Newton's Law of Universal Gravitation

The formula $\mathbf{F_g = mg}$ is used to describe the *weight* of an object, in Newtons. Looking at the formula more closely, we see:

- The term 'weight' is also known as *force of gravity.*
- Like all forces, $\mathbf{F_g}$ depends on two objects:
  - our planet Earth, which exerts a *gravitational field strength* '**g**' all around it, in N/kg.
  - some other object, such as you, that is being pulled to Earth by its field strength.
- The field strength '**g**' is equal to 9.8 N/kg at or near Earth's surface, but is much weaker as you move away a significant distance from the surface.
- Other objects in space, like stars, planets and moons, each have their own gravitational field strengths '**g**' that may be stronger or weaker than that of Earth, depending on their mass and the distance away from them.

Although it may not seem so, '**g**' exists everywhere in space! Its value varies from one location to another, so that $\mathbf{F_g = mg}$ is not very useful for finding the force of gravity that exist between two objects.

Through experimental research, it was Isaac Newton who first determined how the force of gravity is affected in space:

- mass attracts mass, and the size of each mass directly affects the amount of attraction between them, so that $\mathbf{F \propto Mm}$
- the size of the force is inversely dependent on the square of the distance between the centers of mass of the two objects, so that:

$$\mathbf{F \propto \frac{1}{R^2}}$$

- from graphical analysis, an equation was established, with the slope being the universal constant '**G**', and the equation being

$$\boxed{\mathbf{F_g = G\frac{Mm}{R^2}}} \qquad \text{where} \qquad \mathbf{G = 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}}$$

This new formula is useful, because it allows us not only to determine the force of attraction between two objects, but also calculate weight on other planets, or at great altitudes above Earth, where '**g**' is *not* 9.8 N/kg.

Example 1: Determine the force of attraction between a 35 kg dog and a 7.6 kg cat, watching each other from a distance of 4.8 m. (7.7×10^{-10} N)

(see Gravitation Ex 1 for answer)

Example 2:

(a) Find the weight of a 50 kg person on Earth, using $F_g = mg$.

(b) Find the same weight on Earth, using $F_g = G\frac{Mm}{R^2}$.

(c) Find the weight of this person at an altitude of 170 km.

(see Gravitation Ex 2 for answer)

Consider the answers to Example 2: why is this weight less than at Earth's surface? Because the gravitational field strength of the Earth is weaker as you move further away from its center of mass.

Gravity in Space

Recall: the space around a mass in which it exerts a gravitational influence is called its *field*. In this space it can exert a force on another mass because of its *gravitational field strength* '**g**', measured in N/kg. On Earth's surface, that field strength is 9.8 N/kg, but elsewhere the value for '**g**' is quite different.

To find the gravitational field strength '**g**' at *any* location, combine both versions of $\mathbf{F_g}$:

$$\mathbf{F_g = mg} \quad \text{and} \quad \mathbf{F_g = G\frac{Mm}{R^2}}$$

→equate these two to get $\mathbf{mg = G\frac{Mm}{R^2}}$

→cancel out small mass **m** and $\boxed{\mathbf{g = \frac{GM}{R^2}}}$

Here **M** is the mass of the Earth (or any *central* mass), and **R** is the distance you are from the ***center*** of that mass. Note that

R = radius of the Earth + altitude

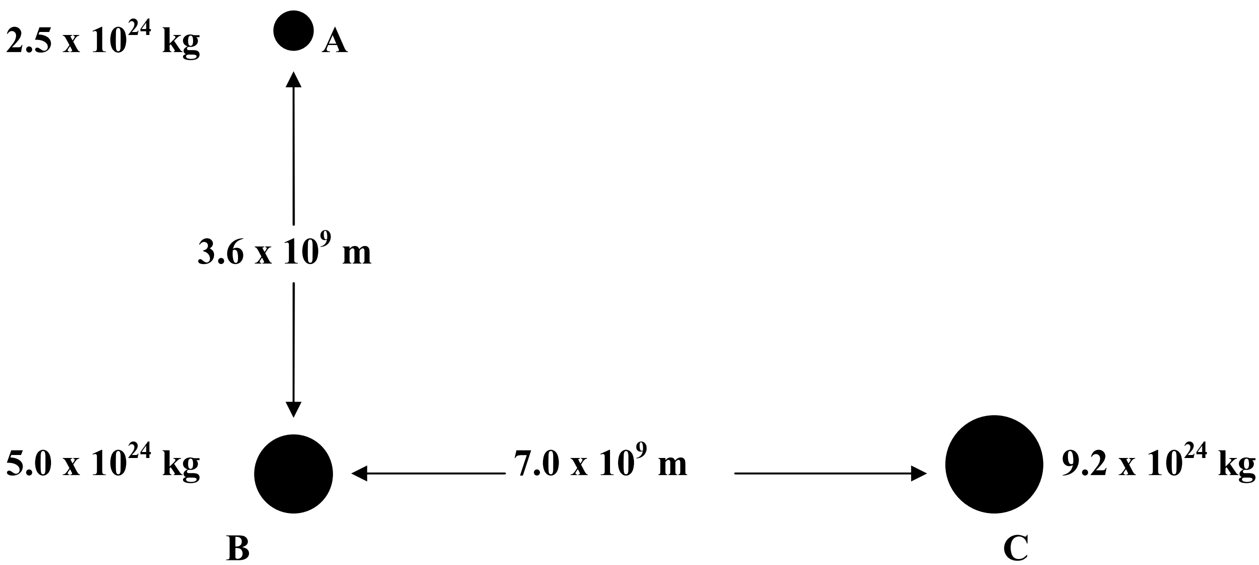
Example 3: Find 'g' at an altitude of 100 km.

