$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ OR } \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

Introduction Exponent Laws II

a) Negative Exponents

$3^{-5} =$ $\qquad$ $\left(-12\right)^{-4} =$

General Rule:

$\frac{1}{7^{-2}} =$ $\qquad$ $\left(\frac{2}{3}\right)^{-5} =$

b) Rational Exponents

$6^{\frac{1}{2}} =$ $\qquad$ $\left(-5\right)^{\frac{1}{3}} =$

General Rule:

$3^{\frac{4}{5}} =$ $\qquad$ $\sqrt{7^5} =$

Numbers, Radicals, and Exponents
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Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 1

Simplify each of the following expressions. Any variables in your final answer should be written with positive exponents.

a) $(-4)^{-2}$

b) $\left(\frac{3}{2}\right)^{-3}$

c) $\left(\frac{a^2b}{c^3}\right)^{-1}$

d) $\left(3a^3\right)^{-2}$

e) $\left(\frac{3^{-1}}{5}\right)^{-2}$

f) $\frac{5(-4)^0}{2^{-1}}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
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Example 2

Simplify. Any variables in your final answer should be written with positive exponents.

a) $2^3(5)^{-2}$

b) $\dfrac{2^{-3}}{a^4}$

c) $\dfrac{(2a)^3}{(2a)^{-2}}$

d) $\left(a^5\right)^{-\frac{3}{5}}$

e) $\left(\dfrac{a^{-4}}{(ab)^2}\right)^{\frac{3}{2}}$

f) $\left(5a^2\right)^{-\frac{3}{2}}\left(a^{\frac{1}{2}}\right)$

Numbers, Radicals, and Exponents
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Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 3

Simplify each of the following expressions. Any variables in your final answer should be written with positive exponents.

a) $\dfrac{10a^7b^9c^6}{5a^6b^{10}c^8}$

b) $\dfrac{-3a^{-7}b^{-11}}{12a^4b^{-3}}$

c) $\left(\dfrac{2}{5}a^{-3}b^{-1}\right)^{-3}$

d) $\left(\dfrac{4a^2b^3}{8ab^5}\right)^{-2}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
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Example 4

Simplify. Any variables in your final answer should be written with positive exponents. Fractional exponents should be converted to a radical.

a) $\left(a^5\right)\left(a^{-\frac{1}{2}}\right)$

b) $\left(27a^{\frac{1}{2}}\right)^{\frac{2}{3}}$

c) $\left(\frac{9a^{-2}}{16b^{-4}}\right)^{-\frac{3}{2}}$

d) $\left(2^{-\frac{5}{4}}\right)\left(2^{-\frac{4}{3}}\right)$

Numbers, Radicals, and Exponents
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$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 5

Simplify. Any variables in your final answer should be written with positive exponents. Fractional exponents should be converted to a radical.

a) $\dfrac{-20a^{-\frac{2}{3}}b}{4ab^{-\frac{1}{2}}}$

b) $\dfrac{2^{-3}+2^{-4}}{2^{-5}}$

c) $\dfrac{\left(\frac{1}{16}\right)^{\frac{5}{4}}\left(\frac{1}{16}\right)^{-\frac{3}{4}}}{\left(\frac{1}{16}\right)^{-5}\left(\frac{1}{16}\right)^{4}}$

d) $9^{\frac{1}{2}}\left(\dfrac{a^{\frac{3}{4}}}{2b^{-\frac{1}{7}}}\right)^{0}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
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Example 6

Write each of the following radical expressions with rational exponents and simplify.

a) $-\sqrt{a^3}$

b) $\sqrt{\sqrt{a}}$

c) $\sqrt{\sqrt[3]{a}}$

d) $\sqrt{\sqrt[3]{64a^6b^{12}}}$

Numbers, Radicals, and Exponents
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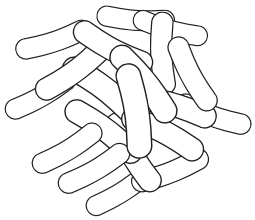
$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 7



A culture of bacteria contains 5000 bacterium cells. This particular type of bacteria doubles every 8 hours. If the amount of bacteria is represented by the letter *A*, and the elapsed time (*in hours*) is represented by the letter *t*, the formula used to find the amount of bacteria as time passes is:

$$A = 5000\left(2\right)^{\frac{t}{8}}$$

a) How many bacteria will be in the culture in 8 hours?

b) How many bacteria will be in the culture in 16 hours?

c) How many bacteria were in the sample 8 hours ago?

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
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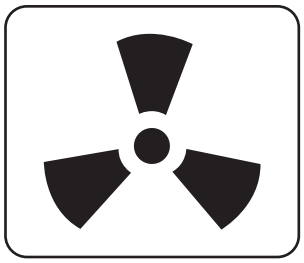
Example 8



Over time, a sample of a radioactive isotope will lose its mass. The length of time for the sample to lose half of its mass is called the *half-life* of the isotope. Carbon-14 is a radioactive isotope commonly used to date archaeological finds. It has a half-life of 5730 years.

If the initial mass of a Carbon-14 sample is 88 g, the formula used to find the mass remaining as time passes is given by:

$$A = 88\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

In this formula, A is the mass, and t is time *(in years)* since the mass of the sample was measured.

a) What will be the mass of the Carbon-14 sample in 2000 years?

b) What will be the mass of the Carbon-14 sample in 5730 years?

c) If the mass of the sample is measured 10000 years in the future, what percentage of the original mass remains?