Equilibrium Notes

3 – Torque Not at 90°

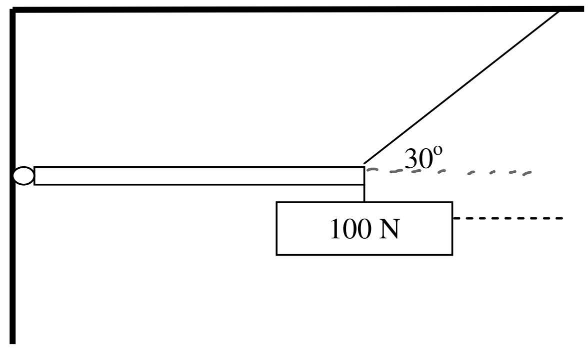
Although we've already learned about torque, we don't quite have the whole story. So far we have only seen torque provided by forces acting perpendicular to the body in equilibrium. **What happens if a force acts in a direction other than perpendicular to the body?**

Ex

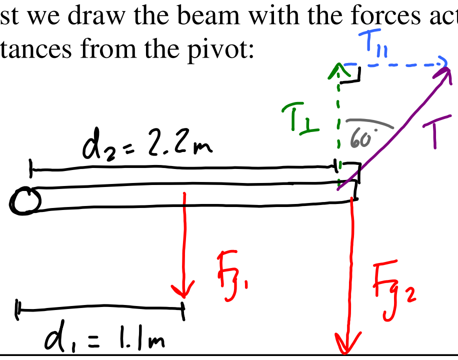
A 2.2 m long 50.0 N uniform beam is attached to a wall by means of a hinge. Attached to the other end of the beam is a 100 N weight. A rope also helps support the beam as shown.

a) What is the tension in the rope?

b) What are the vertical and horizontal components of the supporting force provided by the hinge?



First we draw the beam with the forces acting on it and their distances from the pivot:



Notice that if we break the tension in the rope into component forces, the parallel component does not contribute to the torque in either the clockwise or counterclockwise direction

* Not F_x and F_y
But $F_{\perp}$ and $F_{\parallel}$!!!

So, whenever we are calculating the torque on a body we must ALWAYS use the perpendicular component of the force.

Ok now go solving!

a)

$$\tau_c = \tau_{cc}$$

$$F_{g1} d_1 + F_{g2} d_2 = T_{\perp} d_2$$

$$T_{\perp} = \frac{(50\,N)(1.1) + (100\,N)(2.2)}{2.2} = 125\,N$$

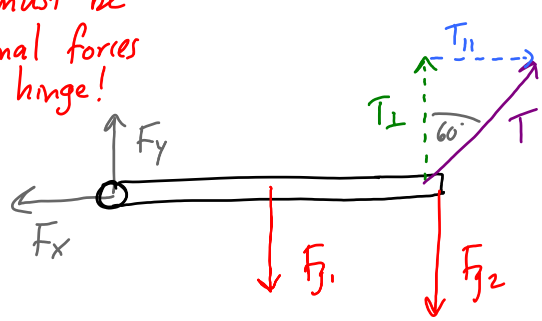
$$\cos 60° = \frac{T_{\perp}}{T}$$

$$T = \frac{T_{\perp}}{\cos 60°} = \frac{125}{\cos 60°}$$

$$= \boxed{250\,N}$$

b)

There must be additional forces on the hinge!



