Impulse and Momentum

Recall from Newton's 1st Law: *inertia* is the tendency of an object to keep on doing what its already doing, that is:

- either remaining stationary, or:
- travelling at a constant velocity.

We can describe the inertia of a moving object as its *momentum*. In essence, momentum is the quantity of motion possessed by an object. It is like the 'unstopability' of the mass, proportional to both the mass of a moving object and its speed, and described by the equation

$$\mathbf{p = mv} \qquad \text{with units being } \frac{\mathbf{kg\ m}}{\mathbf{sec}}.$$

For example, if a **1000 kg** car moves at **40 m/s**, its momentum is

$$\mathbf{p = mv = 1000(40) = 40\ 000}\ \frac{\mathbf{kg\ m}}{\mathbf{sec}}.$$

Impulse simply refers to a change in momentum, and is usually caused by a change in velocity, as described by

$$\mathbf{\Delta p = m\Delta v}.$$

For example, if a **200 kg** cart speeds up from **20 m/s** to **25 m/s**, the change in momentum of the cart is

$$\mathbf{\Delta p = m\,(v_f - v_i) = 200(25 - 20) = 1000}\ \frac{\mathbf{kg\ m}}{\mathbf{sec}}.$$

Since impulse involves a change in velocity, an acceleration and net force must take place.

From $\mathbf{F_{Net} = ma}$ and substituting $\mathbf{a = \frac{\Delta v}{\Delta t}}$ for acceleration,

we get $\mathbf{F_{Net} = m\frac{\Delta v}{\Delta t}}$ which simplifies to produce

$$\mathbf{F\Delta t = m\Delta v = \Delta p}.$$

In describing impulse, $\mathbf{F\Delta t}$ measures how long and how hard we must push to change a motion. Meanwhile, $\mathbf{m\Delta v}$ is the resultant change in the quantity of motion possessed by the mass.

An important note: impulse can be measured in both $\mathbf{\frac{kg\ m}{sec}}$ or in **Newton-seconds** (**N-s**). However, momentum can't be measured in terms of force-time, because an object can travel at constant momentum without any net force acting on it. This means that units of force (Newtons) can't be used to help describe it.

Example #1: If a force of 250 N acts on a 50 kg mass for 10.0 seconds, what is the increase in velocity?

(see Momentum Ex 1 for answer)

Example #2: If a 2000 kg car is travelling 22 m/s along the highway and applies a force of 500 N for 12 sec to pass another car, what is the new velocity?

(see Momentum Ex 2 for answer)

Example #3: A 1000 kg car, travelling at 22 m/s strikes a concrete bridge support and comes to a complete halt in 0.50 seconds.

a) What average force acts on the car?

b) If the support was cushioned so that it took 3.0 sec. to stop, what is the force now?

c) Explain the significance of these two very different values.

(see Momentum Ex 3 for answer)

Example #4:

a) Calculate the impulse suffered by a 105 kg man who lands on firm ground after jumping from a height of 1.5 m.

b) What force would be exerted on the man if he bent his knees and absorbed the fall in 0.40 s ?

(see Momentum Ex 4 for answer)

Example #5: A 0.0030 kg bullet travelling with a velocity of 800. m/s is fired into a 0.50 kg box of sand that is at rest on a horizontal, frictionless surface. The bullet passes through the sand and emerges with a velocity of 200. m/s at the other side.

a) Find the impulse delivered to the sand.
b) Find the velocity of the box of sand after the collision.
c) If the bullet was inside the box for 0.020 s, find the average force exerted on the block.

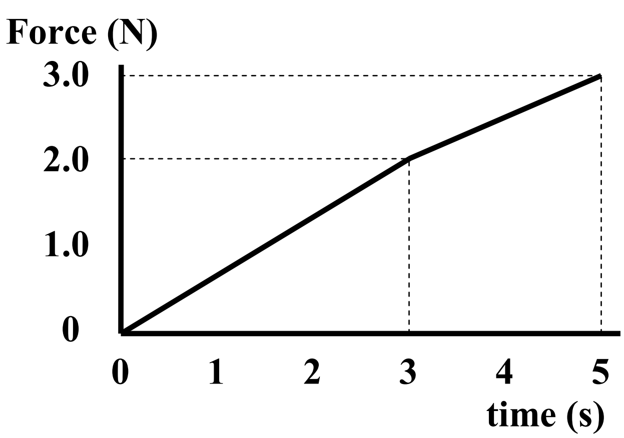
(see Momentum Ex 5 for answer)

Example #6: A 1.2 kg ball is thrown towards a brick building at 23 m/s. Find the impulse delivered to the ball if:

a) the ball shatters and goes straight through a window, slowing to 17 m/s.
b) the ball hits the brick wall and rebounds straight back at 19 m/s.

(see Momentum Ex 6 for answer)

Example #7: A changing net force acts on a 3.5 kg cart for 5.0 s, and is recorded on a Force-Time graph:



a) What is the total impulse?
b) What final velocity would the 3.5 kg mass have, travelling in a straight line, if its initial speed was 4.0 m/s?