(see Gravitation Ex 3 for answer)

All masses have a gravitational field strength surrounding them. This means the force of gravity can act on you anywhere in space, due to any number of objects that pull on you in different directions. For example, the Earth's oceans are spread throughout the planet due to its gravitational field strength pulling on the bodies of water. But the moon also exerts a pulling force of gravity on Earth's oceans, producing the daily high and low tides seen by any coastal region.

With different forces acting in different directions, we can find the *net force* on one mass due to the combined gravitational forces of two other masses in a specific position in space.

Example 4: Determine the net force acting on Planet B by the other two planets, as illustrated below:



To solve this type of problem, it may be helpful to follow this series of steps:
(1) Draw a free-body vector diagram showing the forces acting on Planet B.
(2) Calculate force $\mathbf{F_{AB}}$ using information on those two planets only.
(3) Calculate force $\mathbf{F_{BC}}$ in the same way as in (2).
(4) Determine the resultant force from the vector addition of $\mathbf{F_{AB}}$ and $\mathbf{F_{BC}}$.

(see Gravitation Ex 4 for answer)

Ratio solutions for the Gravitation Law.

Recall from math relationships:

- if $\mathbf{a} \propto \mathbf{b}$ → then $\dfrac{\mathbf{a_2}}{\mathbf{a_1}} = \dfrac{\mathbf{b_2}}{\mathbf{b_1}}$
- if $\mathbf{x} \propto \dfrac{\mathbf{1}}{\mathbf{y}}$ →then $\dfrac{\mathbf{x_2}}{\mathbf{x_1}} = \dfrac{\mathbf{y_1}}{\mathbf{y_2}}$

Therefore,in a two-planet system:

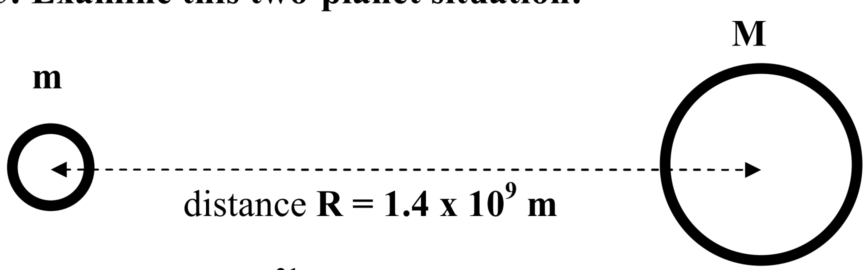
$$\mathbf{F = G\frac{Mm}{R^2}} \quad \text{becomes} \quad \mathbf{\frac{F_B}{F_A} = \left(\frac{M_B}{M_A}\right)\left(\frac{m_B}{m_A}\right)\left(\frac{R_A^2}{R_B^2}\right)}$$

→ or, $\mathbf{F_B = F_A\left(\frac{M_B}{M_A}\right)\left(\frac{m_B}{m_A}\right)\left(\frac{R_A^2}{R_B^2}\right)}$

This appears complex, but is in fact based on some very simple math principles:

- if one mass changes, the force changes proportionally; e.g., if the mass of *one* of the two planet doubles, so does the $\mathbf{F_g}$ between them.
- if <u>both</u> masses change, the force changes proportionally for each mass; e.g., if the mass of *both* planets double, the $\mathbf{F_g}$ between them increases by **2 x 2 = 4** times
- if the distance between the two planets changes, the force between them *decreases* by that factor squared; e.g., if the distance between the planets doubles, the $\mathbf{F_g}$ between them is decreased to $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Example 5: Examine this two-planet situation:



$\mathbf{F_g = 4.1 \times 10^{21}\ N}$ between the two planets

The above is now changed, as follows; find the new gravitational force in each case:

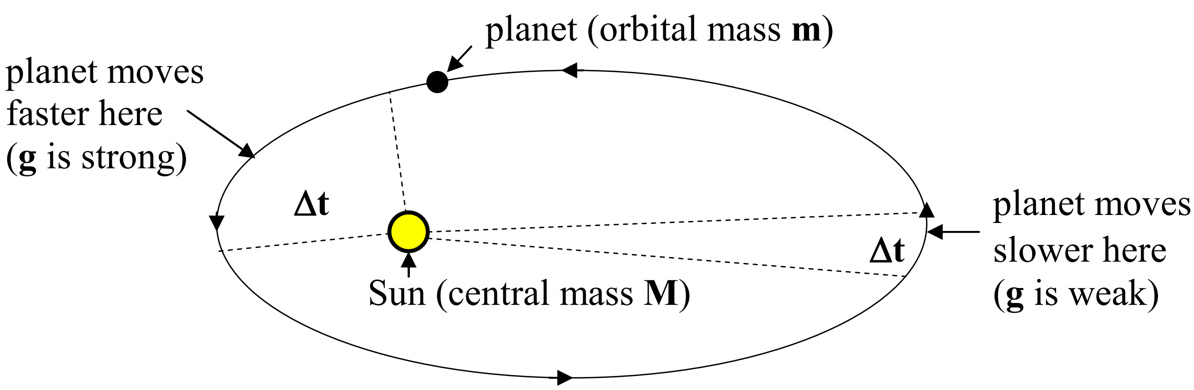
(a) m is tripled.
(b) M is tripled, and m is reduced by half.
(c) M is one-tenth as large, and R is tripled.
(d) m is quadrupled, and R = 9.5 x 10^8 m.

(see Gravitation Ex 5 for answer)

Kepler's Three Laws of Planetary Motion

In the 1700's, Johannes Kepler used Newton's work to establish the nature of a body's orbital path around a central mass. This is what he found:

1. Planetary orbits are *elliptical*, with the sun at one focus.
   - Note that many orbits are very nearly circular; for such cases, we can state that $\mathbf{F_c = F_g}$
2. A line joining the center of any planet with the sun, traces out equal areas in equal times.
   - This statement tells us that for very elliptical paths, a planet travels at greatest speed *closest* to the Sun (where '**g**' is greatest) and travels slowest at its *furthest* point from the Sun (where 'BgB' is weakest).
3. For a given central mass, if **R = the average orbital radius**, and **T** = **the orbital period**, then the ratio of $\mathbf{\frac{R^3}{T^2}}$ is a constant.



Newton's Synthesis

We can derive Kepler's Third Law from Newton's Second Law and find a way to get Kepler's Constant. Start by examining a satellite of mass '**m**' travelling in a stable orbit with period of revolution '**T**' around a central planetary mass '**M**'.

If we assume the elliptical orbit is very nearly circular, then once again $\mathbf{F_c = F_g}$.

- combine $\mathbf{F_g = \frac{GMm}{R^2}}$ and $\mathbf{F_c = m\frac{4\pi^2 R}{T^2}}$: $\mathbf{\frac{GMm}{R^2} = m\frac{4\pi^2 R}{T^2}}$
- cancel orbital mass '**m**' and rearrange to get $\mathbf{\frac{R^3}{T^2} = \frac{GM}{4\pi^2}}$
- **G**, **M**, and $\mathbf{4\pi^2}$ are all constant (remember, '**M**' is the central mass), so Kepler's Constant is therefore $\mathbf{k = \frac{GM}{4\pi^2}}$.

Example 7: A telecommunications satellite orbits the Earth once every 24 hours in what is called a <u>geosynchronous</u> orbit. What is the altitude of this satellite?

(see Gravitation Ex 7 for answer)

To use Kepler's Third Law, you must have **R** and **T** data for one satellite so that you can find the constant **k**, or you can use ratio solutions.

If $\mathbf{\frac{R^3}{T^2} = k}$, then $\mathbf{R^3 = kT^2}$

We now have $\mathbf{\left(\frac{R_B}{R_A}\right)^3 = \left(\frac{T_B}{T_A}\right)^2}$ → use this to solve using ratios

Note: In order to simplify meaning, the period (year) and radius measurements of planets are often made in terms of Earth measurements. A planet's year may be given in terms of Earth days or years. A planet's distance from the Sun may be given in terms of Earth's distance from the Sun. This distance is called one **astronomical unit** (AU).

Example 8: Mercury's Year = 88 Earth days and its distance from the Sun is 0.37 AU. Find the orbital radius of Venus if its year = 224.7 Earth days.

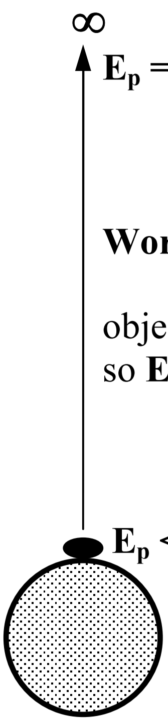
(see Gravitation Ex 8 for answer)

Gravitational Potential Energy in Space

When we are near the Earth's surface, we calculate the $\Delta \mathbf{E_p}$ of an object as the **work done** to lift the object to its new height, where $\mathbf{E_p = mgh}$ near the Earth's surface. The value of "**h**" is measured relative to some point that is chosen by you to be where the object will not fall any further. For example:

- For a pendulum, $\mathbf{h = 0}$ at the bottom of its swing.
- For an object that is dropped, $\mathbf{h = 0}$ where the object lands (usually on the ground).

