf{q}$ doubles, then so does $\mathbf{F}$.
$>$ if both charges $\mathbf{q}_{1}$ and $\mathbf{q}_{2}$ double, $\mathbf{F}$ increases by $\mathbf{4 x}$.
$>$ if distance $\mathbf{r}$ between the charges doubles, then $\mathbf{F}$ is reduced to $1 / 4$ of the original amount.

Example 6. Two unknown charges have a force between them of 5.6 N. How will that force change if:
a) one of the charges is tripled?
b) one charge is halved and the other quadrupled?
c) the distance between them halved?
d) both charges are doubled and the distance tripled?

## Electric Field Strength

The electric field strength $\mathbf{E}$ for any charged object is equal to the force experienced by a unit positive test charge placed at that point. The field around point charges is radial and has the direction that an imaginary positive test charge would take if released near the charge.

If a line of force is the path a positive charge will take in the field, then the following must be true:
$>$ lines must repel each other.
> the closer together two lines are, the more they repel each other.
$>$ many lines close together suggests a strong field.
$>$ lines never cross.


There are two ways of expressing electric field strength:

- by definition, the field strength is equal to the force acting per coulomb of charge, so

$$
\mathbf{E}=\frac{\mathbf{F}}{\mathbf{q}} \quad \text { with units being } \mathrm{N} / \mathrm{C}
$$

- by combining the above formula with the equation for determining electrostatic force,

$$
E=\frac{F}{q}=k \frac{q Q}{R^{2}} \times \frac{1}{q} \quad \rightarrow \quad E=k \frac{Q}{R^{2}}
$$

Note that electric field strength is sometimes called electric intensity.

Example 7. A $6.0 \mu \mathrm{C}$ charge and a $4.5 \mu \mathrm{C}$ charge are positioned 1.6 cm apart. If the smaller charge is removed, what is the electric field strength at the location of the $4.5 \mu \mathrm{C}$ charge, due to the larger charge?
(see Electrostatics Ex 7 for answer)

When two or more electric fields act at one location the resultant field is given by the vector sum of the individual fields. This is the superposition principle.

Example 8. Find the resultant field at point $B$ due to the two charges.


Since the electric field is a vector quantity, you need to utilize the same principles that were used to find electric force:
$>$ draw a free-body diagram showing the vectors acting at $\mathbf{B}$.
$>$ calculate electric fields $\mathbf{E}_{\mathbf{A}}$ and $\mathbf{E}_{\mathbf{B}}$ (one at a time).
$>$ use vector addition to find the net electric field.

## Electric Potential Energy

If two attracting charges were put in an electric field, side by side, and then a force was applied on the negative charge in order to separate them, the work done to do this separation would be the electric potential energy difference (measured in Joules) between the two charges in their new location.


Now suppose that the electron were moved far enough away so that the electrostatic force between both charges becomes weaker and weaker until finally, $\mathrm{F}_{\mathrm{E}}=\mathbf{0}$.
From graphical analysis of electric force $\mathbf{F}_{\mathbf{E}}$ vs. $\mathbf{R}$ : (remember that $\mathbf{F}_{\mathbf{E}}=\frac{\mathbf{k Q q}}{\mathbf{R}^{2}}$ )

$$
\mathbf{F}_{\mathrm{E}} \underbrace{}_{\mathbf{R}} \quad \begin{gathered}
\rightarrow \text { from any point } \mathbf{R} \text { out to } \infty, \\
\text { work done }=\text { area under the graph }
\end{gathered}
$$

Like gravity, at infinity there is no electrostatic force, so potential energy is zero. Therefore, similar to gravitational potential energy:

$$
\mathbf{E}_{\mathrm{p}}=\frac{\mathbf{k Q q}}{\mathbf{R}}
$$

Some important points about this formula:

- potential energy is a scalar quantity. No vector analysis is required
- for two oppositely charged particles, the potential energy between them is always negative. This is because of the attractive force between them:
$>$ two opposite charges have more potential energy when they are further apart (they can potentially move further toward each other).
$>$ Since the greatest distance apart is at $\mathbf{R}=\infty$ (where $\mathbf{E}_{\mathbf{p}}=\mathbf{0}$ ), at any separation distance that is smaller, $\mathbf{E}_{\mathbf{p}}<\mathbf{0}$ and must be negative.
- For two similarly charged particles, the potential energy between them is always positive. This is because of the repulsive force between them:
$>$ Two similar charges have less potential energy when they are further apart. They attempt to push each other apart to a distance $\mathbf{R}=\infty$, and have less distance to get there (thus, less potential).
$>$ Since the greatest distance apart is at $\mathbf{R}=\infty$ (where $\left.\mathbf{E}_{\mathbf{p}}=\mathbf{0}\right)$, at any separation distance that is smaller, $\mathbf{E}_{\mathbf{p}}>\mathbf{0}$ (more potential to move apart) and must be positive.

The bottom line is this: use the signs of the charges (,+- ) when calculating potential energy $\mathbf{E}_{\mathbf{p}}$. Opposite charges will reveal a negative $\mathbf{E p}$, while like charges will reveal a positive $\mathbf{E}_{\mathbf{p}}$.

Finally, we can determine the work done on the system when one charge is brought in from infinity towards another charge:

$$
\begin{aligned}
& \rightarrow \quad \mathbf{W}=\Delta \mathbf{E}_{\mathbf{p}}=\mathbf{E}_{2}-\mathbf{E}_{1} \quad \quad-->\text { but } \mathbf{E}_{1} \text { at infinity }=\mathbf{0} \\
& \rightarrow \quad \therefore \mathbf{W}=\mathbf{E}_{2}=\frac{\mathbf{k Q q}}{\mathbf{R}_{2}}
\end{aligned}
$$

Example 9. Find the work done to move a charge (q) from position \#1 to \#2 under the influence of the field of charge $\mathbf{Q}$. $\quad(0.90 \mathrm{~J})$


Example 10. Re-examine the diagram from Example 4 (see below). Find the potential energy of particle $B$ due to the other charges.
$A \bigodot 50 \mu \mathrm{C}$

$50 \mu \mathrm{C}$

This is somewhat similar to the forces question, but since energy is a scalar quantity, you don't use vector diagrams to solve. Instead:

1) find the potential energy between $\mathbf{A}$ and $\mathbf{B}$;
2) find the potential energy between $\mathbf{B}$ and $\mathbf{C}$ : (note: in this case you must include the sign of the negative charge)
3) add the two quantities together to find the total potential energy contained in $\mathbf{B}$.

## Potential Difference between two Point Charges

Now consider a new quantity, the electric potential (V) of a charge $\mathbf{q}$ at a distance $\mathbf{R}$ away from another charge $\mathbf{Q}$. Electric potential is simply the potential energy of the charge $\mathbf{q}$ per elementary charge due to the electric field of $\mathbf{Q}$.

At any location away from a point charge, the electric potential can be determined by

$$
V=\frac{E_{p}}{q}=k \frac{q \mathbf{Q}}{R} \times \frac{\mathbf{1}}{q} \quad \rightarrow \quad V=k \frac{\mathbf{Q}}{R}
$$

$\rightarrow$ where $\mathbf{V}$ is measured in Volts or Joules/Coulomb
Note that electric potential is a scalar quantity, much like potential energy. Its magnitude depends on:
$>$ the size of the charge $\mathbf{Q}$ you are examining;
$>$ the sign of the charge ( + or - );
$>$ the location away from the charge, i.e., distance $\mathbf{R}$.
A similar quantity, potential difference, is defined as the work needed to move a charge of one Coulomb from one location to another in the presence of a field or the work released if the field acts to move the one Coulomb of charge.