(see Momentum Ex 7 for answer)

The Law of Conservation of Momentum

Recall Physics 11: you learned (and perhaps forgot?) that the sum of momentums of all objects before an event such as an explosion or a collision = the sum of momentums of all objects after the event has occurred. Even though momentum is a vector quantity, this was an easy task, since only motion in a straight line was considered.

Formally, the Law of Conservation of Momentum states that the total vector momentum of all bodies before an event = the total vector momentum of all the bodies after the event. As with all vector problems in this course, components need to be considered in order to deal with this situation.

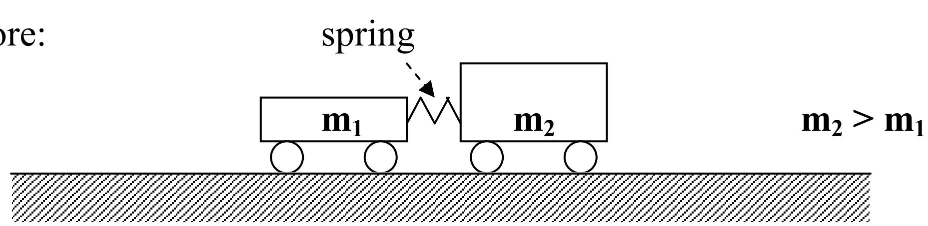
For now, we'll review the positive-negative types of momentum from Physics 11.

Explosions

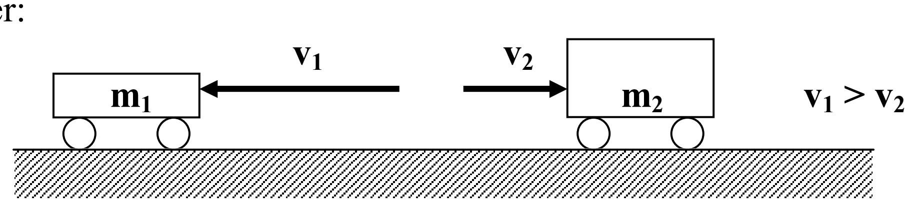
In an explosion involving a system that is initially stationary, the total momentum before the explosion event is zero. Therefore, the vector adding of all particles after the explosion must also = 0.

When only 2 particles explode apart, they must move in opposite directions along the same straight line. This is because the momentum of one particle must be opposite to the momentum of the other particle. Note that if one particle is larger than the other, its opposing velocity must be smaller than the other.

- Before:



- After:



total momentum before = total momentum after, so

$$0 = m_1v_1 + m_2v_2$$

Example #8: A 3.5 kg cart explodes away from a 5.0 kg cart at 2.0 m/s. What is the velocity of the larger cart?

(see Momentum Ex 8 for answer)

Example #9: The nucleus of a certain atom has a mass of 3.8×10^{-25} kg and is at rest. The nucleus is radioactive and ejects a particle of mass 6.6×10^{-27} kg and speed 1.5×10^{7} m/s. Find the recoil velocity of the remaining nuclear mass left behind.

(see Momentum Ex 9 for answer)

Collisions (still one-dimensional motion)

There are numerous types of collision problems to consider; some examples are shown below. Note that negative velocities must be indicated when objects approach from opposite directions, or change directions because of the impact.

- For a moving object colliding with a stationary object, the moving object $\mathbf{m_1}$ has the only momentum before the collision, and both objects (usually) have momentum after the collision.

total momentum before = total momentum after

$$m_1v_1 = m_1v_1' + m_2v_2'$$

Example #10: A 3.0 kg lab cart going 15 m/s runs into a 10 kg stationary lab cart so that it takes a speed of 6.0 m/s. What is the velocity of the 3.0 kg lab cart after the collision?

(see Momentum Ex 10 for answer)

- When both objects are moving before the collision, they both have a 'before' momentum. This is true when one object catches up to and rear-ends another, or when both objects collide head-on. If both objects bounce off each other after the collision, each one will have an 'after' momentum.

total momentum before = total momentum after

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Example #11: A 2.0 kg lab cart moving at 3.0 m/s catches up to and rear ends a 1.0 kg cart moving at 0.50 m/s. After the collision, the 2.0 kg cart follows the 1.0 kg cart at 1.2 m/s. Find the new velocity of the 1.0 kg cart.

(see Momentum Ex 11 for answer)

- When two objects collide in a completely *inelastic* collision, they will stick and move off together, acting as a single, combined mass.

total momentum before = total momentum after

$$\mathbf{m_1v_1 + m_2v_2 \quad = \quad (m_1 + m_2)v'}$$

→where **v'** is the common velocity after the collision.

Example #12: A 5000. kg truck going at 15 m/s makes a head on collision with a 1000. kg Honda that was going 20 m/s. The wreckage sticks together. Find the velocity of the wreckage just after the collision.