However, because **g** changes with altitude, we can't use this formula in space. In space, if we move one mass away from another far enough (so that $\mathbf{R} = \infty$), we will move it out of that mass's field and so $\mathbf{E_p}$ will finally equal zero.



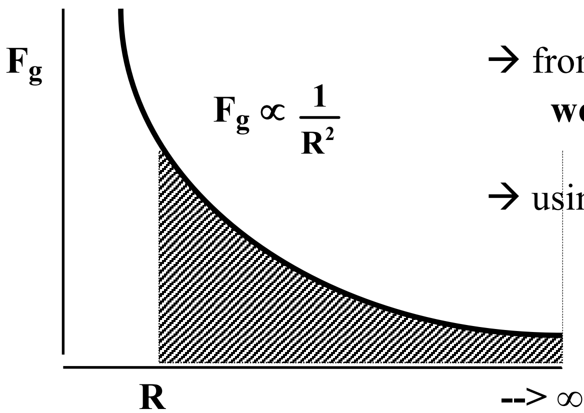
Whenever an object is lifted, potential energy is increased, resulting in *positive work done.*

If that object is lifted to *infinity*, where $\mathbf{E_p = 0}$, then the potential energy of the object must be *less than zero* at any height that is closer than infinity; this includes the earth's surface.

Put another way, when an object falls, $\mathbf{E_p}$ decreases. Since the $\mathbf{E_p}$ decreases as the object falls from infinity to Earth, and since $\mathbf{E_p = 0}$ *at* $\mathbf{R} = \infty$, the $\mathbf{E_p}$ becomes increasingly *negative*.

We need to find an expression for $\mathbf{E_p}$ of an object at the surface of the Earth, compared with $\mathbf{E_p = 0}$ at $\mathbf{R} = \infty$.

From graphical analysis of $\mathbf{F_g}$ vs. $\mathbf{R}$: (remember that $\mathbf{F_g} = \frac{\mathbf{GMm}}{\mathbf{R^2}}$)



→ from any point $\mathbf{R}$ out to ∞,
work done = area under the graph

→ using calculus, $\mathbf{W} = \Delta \frac{\mathbf{GMm}}{\mathbf{R}}$

- but $\mathbf{W} = \Delta \mathbf{E_p} = \mathbf{E_{p\,(f)}} - \mathbf{E_{p\,(i)}} = \mathbf{0} - \mathbf{E_{p\,(i)}}$
- $\mathbf{E_p}$ is negative for <u>any value</u> of $\mathbf{R} < \infty$; that is, $\boxed{\mathbf{E_p} = -\frac{\mathbf{GMm}}{\mathbf{R}}}$

If this makes no sense to you, don't panic! The above is only a short, non-calculus explanation for the new formula for $\mathbf{E_p}$. What you <u>must</u> know is this: the formula $\mathbf{E_p} = -\frac{\mathbf{GMm}}{\mathbf{R}}$ is used to find potential energy for an object '**m**' in space relative to:

- the central planetary mass '**M**'.
- $\mathbf{E_p} = \mathbf{0}$ at infinity.

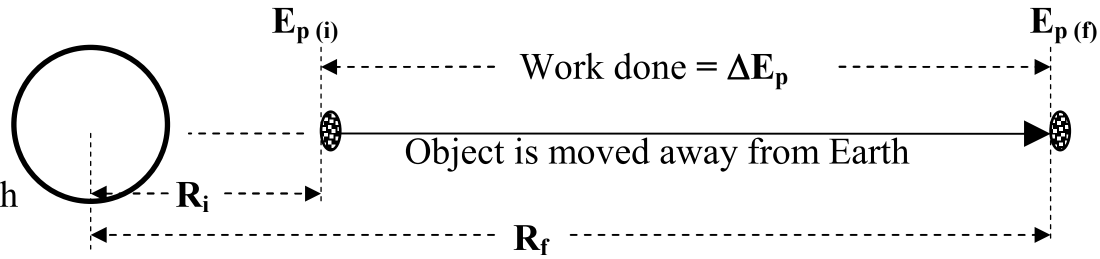
Example 9: Find the potential energy (relative to infinity) of a 50. kg person flying at an altitude of 1.0 x 10^4 m above Earth's surface, relative to infinity.

(see Gravitation Ex 9 for answer)

Example 10: If that 50. kg person was in a space shuttle orbiting at an altitude of 250 km, what would be her new potential energy, relative to infinity?

