### 3.6 The Inverse and the Contrapositive

## of Conditional Statements <br> p. 208

Name $\qquad$
Date $\qquad$

Goal: Understand and interpret the contrapositive and inverse of a conditional statement.

1. inverse: A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2 ," the inverse is "If a number is not even, then it is not divisible by 2. ."
2. contrapositive: A statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the contrapositive is "If a number is not divisible by 2 , then it is not even."

| $\mathbf{p}$ | $\mathbf{q}$ | $\boldsymbol{p} \Rightarrow \boldsymbol{q}$ |
| :--- | :--- | :--- |
| T | T | T |
| F | F | T |
| F | T | T |
| T | F | F |

## Communication Notation

In logic notation, the inverse of "if $p$, then $q$ " is written as "If $\neg p$, then $\neg q$."

## Communication Notation

In logic notation, the contrapositive of "if $p$, then $q$ " is written as "If $\neg q$, then $\neg p$."

Example 1: Verifying the inverse and contrapositive of a conditional Statement (p. 209) Consider the following conditional statement: "If today is February 29, then this year is a leap year."
a) Verify the statement, or disprove it with a counterexample.

Hypothesis $(p)$ : $\qquad$ Conditional statement: if $p$, then $q$.
Conclusion (q): $\qquad$

| $p$ | $q$ | $p \Rightarrow q$ |
| :--- | :--- | :--- |
|  |  |  |

b) Verify the converse, or disprove it with a counterexample.
converse: $\qquad$

Hypothesis $(p)$ : $\qquad$ Converse: if $q$, then $p$.
Conclusion (q): $\qquad$

| $q$ | $p$ | $q \Rightarrow p$ |
| :--- | :--- | :--- |
|  |  |  |

c) Verify the inverse, or disprove it with a counterexample.

Inverse:

Hypothesis $(\neg p)$ : $\qquad$ Inverse: if $\qquad$ , then $\qquad$
Conclusion $(\neg q)$ : $\qquad$

| $\neg p$ | $\neg q$ | $\neg p \Rightarrow \neg q$ |
| :--- | :--- | :--- |
|  |  |  |

d) Verify the contrapositive, or disprove it with a counterexample.

Contrapositive: $\qquad$

Hypothesis $(\neg q)$ : $\qquad$
Conclusion $(\neg p)$ : $\qquad$

Contrapositive: if __, then $\qquad$ .

| $\neg q$ | $\neg p$ | $\neg q \Rightarrow \neg p$ |
| :--- | :--- | :--- |
|  |  |  |

Example 2: Examining the relationship between a conditional statement and its contrapositive (p. 210)

Consider the following conditional statement: "If a number is a multiple of 10 , then it is a multiple of 5."
a) Write the contrapositive of this statement.
b) Verify that the conditional and contrapositive statements are both true.

## In Summary:

- You form the inverse of a conditional statement by $\qquad$ the hypothesis and the conclusion.
- You form the converse of a conditional statement by $\qquad$ the hypothesis and the conclusion
- You form the contrapositive of a conditional statement by $\qquad$ the hypothesis and the conclusion of it's $\qquad$ .
- If a conditional statement is true, then it's $\qquad$ is true, and vice versa
- If the inverse of a conditional statement is true, then the $\qquad$ of the statement is also true, and vice versa.

HW: 3.6 p. 214-216 \#1, 5, 6, 7, 9 \& 12

