Work, Energy and Momentum Notes

2 – Power and Efficiency

In everyday language we often use the words WORK, ENERGY and POWER synonymously. However this makes the physics gods extremely furious because we should all know that:

POWER is…the rate of doing Work

Mathematically we define power as:

$$\boxed{P = \frac{W}{t} = \frac{\Delta E}{t}}$$

The unit of power is J/s or Watts (W)
(this is sometimes confusing because W is also the *symbol* for work)

Example:
A physics student is setting up a wicked body slam on a biology student. He lifts the 75 kg student clear over his head to a height of 2.2 m in 0.675 s. How much power did the physics student generate?

$$P = \frac{\Delta E_p}{t} = \frac{mgh}{t} = \frac{(75\,kg)(9.8\,m/s^2)(2.2\,m)}{0.675\,s}$$
$$= \boxed{2400\,W}$$

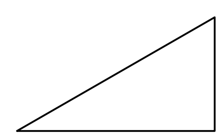
Example:
While cruising along level ground in a one horse open sleigh at 4.0 m/s, Mr Trask cracks the whip and speeds up to 12.0 m/s in 4.5 s. If the sleigh has a mass of 850 kg, how much power did it generate? Ignore friction.

$$P = \frac{\Delta E_K}{t} = \frac{\frac{1}{2}m\Delta v^2}{t} = \frac{\frac{1}{2}m(v_f^2 - v_i^2)}{t}$$
$$= \frac{\frac{1}{2}(850\,kg)(12.0^2 - 4.0^2)}{4.5} = \boxed{12\,000\,W}$$

Another useful equation for power can be derived:

$$P = \frac{W}{t} \qquad W = Fd$$
$$= \frac{Fd}{t} \qquad v = \frac{d}{t}$$
$$\boxed{P = Fv} \quad * \text{ const } \vec{v}$$

Example:
A student pushes 14 kg of their physics homework up a 40° ramp at a constant velocity of 3.2 m/s. The friction force is 26 N. How much power must the student exert?



Whenever we use a machine to do work some of the energy we put into the machine is always lost, mainly due to **friction**.

For example
an electric heater is ~ 95% efficient
a car is ~ 30% efficient
a lightbulb is ~ 3% efficient

We can define efficiency in two ways:

$$\text{Efficiency} = \frac{W_{out}}{W_{in}} \times 100\% = \frac{P_{out}}{P_{in}} \times 100\%$$

The most common source of confusion when calculating efficiency is in understanding which values applies to work/power IN and which applies to work/power OUT.

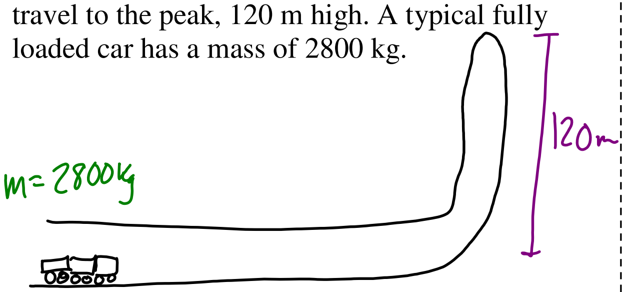
Work/Power In: total energy/power used by the process

Work/Power Out: what was actually accomplished by process

Remember that energy is always LOST somewhere in using the machine, so

Work IN $>$ Work OUT and Efficiency $\leq$ 100%

Example:
The Top Thrill Dragster is one of the tallest roller coaster in the world. The car is accelerated along a level track until they take a 90° vertical turn and travel to the peak, 120 m high. A typical fully loaded car has a mass of 2800 kg.



a) Calculate the minimum amount of work done on the car in order for it to reach the peak.

$$W = \Delta E_p = mgh = (2800\,kg)(9.8\,m/s^2)(120\,m)$$
$$= 3\,290\,000\,J$$
$$= 3.29\times10^6\,J$$

b) In reality the roller coaster is accelerated from 0 to 193 km/h in 3.8 s. Find the actual power input of the ride. 193 km/h ÷ 3.6 = 53.61 m/s

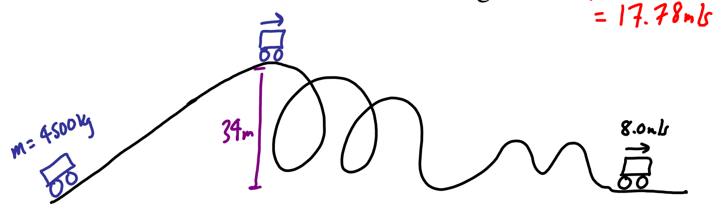
$$P = \frac{W}{t} = \frac{4\,024\,000\,J}{3.8\,s} = 1.06\times10^6\,W$$

$$W = \Delta E_K = \tfrac{1}{2}m\Delta v^2 = \tfrac{1}{2}(2800\,kg)(53.61\,m/s)^2$$
$$= 4\,024\,000\,J = \underline{4.02\times10^6\,J}$$

c) Determine the efficiency of the ride from start to peak.

$$E_{ff} = \frac{W_{out}}{W_{in}}\times100\% = \frac{3.29\times10^6\,J}{4.02\times10^6\,J}$$
$$= \boxed{82\%}$$

Example:
On the Incredible Hulk roller coaster the car is initially launched up a hill 34 m high, traveling from 0 to 64 km/h in 2.0 s. A full car has a mass of 4500 kg. ÷3.6 = 17.78 m/s



a) Find the power output of the ride.

$$W_{out} = \Delta E_p + \Delta E_K = 1.499\times10^6 + 7.11\times10^5 = \underline{2.21\times10^6\,J}$$
$$\Delta E_p = mgh = (4500\,kg)(9.8\,m/s^2)(34\,m) = 1.499\times10^6\,J$$
$$\Delta E_K = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(4500\,kg)(17.78\,m/s)^2 = 7.11\times10^5\,J$$
$$P_{out} = \frac{W_{out}}{t} = \frac{2.21\times10^6\,J}{2.0\,s} = \underline{1.105\times10^6\,W}$$

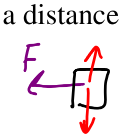
b) The power consumption during the initial launch is actually 1.45 MW. Determine the efficiency of the ride during the initial launch.

$$E_{ff} = \frac{P_{out}}{P_{in}}\times100\% = \frac{1.105\times10^6\,W}{1.45\times10^6\,W} = \boxed{76\%}$$

c) If the car pulls in to the station at 8.0 m/s. How much heat has been generated? since the 1st peak?

$$E_i = E_f$$
$$E_i = E_{K_f} + E_H$$
$$E_H = E_i - E_{K_f}$$
$$= 2.21\times10^6 - \tfrac{1}{2}(4500)(8.0)^2$$
$$= \boxed{2.07\times10^6\,J}$$

d) The car is finally brought to rest over a distance of 2.0 m. How much force is required?



$$W = \Delta E_K = F_{net}d$$
$$F_{net} = \frac{\Delta E_K}{d} = \frac{\cancel{E_{K_f}}^{\,0} - E_{K_i}}{d} = \frac{-E_{K_i}}{d}$$
$$= \frac{-\tfrac{1}{2}mv_i^2}{d} = \frac{-\tfrac{1}{2}(4500)(8.0)^2}{2.0} = \boxed{-72\,000\,N}$$

opposite motion