(see Gravitation Ex 10 for answer)

Now examine the situation below where work is done to move an object further away from Earth:



Recall that work is equal to change in energy. In this example, the **work done** = the difference between the initial $\mathbf{E_p}$ and the final $\mathbf{E_p}$.

$$\mathbf{W = \Delta E_p = E_{p\,(f)} - E_{p\,(i)} = -\frac{GMm}{R_f} - \left(-\frac{GMm}{R_i}\right)}$$

$$\mathbf{W = GMm\left(\frac{1}{R_i} - \frac{1}{R_f}\right)}$$ → energy added is *positive* energy.

Example 11: Determine the work done to move a 3.5 x 10^4 kg cargo of space junk from an altitude of 430 km above the Moon's surface to a radial distance of 2.8 x 10^6 m away.

(see Gravitation Ex 11 for answer)

Conservation of Energy in Space

Recall that energy cannot be created or destroyed; it can only be converted from one form to another. In other words, regardless of what energy is being used or produced, the total energy contained in any system stays the same.

In space, any object:
- will always have gravitational potential energy relative to some galaxy, star, planet, or moon.
- may have kinetic energy, if it is moving relative to some galaxy, star, planet or moon.
- will not likely produce any significant heat or other form of wasted energy in its movement, due to the lack of friction from wind resistance.

Therefore, to calculate the total energy of a moving mass in space, we can state that:

$$\mathbf{E_t = E_k + E_p}$$

- substitute in the proper formulas to obtain

$$\mathbf{E_t = \frac{1}{2}mv^2 - \frac{GMm}{R}}$$

where: → **m** is the mass of the moving object
→**M** is the mass of the object exerting the gravitational pull (a planet, moon, etc.)
→**R** is the distance between the two objects
→**v** is the speed of the object

Use total energy to find unknown quantities for any mass that changes its speed or distance in space from a planetary or stellar object.

Example 12: A 12 000 kg spaceship is 7.2 x 10^8 m from the center of a planet that has a mass of 5.1 x 10^{25} kg. As it "falls" back to the planet's surface, the spaceship gains 9.0 x 10^{11} J of kinetic energy. What is the radius of the planet?

(see Gravitation Ex 12 for answer)

Conservation of energy can be used to determine a value called the *escape velocity* of a spacecraft. The escape velocity is the minimum velocity required for the craft to escape a planet's gravitational field. This means that:

- A body has "escaped" from a gravitational field if it is so far from the mass that generates the field that once it has *stopped*, its $\mathbf{E_p = 0}$.
- At a minimum speed, the spacecraft will (in theory) reach infinity and stop, so that $\mathbf{E_k = 0}$.

Therefore, in this system where the minimum velocity is required, the total energy $\mathbf{E_t = 0}$! In terms of conservation of energy:

total energy before = total energy after: $\mathbf{E_k + E_p = 0}$

Here the speed of the escaping object at the planet's surface must be great enough to provide the $\mathbf{E_k}$ needed so that $\mathbf{E_k = - E_p}$.

At the planet's surface, $\mathbf{E_p = -\dfrac{GMm}{R_p}}$ where $\mathbf{R_p}$ **= radius of the planet**

- $\therefore \quad \mathbf{\dfrac{GMm}{R_p} = \dfrac{1}{2}mv^2}$ ---> cancel spacecraft's mass and solve for **v**

Example 13: Determine the escape velocity for any spacecraft launched from the surface of Mars, which has a planetary mass of 6.40 x $\mathbf{10^{23}}$ kg and a radius of 3.4 x $\mathbf{10^{6}}$ m.

(see Gravitation Ex 13 for answer)

Finally, consider the total energy possessed by a satellite in a stable orbit.

Total energy = kinetic energy at *orbit speed* + potential energy at orbital altitude

Don't forget that **R = radius of planet + altitude**

To begin, the orbital speed must be determined. Since the satellite is orbiting in a circle, use $\mathbf{F_c = F_g}$ to find this speed.

Then, solve using $\mathbf{E_T = E_k + E_p}$ where $\mathbf{E_p = -\dfrac{GMm}{R}}$ (relative to infinity)

Example 14: Determine the total energy possessed by the Moon as it orbits the Earth.

(see Gravitation Ex 14 for answer)

Note that the total energy possessed by *any* object in a stable orbit is always equal to ***half*** of the potential energy contained in that object, or

$$\mathbf{E_{T\,(stable\ orbit)} = -\frac{1}{2}\frac{GMm}{R}}$$

This can be proven with algebraic substitution.