(see Momentum Ex 12 for answer)

Impulse in 2-Dimensions

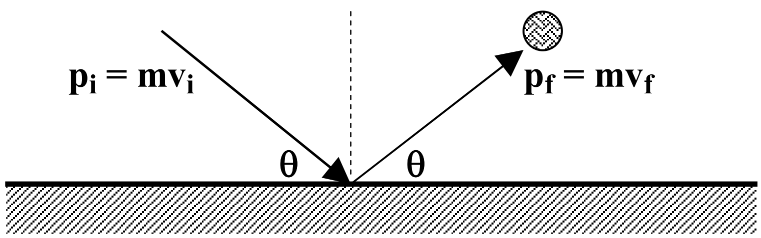
Momentum is a vector quantity. When the velocities are not along the same line (i.e., positive or negative), vector adding must be done. In the following case, a ball with an initial momentum is given an impulse by the wall that it strikes, and this results in a new final momentum as well as a change in the ball's direction.

Remember that impulse is equal to change in momentum, so

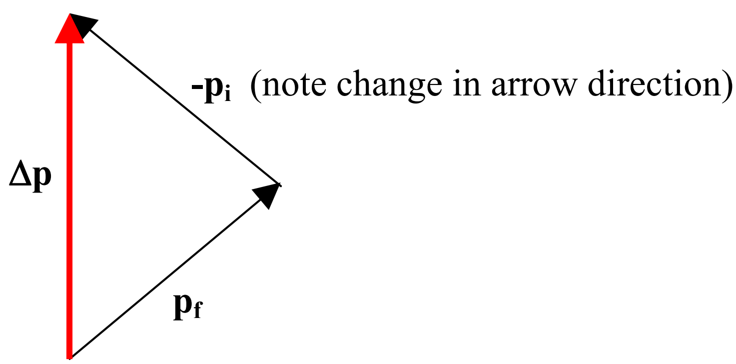
$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{p}_f + -\mathbf{p}_i$$

Using vector analysis, this equation shows that impulse is the resultant of the vector addition of $\mathbf{p}_f$ and $-\mathbf{p}_i$.

Start with a diagram of the situation, showing initial and final momentum vectors:



Next, vector-add $\mathbf{p}_f + -\mathbf{p}_i$ and draw the resultant vector $\Delta\mathbf{p}$:



Note that the impulse acts perpendicular to the surface of impact. In fact, the impulse acts in the same direction as the force of the surface on the object. This should not be surprising, since

$$\Delta \mathbf{p} = \mathbf{F}\Delta \mathbf{t}$$

Example #13: A 2.0 kg ball going 10 m/s bounces off a wall at an angle of 40º to the wall. (Both incoming and outgoing angles are 40º). After the bounce the speed is still 10 m/s.

a) What is the impulse on the ball?

b) What is the change in velocity?

(see Momentum Ex 13 for answer)

Finally, the formula $\Delta \mathbf{p} = \mathbf{p_f} - \mathbf{p_i}$ can be re-arranged to read $\mathbf{p_f} = \mathbf{p_i} + \Delta\mathbf{p}$. In other words, the vector addition of initial momentum and impulse produces the resultant final momentum. Use this approach on problems where both initial momentum and impulse are given.

<u>Conservation of Momentum in Two Dimensions</u>

Note the slight difference in the conservation of momentum statement:

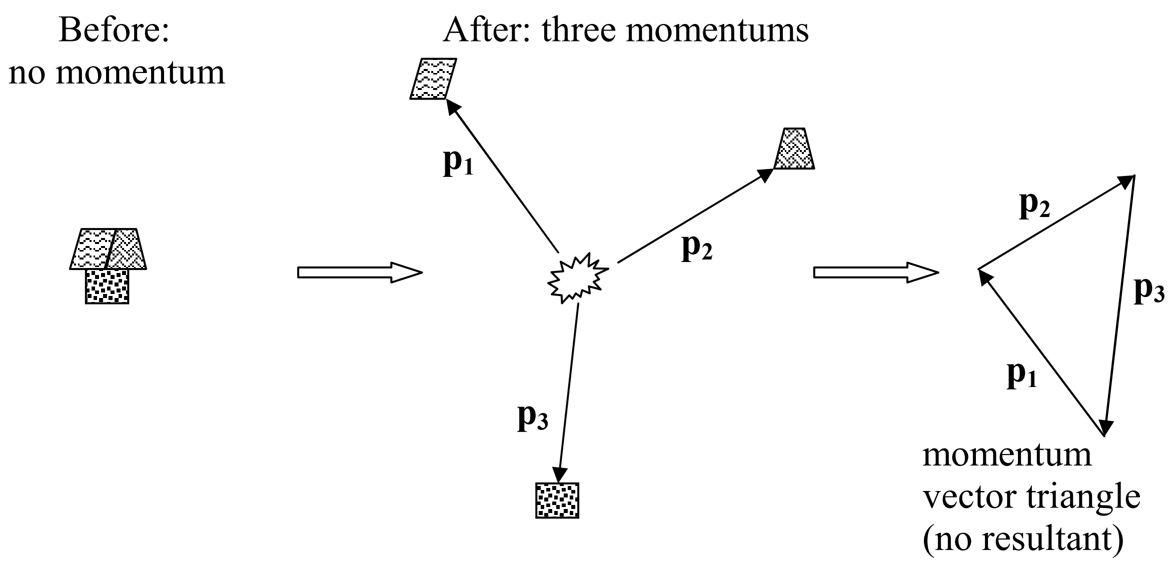
total <u>vector</u> momentum before = total <u>vector</u> momentum after

In other words, vector addition of momentum quantities is needed to solve for unknown values.

