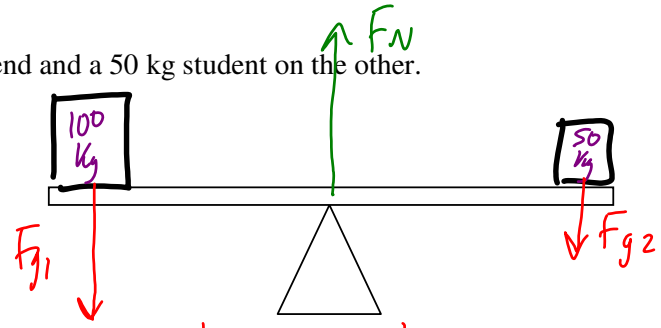
Equilibrium Notes

2 – Torque at 90°

A body in translational equilibrium will have no acceleration in the x or y directions. However it still could be rotating.

Consider a teeter-totter, with a 100 kg student on one end and a 50 kg student on the other.



What are the net translational forces in:

The x-direction? $\sum F_x = 0$

The y-direction? $\sum F_y = 0$

Although the net translational forces are zero, the system has a net torque - so it is not in equilibrium.

An object in equilibrium must have both translational and rotational equilibrium.

The second condition of equilibrium is that in order to have no rotation, there must be no net torque.

Torque is defined as: **force x distance to pivot**

$\tau = Fd$

Unit of torque: N·m

Imagine trying to loosen the lug nuts to remove a tire from your car. The longer the wrench you use, the easier it will be.

Ex:

A torque of 24.0 Nm is needed to tighten a nut. If a person can apply a force of 100 N, what is the minimum length of wrench that is required?

$\tau = Fd \quad d = \frac{\tau}{F} = \frac{24.0 N \cdot m}{100 N} = 0.24 m$

Torque is a vector quantity, which must work in either the clockwise (c) or counterclockwise (cc) directions.

If an object is in rotational equilibrium then:

$\sum \tau = 0$ or $\tau_c = \tau_{cc}$

A few more terms we need to learn before we go on…

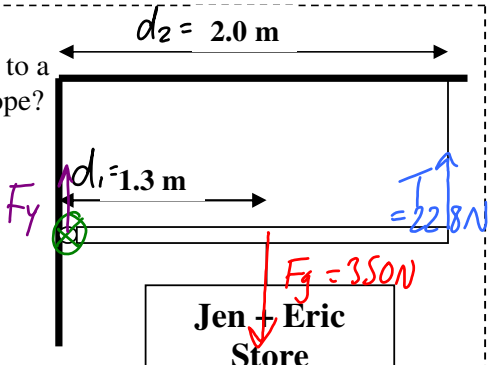
Centre of Gravity: Where the average mass acts. Where we draw Fg!

Uniform Beam: Constant shape and density

Arbitrary Position of Rotation: You choose the location of pivot!

Ex:

A 350 N store sign hangs from a pole of negligible mass. The pole is attached to a wall by a hinge and supported by a vertical rope. What is the tension in the rope?

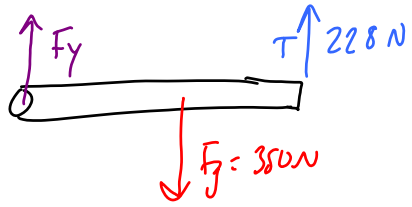


$\tau_c = \tau_{cc}$

$F_g d_1 = T d_2$

$T = \frac{F_g d_1}{d_2} = \frac{350(1.3)}{2.0} = 227.5 N = 228 N$

Extension:
What are the vertical and horizontal components of the supporting force provided by the hinge in the last question?



$$\sum F_y = F_y + T - F_g = 0$$

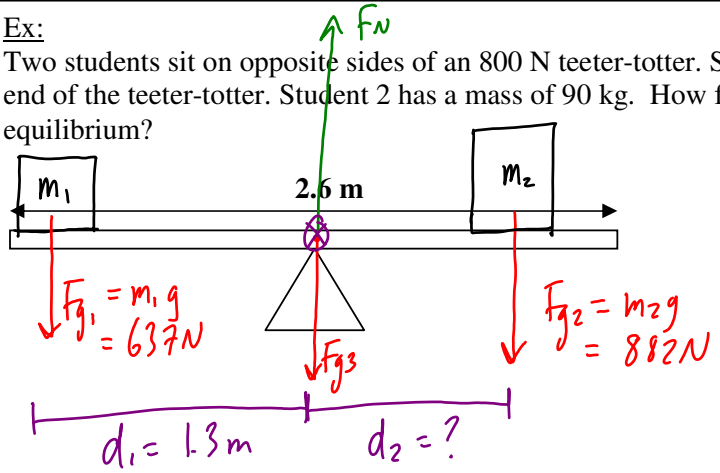
$$F_y = F_g - T$$

$$= 350 - 228$$

$$= \boxed{122\ N}$$

$$F_x = 0$$

Ex:
Two students sit on opposite sides of an 800 N teeter-totter. Student 1 has a mass of 65 kg and sits at the very end of the teeter-totter. Student 2 has a mass of 90 kg. How far from the pivot should he sit in order to achieve equilibrium?



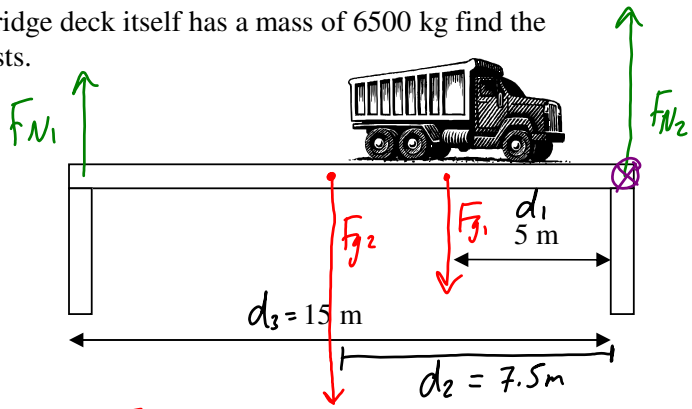
$$\tau_c = \tau_{cc}$$

$$F_{g2} d_2 = F_{g1} d_1$$

$$d_2 = \frac{F_{g1} d_1}{F_{g2}} = \frac{(637N)(1.3m)}{(882N)}$$

$$= \boxed{0.94m}$$

Ex:
A 3500 kg truck is parked on a bridge as shown. If the bridge deck itself has a mass of 6500 kg find the supporting force provided by each of the two support posts.



$$\tau_c = \tau_{cc}$$

$$F_{N1} d_3 = F_{g1} d_1 + F_{g2} d_2$$

$$F_{N1} = \frac{F_{g1} d_1 + F_{g2} d_2}{d_3}$$

$$= \frac{(34300)(5) + (63700)(7.5)}{15}$$

$$= \boxed{43300N}$$

$$F_{g1} = m_1 g = (3500)(9.8) = 34300\ N$$

$$F_{g2} = m_2 g = (6500)(9.8) = 63700\ N$$

$$\sum F_y = F_{N1} + F_{N2} - F_{g1} - F_{g2} = 0$$

$$F_{N2} = F_{g1} + F_{g2} - F_{N1} =$$

$= 34300 + 63700 - 43300\ N$

$= \boxed{54\,700\ N}$