Therefore, potential difference or voltage $\left(\mathbf{V}_{\mathbf{a b}}\right.$ or $\left.\Delta \mathbf{V}\right)$ is simply the change in potential energy (or work done) per coulomb of charge as a charge moves between two locations $\mathbf{a}$ and $\mathbf{b}$ in an electric field. That is,

$$
\Delta \mathbf{V}=\frac{\Delta \mathbf{E}_{\mathrm{p}}}{\mathbf{q}}
$$

Note that since $\Delta \mathbf{E}_{\mathbf{p}}=\mathbf{W}=\mathbf{F d}$, the above equation can be rewritten as

$$
\begin{array}{ll}
\Delta \mathbf{V}=\frac{\mathbf{F d}}{\mathbf{q}} & \rightarrow \text { and since } \mathbf{E}=\frac{\mathbf{F}}{\mathbf{q}} \text { (on your formula sheet), } \\
\Delta \mathbf{V}=\mathbf{E d} & \text { or } \quad \mathbf{E}=\frac{\Delta \mathbf{V}}{\mathbf{d}}
\end{array}
$$

This formula provides an alternate method to calculate electric field strength, with units being Volts/metre ( $\mathrm{V} / \mathrm{m}$ ).

To find the potential difference between two locations $\mathbf{a}$ and $\mathbf{b}$ in an electric field:

$\Delta \mathbf{V}=\mathbf{V}_{\mathrm{ab}}=\mathbf{V}_{\mathrm{b}}-\mathbf{V}_{\mathrm{a}} \quad$ where $\mathbf{V}=\mathbf{k} \frac{\mathbf{Q}}{\mathbf{R}} \quad \rightarrow \Delta \mathbf{V}=\mathbf{V}_{\mathrm{ab}}=\mathrm{k} \frac{\mathbf{Q}}{\mathbf{R}_{\mathrm{b}}}-\mathbf{k} \frac{\mathbf{Q}}{\mathbf{R}_{\mathrm{a}}}$
Remember, to get work done to move another charge $\mathbf{q}$ from $\mathbf{Q}$, use:

$$
\mathbf{W}=\Delta \mathbf{E}_{\mathbf{p}}=\mathbf{q} \Delta \mathbf{V}
$$

Example 11. What is the potential difference between the two positions in the following example?

(see Electrostatics Ex 11 for answer)

Example 12. Re-examine the diagram from Example 8 (see below). Find the electric potential at point $B$ due to the other charges. Hint: remember, electric potential is a scalar quantity. No vector analysis is needed here.

(see Electrostatics Ex 12 for answer)

## Constant Electric Fields

So far, we have looked at electric fields due to small point charges. Like the gravity of planets and stars in space, the electric field strength of any charged particle decreases with distance, as indicated by the equation

$$
\mathbf{E}=\mathbf{k} \frac{\mathbf{Q}}{\mathbf{R}^{2}} \quad \text { where } \quad \mathbf{E} \boldsymbol{\alpha} 1 / \mathbf{R}^{2}
$$

In fact, each of the formulas for $\mathbf{F}_{\mathbf{E}}, \mathbf{E}_{\mathbf{p}}$ and $\mathbf{V}$ show that all of these quantities decrease with increased distance $\mathbf{R}$ from a given charged particle.

Now we will examine the field between two oppositely charged parallel plates. The field is uniform everywhere between the plates. If a charged particle is placed in such a field (as shown in the diagram below), it will "fall" away from the repelling plate and move towards the attracting plate.


Because the force on a charge is $\mathbf{F}=\mathbf{q E}$, and since the electric field E is constant between the plates, the force on a charge is the same wherever that charge is located between the plates. In other words, ' $\mathbf{R}$ ' is irrelevant!

Now examine a single positive test charge moved from plate $\mathbf{A}$ to plate $\mathbf{B}$ as shown in the following diagram:


Since the test charge has been moved across the space between the plates, against the field, work has been done against the field, producing a gain in potential energy, shown as $\mathbf{W}=\Delta \mathbf{E}_{\mathbf{p}}=\mathbf{F d}$.

This knowledge can be used to find the electric field strength between the plates. Recall that potential difference or voltage $\left(\mathbf{V}_{\mathbf{a b}}\right.$ or $\left.\Delta \mathbf{V}\right)$ is simply the change in potential energy per Coulomb of charge as a charge moves between two charged plates in a uniform electric field.

From the definition, voltage $\quad \Delta \mathbf{V}=\frac{\Delta \mathbf{E}_{\mathbf{p}}}{\mathbf{q}}=\frac{\mathbf{F d}}{\mathbf{q}} \quad$ and $\quad \mathbf{E}=\frac{\mathbf{F}}{\mathbf{q}}$
$\rightarrow$ so $\Delta \mathbf{V}=\mathbf{E d}$ or $\quad \mathbf{E}=\frac{\Delta \mathbf{V}}{\mathbf{d}}$
The units for electric field strength: Volts/metre (V/m) ...yeah, I know this was already done but its good review so what the hey...