<u>Explosions</u>

Before exploding, a stationary object has zero momentum. Therefore, the <u>vector sum</u> of the momentums of all particles after the explosion must also equal zero. There is <u>no</u> resultant momentum vector.

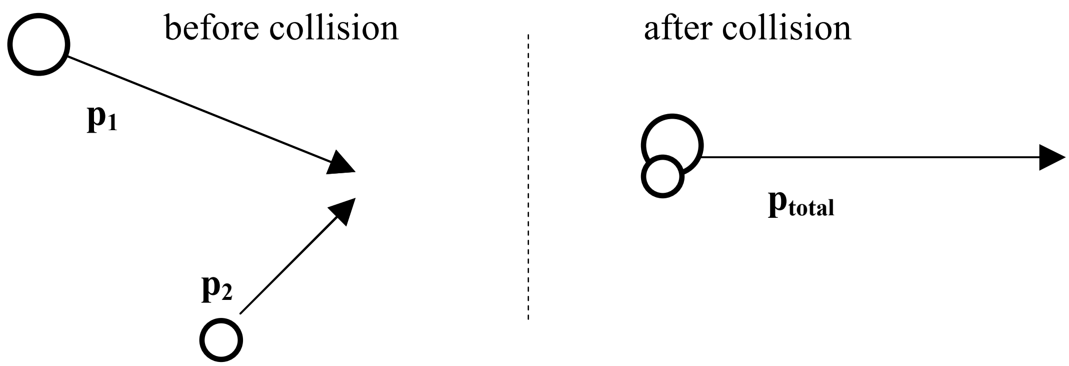
If there are three masses in the explosion, adding the momentums will form a <u>*closed*</u> triangle of momentum vectors.



Example #14: An object, at rest, explodes into three pieces, each travelling parallel to the ground. The first piece has a mass of 3.0 kg and travels at 4.0 m/s (30º N of E). The second piece has a mass of 4.0 kg and travels 3.0 m/s (30º S of E). Find the speed and direction of the third piece if its mass is 5.0 kg.

(see Momentum Ex 14 for answer)

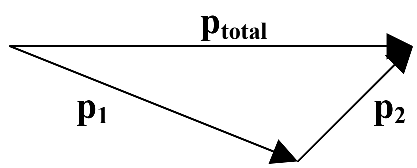
<u>Oblique Collision, two particles sticking together</u>.
When two masses collide and stick together, a third momentum vector is formed. From conservation of momentum, the sum of the two vectors before the collision must equal the single <u>resultant</u> momentum vector after the collision.



total <u>vector</u> momentum before = total <u>vector</u> momentum after

$$\mathbf{p_1 + p_2 \quad = \quad p_{total}}$$

This forms a momentum vector triangle:



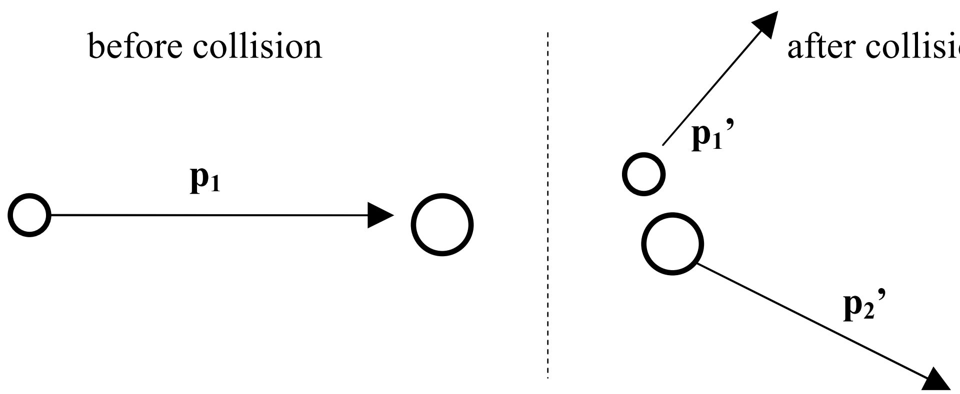
Use sine and/or cosine laws to solve for required values.

Example #15: A 100 kg football player going 3.0 m/s north, tackles another player of mass 150 kg going 1.5 m/s east. The players entangle. What is their combined speed and direction?

(see Momentum Ex 15 for answer)

Oblique collision, moving mass strikes a stationary mass.