$$\sum F_y = F_y + T_{\perp} - F_{g1} - F_{g2} = 0$$

$$F_y = F_{g1} + F_{g2} - T_{\perp} = 50 + 100 - 125$$

$$= \boxed{25\,N}$$

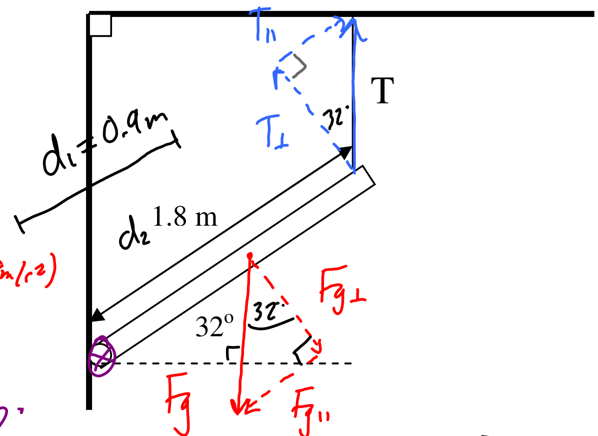
$$\sum F_x = T_{\parallel} - F_x = 0$$

$$F_x = T_{\parallel} = T \sin 60 = 250 \sin 60$$

$$= \boxed{217\,N}$$

RULE NOT TO BREAK LEST YE BE BROKEN:
When we find the torque acting on a body we MUST ALWAYS use the component of the force that is perpendicular to the beam!!!

Ex
A 1.8 m long 12.0 kg bar is attached to a wall by a hinge and supported by a rope as shown. Find the tension in the rope.



$$\tau_c = \tau_{cc}$$

$$F_{g\perp} d_1 = T_{\perp} d_2$$

$$T_{\perp} = \frac{F_{g\perp} d_1}{d_2}$$

$$= \frac{(62.32N)(0.9m)}{1.8m}$$

$$= 31.16N$$

$$T = \frac{T_{\perp}}{\cos 32°} = \boxed{58.8N}$$

$$F_g = m_1 g$$

$$= (12.0kg)(9.8m/s^2)$$

$$= 117.6N$$

$$F_{g\perp} = F_g \cos 32°$$

$$= 62.32N$$

Shortcut!

$$F_{g\perp} d_1 = T_{\perp} d_2$$

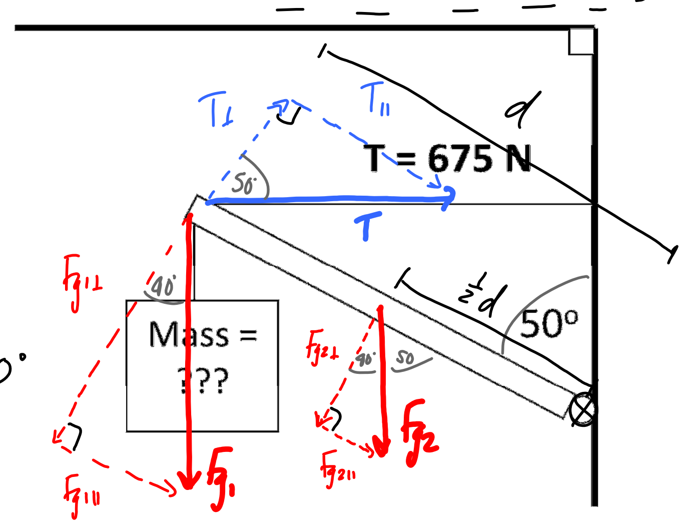
$$F_g \cancel{\cos 32} d_1 = T \cancel{\cos 32} d_2$$

$$T = \frac{F_g d_1}{d_2}$$

$$\vdots$$

Be careful.... this only works when the angles are the same.

Ex
Find the mass of the object given the information in the diagram and that the weight of the uniform beam is 115 N.



$$\tau_c = \tau_{cc}$$

$$T_{\perp} \cancel{d} = F_{g1\perp} \cancel{d} + F_{g2\perp}(\tfrac{1}{2}\cancel{d})$$

$$T\cos 50° = F_{g1}\cos 40° + \tfrac{1}{2} F_{g2}\cos 40°$$

$$F_{g1} = \frac{T\cos 50° - \frac{1}{2} F_{g2}\cos 40°}{\cos 40°}$$

$$= \frac{675\cos 50° - \frac{1}{2}(115)\cos 40°}{\cos 40°}$$

$$= 508.9N$$

$$m = \frac{F_g}{g} = \boxed{52kg}$$