Example 13. A charged particle of $8.0 \times 10^{-19} \mathrm{C}$ is held stationary inside an electric field produced by two electric plates. The voltage between the plates is 270 V and they are separated by a distance of $6.0 \times 10^{-\mathbf{3}} \mathrm{m}$.

a) What constant electric field strength exists between the plates?
b) What is the mass of the particle? Hint: first draw a f.b.d. of the particle to determine its weight.

## Conservation of Energy in a Constant Electric Field

Whenever a body falls, its potential energy is changed to kinetic energy; that is,

$$
\Delta \mathbf{E}_{\mathrm{p}}=\Delta \mathbf{E}_{\mathbf{k}}
$$

An expression for $\Delta \mathbf{E}_{\mathbf{p}}$ when a charge $\mathbf{q}$ "falls" between two charged plates in a uniform field is derived from:

$$
\Delta V=\frac{\Delta \mathbf{E}_{p}}{\mathbf{q}} \rightarrow \Delta \mathbf{E}_{\mathrm{p}}=\mathbf{q} \Delta V
$$

where $\Delta \mathbf{V}$ is the voltage between the charged plates.
Meanwhile: $\Delta \mathbf{E}_{\mathrm{k}}=\frac{1}{2} \mathrm{mv}_{\mathrm{f}}{ }^{2}-\frac{1}{2} \mathrm{mv}_{\mathrm{i}}{ }^{2}$
$\rightarrow$ therefore $\quad \mathbf{q} \Delta V=\frac{1}{2} \mathrm{mv}_{\mathrm{f}}{ }^{2}-\frac{1}{2} \mathrm{mv}_{\mathrm{i}}{ }^{2}$
If the charged particle "falls" from rest from one plate to another, then the initial $\mathrm{E}_{\mathrm{k}}=\mathbf{0}$.
$\rightarrow$ we therefore get $\quad \mathbf{q} \Delta \mathbf{V}=\frac{\mathbf{1}}{\mathbf{2}} \mathbf{m v}_{\mathbf{f}}{ }^{2}$
Formulas (1) and (2) are common derivations for calculating the speed of charged particles in a constant field. Make sure you understand this process.

Also note that the voltage $\Delta \mathbf{V}$ is sometimes referred to as accelerating voltage, because it is responsible for causing particles to accelerate while passing through the electric field.

Example 14. A proton travelling at $3.4 \times 10^{6} \mathrm{~m} / \mathrm{s}$ passes through an electric field as shown below. How fast will the proton be going after it emerges from the field?

(see Electrostatics Ex 14 for answer)

## Projectile Motion in a Constant Electric Field

Now consider what happens when a charged particle enters the field between two charged plates from the side. Recall that the force on the charge is constant no matter where it is between two charged electric plates. If an electron travels horizontally through these plates, the electric force on the charge will act just like gravity acts on a projectile fired horizontally off a cliff - i.e. creating a parabolic path.

Examine the diagram below:


A force is exerted on the electron in the y-direction, upward (due to the electric field that acts downward). To determine the amount of deflection $\mathbf{d}_{\mathbf{y}}$ that takes place, start with $\quad \mathbf{F}_{\mathbf{y}}=\mathbf{m a}_{\mathbf{y}}$.

Note that the electric force is the only significant force causing an acceleration in the $y$-direction, so that

$$
\mathbf{a}_{\mathrm{y}}=\frac{\mathbf{F}_{\mathrm{y}}}{\mathbf{m}}
$$

$\rightarrow$ substitute in

$$
\mathbf{F}_{\mathbf{y}}=\mathbf{q} \mathbf{E} \quad \text { and } \quad \mathbf{E}=\frac{\Delta \mathbf{V}_{\mathrm{d}}}{\mathbf{d}}
$$

where $\Delta \mathbf{V}_{\mathrm{d}}$ is the deflecting voltage.
Solve for $\mathbf{a}_{\mathbf{y}}$ and apply some kinematics (remember to list horizontal and vertical components separately) to determine the deflection $\mathbf{d}_{\mathbf{y}}$.