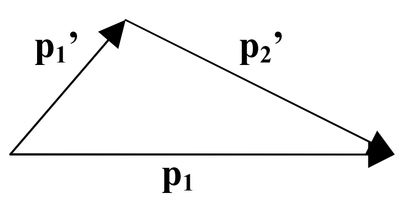
From conservation of momentum, the single vector momentum of the projectile mass before must equal the sum of the vector momentums of the projectile mass and target mass after. This forms another momentum triangle, though the set-up is slightly different from the previous example.



total vector momentum before = total vector momentum after

$$p_1 = p_1' + p_2'$$

Adding the two "after" vectors equals the first projectile vector:



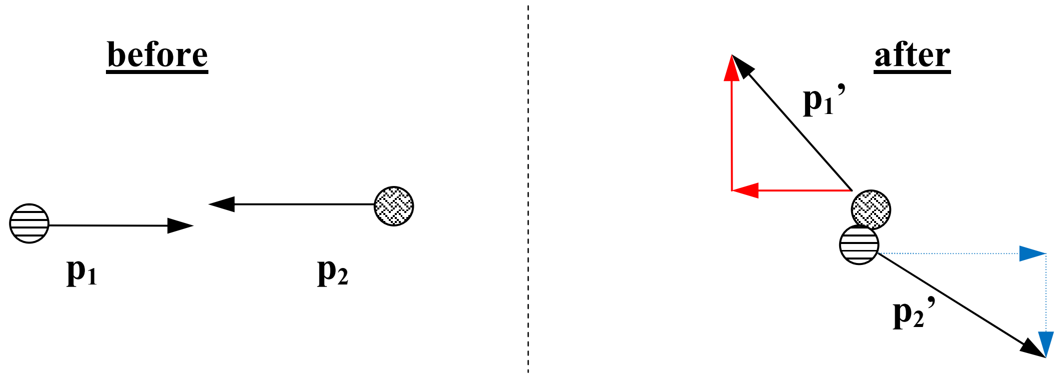
Again, use sine and/or cosine laws to solve for required values.

Example #16: A 2.0 kg ball going 5.0 m/s strikes a stationary 4.0 kg ball. After the collision, the second ball goes off at 1.11 m/s at 54° from the direction of the original ball. What is the speed and direction of the first ball?

(see Momentum Ex 16 for answer)

Note: If *more* than three vectors are involved in a conservation of momentum problem, parallel and perpendicular components are used to solve for the required quantities.

Consider two balls that collide obliquely:



Draw components in
this diagram.

- Start with: **total momentum before = total momentum after**
- Solve for both $\mathbf{p_x}$ components and $\mathbf{p_y}$ components:

X-components	**Y-components**
before = after	**before = after**

- Then use the net components to find the resultant momentum.

Example #1: If a force of 250 N acts on a 50 kg mass for 10.0 seconds, what is the increase in velocity?

$$\Delta p = Ft = 250(10.0)$$

$$= 2500 \text{ N}\cdot\text{s}$$

$$\Delta p = m\Delta v$$

$$2500 = 50\Delta v$$

$$\boxed{\Delta v = 50. \text{ m/s}}$$

Example #2: If a 2000 kg car is travelling 22 m/s along the highway and applies a force of 500 N for 12 sec to pass another car, what is the new velocity?

$$\Delta p = Ft = 500(12)$$

$$= 6000 \ N \cdot s.$$

$$\Delta p = m(v_f - v_i)$$

$$6000 = 2000(v_f - 22)$$

$$\boxed{v_f = 25 \ m/s}$$

Example #3: A 1000 kg car, travelling at 22 m/s strikes a concrete bridge support and comes to a complete halt in 0.50 seconds.

a) **What average force acts on the car?**
b) **If the support was cushioned so that it took 3.0 sec. to stop, what is the force now?**
c) **Explain the significance of these two very different values.**

a) $\Delta p = m(v_f - v_i)$

$= 1000(0-22)$

$= -22000 \text{ N}\cdot\text{s}$

$\Delta p = Ft \qquad -22000 = F(.50)$

$\boxed{F = -4.4 \times 10^4 \text{ N}}$

b) $\Delta p = -22000 \text{ N}\cdot\text{s}$ → unchanged, because Δv is the same.

$\Delta p = Ft \qquad -22000 = F(3.0)$

$\boxed{F = -7.3 \times 10^3 \text{ N}}$

c) Since Δp is constant,

$F \propto \frac{1}{t}$, so as t increases, F decreases.