Example 1: Determine the force of attraction between a 35 kg dog and a 7.6 kg cat, watching each other from a distance of 4.8 m.

$$F_g = \frac{GMm}{r^2}$$

$$= \frac{(6.67\times10^{-11})(35)(7.6)}{4.8^2}$$

$$\boxed{F_g = 7.7\times10^{-10}\ \text{N}}$$

Example 2:

(a) Find the weight of a 50 kg person on Earth, using $F_g = mg$.

$$F_g = mg$$
$$= 50\,(9.8)$$
$$\boxed{F_g = 490\ \text{N}}$$

(b) Find the same weight on Earth, using $F_g = G\dfrac{Mm}{R^2}$.

mass of Earth

$$F_g = \frac{GMm}{r^2}$$
$$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(50)}{(6.38\times10^{6})^2}$$
$$\boxed{F_g = 490\ \text{N}}$$

(c) Find the weight of this person at an altitude of 170 km.

$$r = [6.38\times10^{6}] + [170000]$$
$$= 6.55\times10^{6}\ \text{m}$$

$$F_g = \frac{(6.67\times10^{-11})(5.98\times10^{24})(50)}{(6.55\times10^{6})^2}$$

$$\boxed{F_g = 465\ \text{N}}$$

Example 3: Find 'g' at an altitude of 100 km.

$$r = [6.38 \times 10^6] + [100\,000]$$

$$r = 6.48 \times 10^6 \text{ m}$$

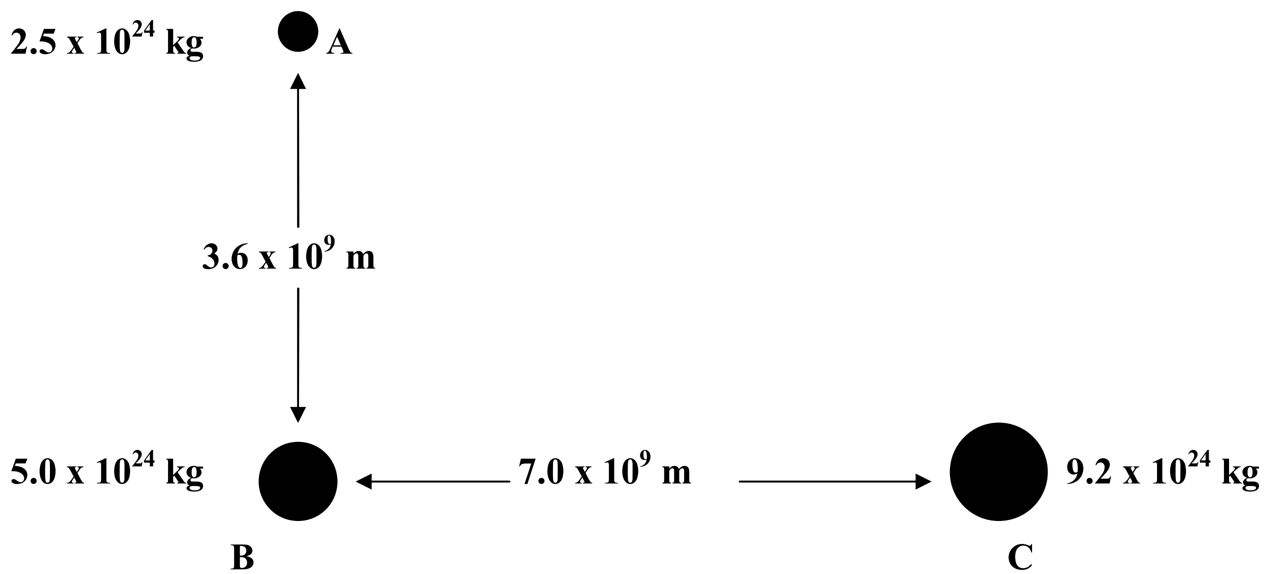
$$F_g = \cancel{m}g = \frac{GM\cancel{m}}{r^2}$$ $\Rightarrow$ cancel out 2nd mass

so $$g = \frac{GM}{r^2}$$ where M is mass of Earth

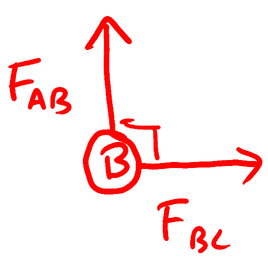
$$= \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.48 \times 10^6)^2}$$

$$\boxed{g = 9.5 \text{ N/kg}}$$

Example 4: Determine the net force acting on Planet B by the other two planets, as illustrated below:

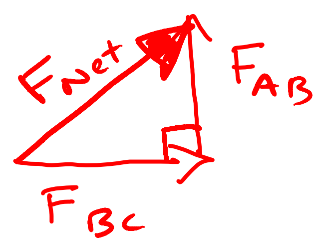


f. b. d. of "B":



$\Rightarrow$

Vector-add to find net grav. force on "B":



