4.3 Permutations When All Objects Are Distinguishable p. 246

Name ________________

Date ________________

Goal: Determine the number of permutations of *n* objects taken *r* at a time, where $0 \le r \le n$.

The number of permutations from a set of n different objects, where r of them are used in each arrangement, can be calculated using the formula:

$$_nP_r = \frac{n!}{(n-r)!}, \text{ where } 0 \le r \le n$$

Communication | *Notation*

$_nP_r$ is the notation commonly used to represent the number of permutations that can be made from a set of *n* different objects where only *r* of them are used in each arrangement, and $0 \le r \le n$.

When all available objects are used in each arrangement, *n* and *r* are equal, so the notation $_nP_n$ is used.

Example 1: Solving a permutation problem where only some of the objects are used in each arrangement (p. 247)

Matt has downloaded 10 new songs from an online music store. He wants to create a playlist using 6 of these songs arranged in any order. How many different 6-song playlists can be created from his new downloaded songs?

Example 2: Solving a permutation problem involving cases (p. 250)

Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and/or any letters of the alphabet. The password is case sensitive, so she can use both lower- and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

Example 3: Solving a permutation problem with conditions (p. 251)

At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a. The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

b. The three red cars must be parked side by side. How many ways can the seven cars be parked?

Example 4: Comparing arrangements created with and without repetition (p. 252)

A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if each digit can be repeated?

How many SINs can be created if no repetition is allowed?

In reality, the Canadian government does not use 0, 8, or 9 as the first digit when assigning SINs to citizens and permanent residents, and repetition of digits is allowed. How many nine-digit SINs do not start with 0, 8, or 9?

In Summary

Key Ideas

- The number of permutations from a set of n different objects, where r of them are used in each arrangement, can be calculated using the formula

$$_nP_r = \frac{n!}{(n-r)!}, \text{ where } 0 \le r \le n$$

For example, if you have a set of three objects, a, b, and c, but you use only two of them at a time in each permutation, the number of permutations is

$$_3P_2 = \frac{3!}{(3-2)!} \text{ or } 6$$

	Position 1	Position 2
Permutation 1	a	b
Permutation 2	a	c
Permutation 3	b	a
Permutation 4	b	c
Permutation 5	c	a
Permutation 6	c	b

- When all n objects are used in each arrangement, n is equal to r and the number of arrangements is represented by $_nP_n = n!$.
- The number of permutations that can be created from a set of n objects, using r objects in each arrangement, where repetition is allowed and $r \le n$, is n^r. For example, the number of four-character passwords using only the 26 lower-case letters, where letters can repeat, is $26 \cdot 26 \cdot 26 \cdot 26 = 26^4$.

Need to Know

- If order matters in a counting problem, then the problem involves permutations. To determine all possible permutations, use the formula for $_nP_n$ or $_nP_r$, depending on whether all or some of the objects are used in each arrangement. Both of these formulas are based on the Fundamental Counting Principle.
- By definition,

$$0! = 1$$

As a result, any algebraic expression that involves factorials is defined as long as the expression is greater than or equal to zero. For example, $(n + 4)!$ is only defined for $n \ge -4$ and $n \in \mathrm{I}$.

- If a counting problem has one or more conditions that must be met,
  - consider each case that each condition creates first, as you develop your solution, and
  - add the number of ways each case can occur to determine the total number of outcomes.

HW: 4.3 p. 255-257 # 1, 2, 5, 7, 9, 11 & 14