Example #4:

a) Calculate the impulse suffered by a 105 kg man who lands on firm ground after jumping from a height of 1.5 m.

b) What force would be exerted on the man if he bent his knees and absorbed the fall in 0.40 s?

a) → use kinematics to find impact speed with the ground:

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2ad$$

$$v_f = \sqrt{2(-9.8)(-1.5)} = 5.42 \text{ m/s}$$

$$\Delta p = m(v_f - v_i)$$

$$= 105(0 - 5.42)$$

note! initial speed of impulse delivered by the ground.

$$\boxed{\Delta p = -5.7 \times 10^2 \ \frac{\text{kg}\cdot\text{m}}{\text{s}}}$$

b) $$\Delta p = Ft$$

$$-569 = F(.40)$$

$$\boxed{F = -1.4 \times 10^3 \text{ N}}$$

Example #5: A 0.0030 kg bullet travelling with a velocity of 800. m/s is fired into a 0.50 kg box of sand that is at rest on a horizontal, frictionless surface. The bullet passes through the sand and emerges with a velocity of 200. m/s at the other side.

a) **Find the impulse delivered to the sand.**
b) **Find the velocity of the box of sand after the collision.**
c) **If the bullet was inside the box for 0.020 s, find the average force exerted on the block.**

a) $\Delta p = m(v_f - v_i)$

$= .0030(200 - 800)$

$$\boxed{\Delta p = -1.8 \frac{kg \cdot m}{s}}$$

b) impulse lost by bullet = impulse gained by block

$\therefore \Delta p_{sand} = 1.8 \frac{kg \cdot m}{s}$

$\Delta p = m(v_f - v_i)$ $\quad 1.8 = .50(v_f - 0)$

$$\boxed{v_f = 3.6 \text{ m/s}}$$

c) $\Delta p = Ft$ $\quad 1.8 = F(.020)$

$$\boxed{F = 90 \text{ N}}$$

Example #6: A 1.2 kg ball is thrown towards a brick building at 23 m/s. Find the impulse delivered to the ball if:

a) **the ball shatters and goes straight through a window, slowing to 17 m/s.**
b) **the ball hits the brick wall and rebounds straight back at 19 m/s.**

a) $\Delta p = m(v_f - v_i)$

$= 1.2(17 - 23)$

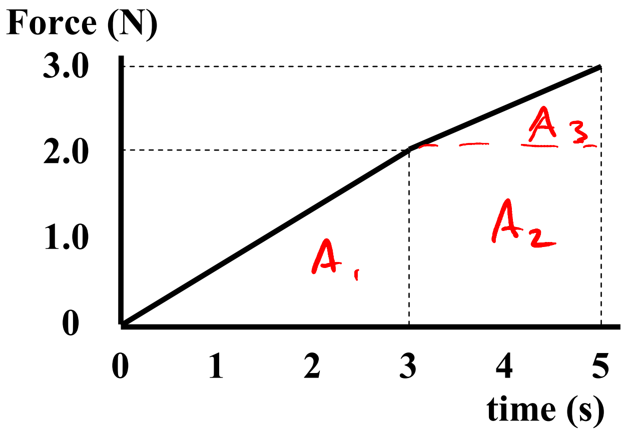
$$\boxed{\Delta p = -7.2 \ \frac{kg \cdot m}{s}}$$

NOTE!!

b) $\Delta p = 1.2(-19 - 23)$

$$\boxed{\Delta p = -5.0 \times 10 \ \frac{kg \cdot m}{s}}$$

Example #7: A changing net force acts on a 3.5 kg cart for 5.0 s, and is recorded on a Force-Time graph:



a) What is the total impulse?

b) What final velocity would the 3.5 kg mass have, travelling in a straight line, if its initial speed was 4.0 m/s?

a) Since $\Delta p = Ft$, Δp = area under the graph

$$\Delta p = A_1 + A_2 + A_3$$
$$= \frac{1}{2}(3)(2) + 2(2) + \frac{1}{2}(2)(1)$$

$$\boxed{\Delta p = 8.0 \text{ N}\cdot\text{s}}$$

b) $\Delta p = m(v_f - v_i)$

$$8.0 = 3.5(v_f - 4.0)$$

$$\boxed{v_f = 6.3 \text{ m/s}}$$

Example #8: A 3.5 kg cart explodes away from a 5.0 kg cart at 2.0 m/s. What is the velocity of the larger cart?

→ use conservation of momentum:

before

$P_T = 0$

after

$P_T = 3.5(2) + 5.0v$

$0 = 7 + 5v$

$\boxed{v = -1.4 \text{ m/s}}$

(1.4 m/s in the opposite direction)

Example #9: The nucleus of a certain atom has a mass of 3.8×10^{-25} kg and is at rest. The nucleus is radioactive and ejects a particle of mass 6.6×10^{-27} kg and speed 1.5×10^{7} m/s. Find the recoil velocity of the remaining nuclear mass left behind.

before

$p_T = 0$

after

$$p_T = (6.6\times10^{-27})(1.5\times10^{7}) + \underbrace{\left[(3.8\times10^{-25}) - (6.6\times10^{-27})\right]}_{\text{the remaining mass}} v$$

$$0 = 9.9\times10^{-20} + 3.734\times10^{-25}\, v$$

$$\boxed{v = -2.7\times10^{5}\ \text{m/s}}$$

Example #10: A 3.0 kg lab cart going 15 m/s runs into a 10 kg stationary lab cart so that it takes a speed of 6.0 m/s. What is the velocity of the 3.0 kg lab cart after the collision?