Use $F_g = \frac{GMm}{r^2}$ to find each force:

$$F_{AB} = \frac{(6.67\times10^{-11})(2.5\times10^{24})(5\times10^{24})}{(3.6\times10^{9})^2}$$

$$= 6.43\times10^{19}\ \text{N}$$

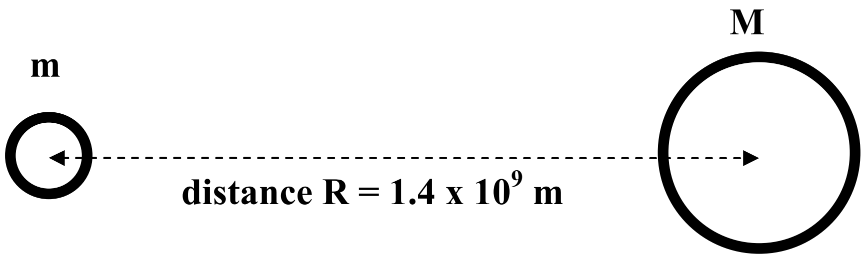
$$F_{BC} = \frac{(6.67\times10^{-11})(5\times10^{24})(9.2\times10^{24})}{(7\times10^{9})^2}$$

$$= 6.26\times10^{19}\ \text{N}$$

Use pythagoras and $\tan^{-1}\theta$ to get

$$\boxed{F_{Net} = 9.0\times10^{19}\ \text{N},\ 46^\circ \text{ up from line BC}}$$

Example 5: Examine this two-planet situation:



$F_g = 4.1 \times 10^{21}$ N

The above is now changed, as follows; find the new gravitational force in each case:

(a) m is tripled.
(b) M is tripled, and m is reduced by half.
(c) M is one-tenth as large, and R is tripled.
(d) m is quadrupled, and R = 9.5 x 10^8 m.

a) $F \propto m$ so $F_{New} = F_{old} \times \left[\frac{m_{new}}{m_{old}}\right]$

$$F_{New} = 4.1 \times 10^{21}\left(\frac{3\cancel{m}}{\cancel{m}}\right)$$

$$\boxed{F_{New} = 1.2 \times 10^{22}\ \text{N}}$$

b) $F \propto M$, $F \propto m$

so $F_{New} = F_{old} \times \left[\frac{M_{new}}{M_{old}}\right] \times \left[\frac{m_{new}}{m_{old}}\right]$

$$F_{New} = 4.1 \times 10^{21}\left(\frac{3\cancel{M}}{\cancel{M}}\right) \times \left(\frac{.5\cancel{m}}{\cancel{m}}\right)$$

$$\boxed{F_{New} = 6.2 \times 10^{21}\ \text{N}}$$

continued next page . . .

c) $F \propto M, \quad F \propto \frac{1}{r^2}$

so $F_{New} = F_{old}\left[\frac{M_{new}}{M_{old}}\right]\left[\frac{r_{old}^2}{r_{new}^2}\right]$

NOTE!

$= 4.1\times10^{21}\left[\frac{.1M}{M}\right]\left[\frac{r^2}{(3r)^2}\right]$

$= 4.1\times10^{21}\left[.1\right]\left[\frac{1}{9}\right]$

$\boxed{F_{New} = 4.6\times10^{19}\ N}$

d) $F_{New} = 4.1\times10^{21}\left[\frac{4\cancel{M}}{\cancel{M}}\right]\left[\frac{(1.4\times10^{9})^2}{(9.5\times10^{8})^2}\right]$

$\boxed{F_{New} = 3.6\times10^{22}\ N}$

Example 6:

(a) **Determine the stable parking orbit velocity for a surveying satellite located 230 km above the moon's surface.**

orbit ⇒ circular motion ∴ $F_c = F_g$

sub. in Formulas $\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$

$$v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(7.35\times10^{22})}{[(1.74\times10^{6}) + (230\ 000)]}}$$

$$\boxed{v = 1.58\times10^{3}\ \text{m/s}}$$

(b) **If that orbital radius were reduced by one-tenth, by what factor would the orbiting speed increase?**

→ $\frac{1}{10}$ less means $r_{new} = \frac{9}{10} r_{old}$

→ looking at $v = \sqrt{\frac{GM}{r}}$ (see above),

$$v \propto \frac{1}{\sqrt{r}}$$

→ So $v_{new} = v_{old}\left[\sqrt{\frac{r_{old}}{r_{new}}}\right] = v_{old}\left[\sqrt{\frac{\cancel{r}}{.9\cancel{r}}}\right]$

$$\boxed{v_{new} = 1.05\ v_{old}}$$

Example 7: A telecommunications satellite orbits the Earth once every 24 hours in what is called a <u>geosynchronous</u> orbit. What is the altitude of this satellite?

$$T = 24\,\cancel{h} \times \frac{3600\,s}{1\,\cancel{h}} = 8.64 \times 10^4\ s$$

→ use $F_c = F_g$ for orbiting satellite

$$\frac{\cancel{m}4\pi^2 r}{T^2} = \frac{GM\cancel{m}}{r^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

note →

$$r = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(8.64\times10^4)^2}{4\pi^2}}$$

$$r = 4.225 \times 10^7\ m$$

so altitude $= (4.225\times10^7) - (6.38\times10^6)$

$$\boxed{alt = 3.6 \times 10^7\ m}$$

Example 8: Mercury's Year = 88 Earth days and its distance from the Sun is 0.37 AU. Find the orbital radius of Venus if its year = 224.7 Earth days.