Example 15. An electron travelling at $4.6 \times 10^{7} \mathrm{~m} / \mathrm{s}$ enters a constant electric field between two charged plates spread 0.0085 m apart, as shown below. The voltage between the plates is 650 V and the plates are 0.036 m long.

a) What is the electric force acting on the electron?
b) How much time is taken for the electron to pass through the plates?
c) How far will the electron "fall" from its path while in-between the two plates?
Hint: for b) and c), you'll have to examine horizontal and vertical components, just like for objects fired horizontally off a cliff.
(see Electrostatics Ex 15 for answer)

## The Cathode Ray Tube (CRT)

This device is a simplified version of a television tube that sends a beam of electrons (a cathode ray) from one end of the tube to another, striking a phosphorescent screen on the other end. In cross-section, the CRT appears as follows:


Parts of the CRT and their functions include the following:

## The Electron Gun

At one end, an electron gun produces an electron beam, also called a cathode ray in the following way:
Charged parallel plates have an accelerating electric field between them. The negative plate may be called a cathode, the positive plate, an anode. If the cathode is heated to a high temperature, electrons will "boil" off its surface, creating a cloud of electrons (called a space charge) over the surface of the hot cathode. The electric field between anode and cathode will subsequently accelerate the electrons in the space charge towards the anode. If there is a hole in the anode, the electrons will not stop but pass through, creating an electron beam. All TV's and cathode ray tubes use this electron gun.


Between anode and cathode, potential energy is converted to kinetic energy so that

$$
\mathbf{q} \Delta \mathbf{V}=\frac{1}{2} \mathbf{m v}^{2} \quad \ldots->\text { which becomes } \quad \mathbf{v}=\sqrt{\frac{2 \mathbf{q} \Delta \mathbf{V}_{\mathbf{a}}}{\mathbf{m}}}
$$

Note that $\Delta \mathbf{V}_{\mathbf{a}}$ stands for accelerating voltage, the voltage between cathode and anode in the electron gun.

Once through the anode hole, $\mathbf{v}$ remains constant until some other force acts on the beam of electrons. This force is caused by a second deflecting voltage $\Delta \mathbf{V}_{\mathbf{d}}$ acting on another set of plates, oriented so that the electrons are deflected up or down as they pass through the plates (similar to what you saw in the last section).


The last two sections explained how to solve relevant problems involving accelerating voltage plates and deflecting voltage plates. Also, a relationship between deflection $\mathbf{d}_{\mathbf{y}}$ and the two voltages involved can be derived, and proven through experimental analysis to be

$$
\mathbf{d}_{\mathbf{y}}=\mathbf{k} \frac{\Delta \mathbf{V}_{\mathrm{d}}}{\Delta \mathbf{V}_{\mathrm{a}}} \rightarrow \text { be sure to memorize this relationship! }
$$

Example 16. Given this information:
$\mathrm{V}_{\mathrm{a}}=100 \mathrm{~V} \quad$ distance between $Y$-plates $=\mathbf{0 . 0 4 0} \mathrm{m}$
$V_{d}=10.0 \mathrm{~V} \quad$ length of $Y$-plates $=0.100 \mathrm{~m}$
a) use accelerating voltage $V_{a}$ to find electron velocity in the $x$ direction $v_{x}$ after leaving the anode.
b) since $v_{\mathbf{x}}$ is constant after leaving the anode, calculate the time taken for an electron to pass through the deflecting Y-plates.
c) use deflecting voltage $\mathbf{V}_{\mathbf{d}}$ to find the force $\mathrm{F}_{\mathbf{y}}$ on the electron between the Y-plates.
d) find the acceleration $a_{y}$ of the electron between the Y-plates.
e) At this point, you have enough kinematics information to find the $y$-deflection $d_{y}$ between the $Y$-plates.
f) If the accelerating voltage is now doubled, while the deflecting voltage is reduced to $3 / 4$ of its original value, what is the new magnitude for $\mathbf{d}_{\mathbf{y}}$ ?