<u>before</u>

$p_T = 3(15)$
$= 45$

<u>after</u>

$p_T = 10(6) + 3v$
$= 60 + 3v$

$45 = 60 + 3v$

$\boxed{v = -5.0 \text{ m/s}}$

Example #11: A 2.0 kg lab cart moving at 3.0 m/s catches up to and rear ends a 1.0 kg cart moving at 0.50 m/s. After the collision, the 2.0 kg cart follows the 1.0 kg cart at 1.2 m/s. Find the new velocity of the 1.0 kg cart.

<u>before</u>

$$p_T = 2(3) + 1(.5)$$
$$= 6.5$$

<u>after</u>

$$p_T = 2(1.2) + 1v$$
$$= 2.4 + v$$

$$6.5 = 2.4 + v$$

$$\boxed{v = 4.1 \text{ m/s}}$$

Example #12: A 5000. kg truck going at 15 m/s makes a head on collision with a 1000. kg Honda that was going 20 m/s. The wreckage sticks together. Find the velocity of the wreckage just after the collision.

"head-on" in opposite direction

before

$$P_T = 5000(15) + 1000(-20)$$

$$= 55000$$

after

$$P_T = (5000+1000)v$$

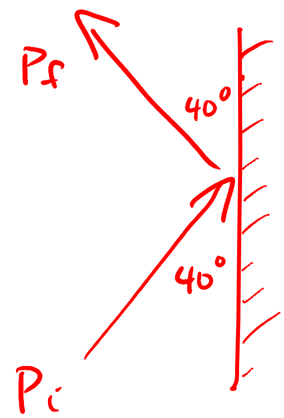
$$= 6000\,v$$

$$55000 = 6000\,v$$

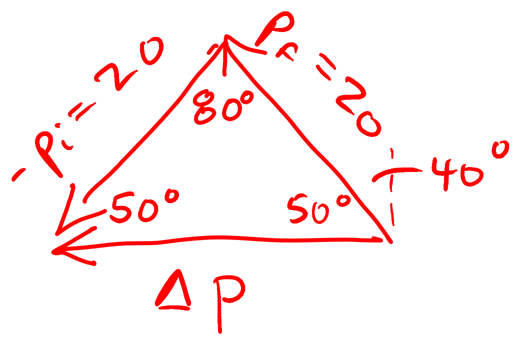
$$\boxed{v = 9.2\ m/s}$$

Example #13: A 2.0 kg ball going 10 m/s bounces off a wall at an angle of 40º to the wall. (Both incoming and outgoing angles are 40º). After the bounce the speed is still 10 m/s.

a) **What is the impulse on the ball?**
b) **What is the change in velocity?**



$$\Delta P = P_f - P_i$$
$$= P_f + -P_i$$



$$P_f = P_i = 2(10)$$
$$= 20$$

$$\frac{\sin 80}{\Delta P} = \frac{\sin 50}{20}$$

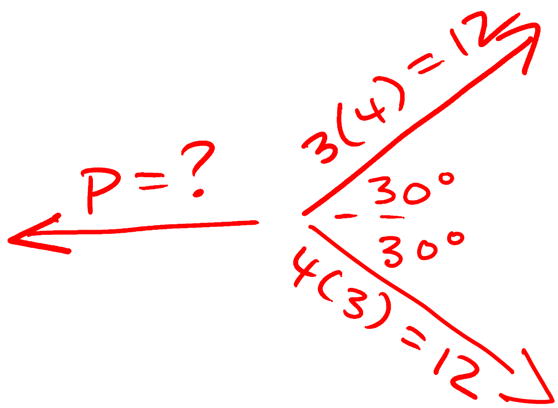
$$\boxed{\Delta P = 26\ \frac{kg \cdot m}{s}}$$

Example #14: An object, at rest, explodes into three pieces, each travelling parallel to the ground. The first piece has a mass of 3.0 kg and travels at 4.0 m/s (30º N of E). The second piece has a mass of 4.0 kg and travels 3.0 m/s (30º S of E). Find the speed and direction of the third piece if its mass is 5.0 kg.

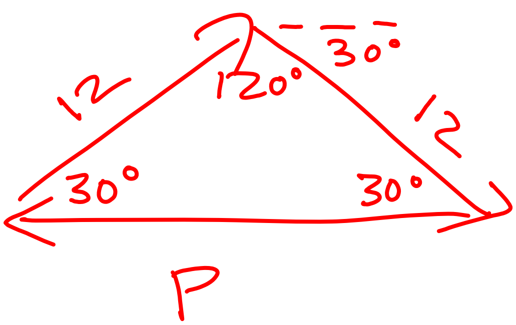
before

$P_T = 0$

after



⇒ vector-sum = a triangle:



⇒ isosceles triangle, so p is due west

$$\frac{\sin 30}{12} = \frac{\sin 120}{P}$$

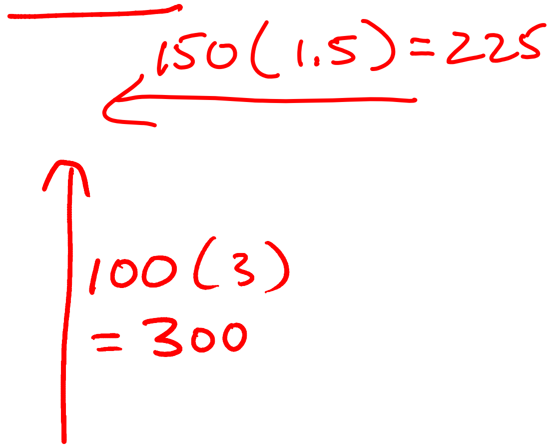
$P = 20.8 \ \frac{kg \cdot m}{s}$

$p = mv \qquad 20.8 = 5v$

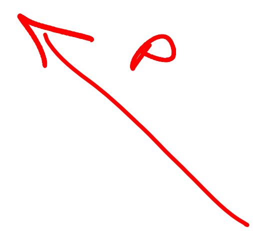
$$\boxed{v = 4.2 \text{ m/s due west}}$$

Example #15: A 100 kg football player going 3.0 m/s north, tackles another player of mass 150 kg going 1.5 m/s east. The players entangle. What is their combined speed and direction?

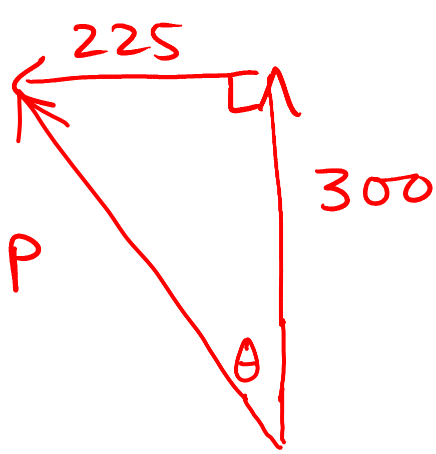
before



after



⇒ vector sum of "before" = 'p' "after"



$p = \sqrt{225^2 + 300^2}$ $\quad p = 375 \ \frac{kg \cdot m}{s}$

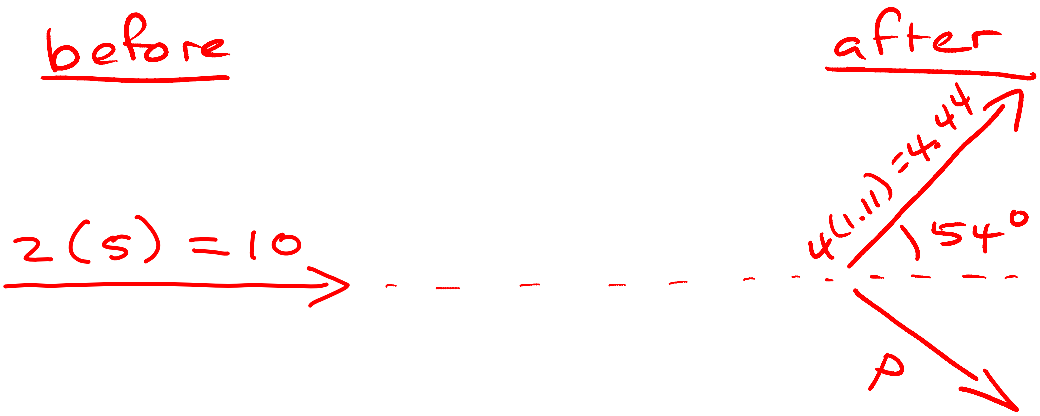
$p = mv \quad 375 = [100 + 150]v \quad v = 1.5 \ m/s$

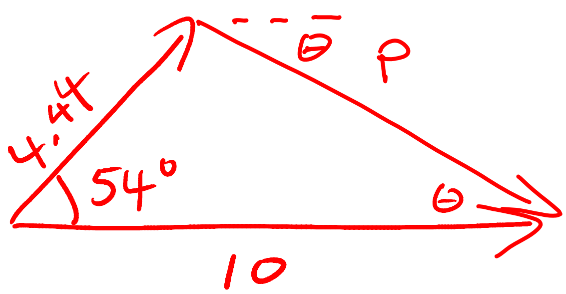
$\theta = \tan^{-1} \frac{225}{300} = 37°$

⇒ $\boxed{v = 1.5 \ m/s \ @ \ 37° \ W \ of \ N}$

Example #16: A 2.0 kg ball going 5.0 m/s strikes a stationary 4.0 kg ball. After the collision, the second ball goes off at 1.11 m/s at 54º from the direction of the original ball. What is the speed and direction of the first ball?

→ since no orientation is given, choose a simple direction for the first moving ball:





$$p^2 = 10^2 + 4.44^2 - 2(10)(4.44)\cos 54$$

$$p = 8.22\ \frac{kg \cdot m}{s}$$

$p = mv$ $\quad 8.22 = 2v \quad v = 4.1\ m/s$

$$\frac{\sin 54}{8.28} = \frac{\sin \theta}{4.44} \qquad \theta = 26°$$

$$\Rightarrow \boxed{v = 4.1\ m/s\ @\ 26°\ \text{from original path}}$$