To derive Kepler's Constant, use

$F_c = F_g$ for orbiting planets

$$\frac{\cancel{m}4\pi^2 r}{T^2} = \frac{GM\cancel{m}}{r^2} \Rightarrow \text{rearrange to}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$ central mass is constant

so $\frac{r^3}{T^2} = k$ (a constant for any units of r & T)

$\Rightarrow$ for Mercury: $\frac{(.37)^3}{(88)^2} = 6.54 \times 10^{-6}$

$\Rightarrow$ for Venus: $\frac{r^3}{(224.7)^2} = 6.54 \times 10^{-6}$

$\boxed{r = 0.69 \text{ AU's}}$

Example 9: Find the potential energy (relative to infinity) of a 50. kg person flying at an altitude of 1.0 x 10^4 m above Earth's surface.

$$E_p = -\frac{GMm}{r}$$

$$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(50)}{[(6.38\times10^{6})+10\,000]}$$

$$\boxed{E_p = -3.1\times10^{9}\ \text{J}}$$

Example 10: If that 50. kg person was in a space shuttle orbiting at an altitude of 250 km, what would be her new potential energy, relative to infinity?

$$E_p = -\frac{GMm}{r}$$

$$= -\frac{6.67\times10^{-11}(5.98\times10^{24})(50)}{[(6.38\times10^{6})+(250\,000)]}$$

$$\boxed{E_p = -3.0\times10^{9}\ J}$$

Example 11: Determine the work done to move a 3.5 x 10^4 kg cargo of space junk from an altitude of 430 km above the Moon's surface to a radial distance of 2.8 x 10^6 m away.

$W = \Delta E_p$ ⇒ changing altitudes

$$= -\frac{GMm}{r_2} - \left[-\frac{GMm}{r_1}\right]$$

$$= \frac{GMm}{r_1} - \frac{GMm}{r_2}$$ ← note the "switch" of r's

$$\therefore \quad W = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= (6.67\times10^{-11})(7.35\times10^{22})(3.5\times10^4)\left[\frac{1}{(1.74\times10^6)+(430000)} - \frac{1}{2.8\times10^6}\right]$$

$$\boxed{W = 1.8\times10^{10}\ \text{J}}$$

→ note that work done to "lift" an object is <u>positive</u>, because E_p is increased.

Example 12: A 12 000 kg spaceship is 7.2 x 10^{8} m from the center of a planet that has a mass of 5.1 x 10^{25} kg. As it "falls" back to the planet's surface, the spaceship gains 9.0 x 10^{11} J of kinetic energy. What is the radius of this planet?

$$\Delta E_k = 9.0 \times 10^{11}\ \text{J}.$$

→ using conservation of energy:

before → $E_T = E_{p(old)}$

after → $E_T = E_{p(new)} + \Delta E_k$

$$\text{before: } E_T = E_{p(old)} = -\frac{6.67\times10^{-11}(5.1\times10^{25})(12\,000)}{7.2\times10^{8}}$$

$$= -5.67\times10^{10}\ \text{J}.$$

$$\text{after: } -5.67\times10^{10} = E_{p(new)} + 9\times10^{11}$$

$$E_{p(new)} = -9.567\times10^{11}\ \text{J}$$

$$-9.567\times10^{11} = -\frac{6.67\times10^{-11}(5.1\times10^{25})(12000)}{r}$$

$$\boxed{r = 4.3\times10^{7}\ \text{m}}$$

Example 13: Determine the escape velocity for any space shuttle launched from the surface of Mars, which has a planetary mass of 6.40 x 10^{23} kg and a radius of 3.4 x 10^{6} m.

$E_T = 0$ → object has enough speed to reach infinity so that

$E_p = 0$ & $E_k = 0$

∴ at launch, $E_p + E_k = 0$

$$-\frac{GM\cancel{m}}{r} + \frac{1}{2}\cancel{m}v^2 = 0$$

$$\frac{1}{2}v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$= \sqrt{\frac{2(6.67\times10^{-11})(6.4\times10^{23})}{3.4\times10^{6}}}$$

$$\boxed{v = 5.0\times10^{3}\ m/s}$$

Example 14: Determine the total energy possessed by the Moon as it orbits the Earth.

$$E_T = E_k + E_p$$

$$= \frac{1}{2}mv^2 + -\frac{GMm}{r}$$

→ must find orbital speed

→ use $F_c = F_g$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$ ⇒ sub. into energy equation

So $$E_T = \frac{1}{2}m\left[\frac{GM}{r}\right] + -\frac{GMm}{r}$$

$$= -\frac{1}{2}\frac{GMm}{r}$$

$$= -\frac{.5(6.67\times10^{-11})(5.98\times10^{24})(7.35\times10^{22})}{3.84\times10^{8}}$$

$$\boxed{E_T = -3.8\times10^{28}\ J}$$