Work, Energy and Momentum Notes

3 – Momentum and Collisions

Momentum is a quantity of motion that depends on both the mass and velocity of the object in question.

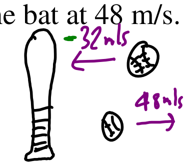
$$\boxed{p = mv}$$

The units of momentum are: kg m/s or N·s

> **Remember**:
> Momentum is a vector quantity, with the same sign as its velocity. As with any vector you can assign any direction as positive and the opposite as negative, but as convention we will refer to **up** or to the **right** as **positive** and **down** or to the **left** as **negative**.

Example: $m = 0.1$ kg

A baseball pitcher hurls a ball at 32 m/s. The batter crushes it and the ball leaves the bat at 48 m/s. What was the ball's change in momentum?



$p = mv$

$\Delta p = m\Delta v \qquad \Delta p = (0.1)(48 - (-32))$

$= \underline{8 \text{ kg m/s}}$

Remember that **momentum** is a VECTOR which means:

left = "−"

right = "+"

Impulse: change in momentum

Recall that momentum is the product of mass and velocity. Since we will not be dealing with changing masses, we can define an object's change in momentum as:

$$\boxed{\Delta p = m\Delta v = F_{net} \cdot t}$$

Whenever a net force acts on a body, an acceleration results and so its momentum must change.

Derivation:

$\Delta p = m\Delta v$

$a = \frac{\Delta v}{t}$

$\Delta v = a \cdot t$

$= mat$

$F_{net} = ma$

$$\boxed{\Delta p = F_{net} \cdot t}$$

Let's try to understand how **forces** relate to **changes in momentum** with a few examples.

A student jumps off a desk. When they land they bend their knees on impact. Why does this help prevent some serious damage to their knees?

$m\Delta v = F_{net} \cdot t$

const. const. ↓ ↑

Coaches for many sports such as baseball, tennis and golf can often be heard telling their athletes to "follow through" with their swing. Why is this so important?

$m\Delta v = F_{net} \cdot t$

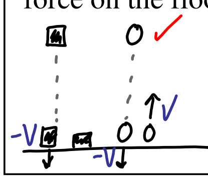
const. ↑ const. ↑

Conventional wisdom suggest that cars should be made tough and rigid to prevent injury during a collision, however newer vehicles are all built with large crumple zones. Why?

$m\Delta v = F_{net} \cdot t$

Const. Const ↓ ↑

A beanbag and a high bounce ball of equal masses are dropped from the same height. The beanbag is brought to a stop in the same time that the ball is in contact with the floor. Which one exerts a greater average force on the floor?



$m\Delta v = F_{net} \cdot t$

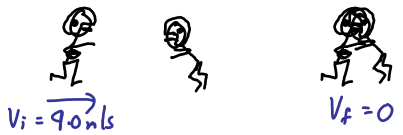
Const. ↑ const.

bouncy ball has larger Δv ∴ larger F_{net}

Example

A 115 kg fullback running at 4.0 m/s East is stopped in 0.75 s by a head-on tackle. Calculate

a) the impulse felt by the fullback.
b) the impulse felt by the tackler.
c) the average net force exerted on the tackler.



a.) $\Delta p = m\Delta v = (115\,kg)(0 - 4.0\,m/s) = \underline{-460\ N\cdot s}$

b.) $\Delta p = F_{net}\cdot t = \underline{460\ N\cdot s}$

same

equal + opposite (Newton's 3rd)

c.) $\Delta p = F_{net}\cdot t$ $\quad F_{net} = \frac{\Delta p}{t} = \frac{460\ N\cdot s}{0.75\,s} = \boxed{613\,N}$

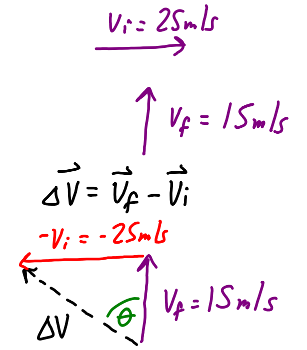
Example

A 1250 kg car traveling east at 25 m/s turns due north and continues on at 15 m/s. What was the impulse of the car exerted while turning the corner?

$\Delta p = m\Delta V$

$\Delta p = (1250\,kg)(29.15\,m/s)$

$= \boxed{36\,000\ N\cdot s \quad 59°\ W\ of\ N}$



$\Delta V = \sqrt{V_f^2 + V_i^2} = 29.15\,m/s$

$\theta = \tan^{-1}\left(\frac{25}{15}\right) = 59°\ W\ of\ N$

The Law of Conservation of Momentum

Momentum is a useful quantity because in a closed system it is always conserved. This means that in any collision, the total momentum before the collision must equal the total momentum after the collision.

There are two ways of thinking about the conservation of momentum:

(1) $p_i = p_f$

$$\boxed{m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}}$$

(2)

$$\boxed{\Delta p_{total} = 0}$$

Collisions can be grouped into two categories,

Elastic Collisions:

E_K is conserved

$E_{Ki} = E_{Kf}$

Inelastic Collisions:

E_K is not conserved

• p is always conserved

• total energy is conserved

In reality collisions are generally somewhere in between perfectly elastic and perfectly inelastic. As a matter of fact, it is impossible for a **macroscopic** collision to ever be perfectly elastic. Perfectly elastic collisions can only occur at the **atomic** or **subatomic level**.

Why can't macroscopic collision ever be truly elastic?

• A change in shape → HEAT
• Sound
• Other vibrations
• Work done

Inelastic

Two rugby players of equal mass collide head on while traveling at the same speed.

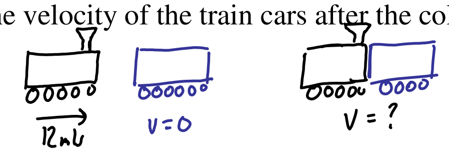
What is their final speed? 0 m/s

Is momentum conserved? Yes

Is energy conserved? Yes

Is kinetic energy conserved? No

A 9500 kg caboose is at rest on some tracks. An 11000 kg engine moving east at 12.0 m/s collides with it and they stick together. What is the velocity of the train cars after the collision?



$$m_1V_{1i} + m_2V_{2i} = m_1V_{1f} + m_2V_{2f}$$

$$m_1V_{1i} + \cancel{m_2V_{2i}}^{0} = m_+V_f$$

$$m_1V_{1i} = m_+V_f$$

$$V_f = \frac{m_1V_{1i}}{m_+} = \frac{(11000\text{ kg})(12.0\text{ m/s})}{(9500 + 11000)} = \boxed{6.4\text{ m/s}}$$

Elastic

A proton traveling at 2×10^3 m/s collides with a stationary proton and comes to rest.

p+ p+ p+ p+

What is the final speed of the other proton? 2×10^3 m/s

Is kinetic energy conserved?

Yes $E_{Ki} = E_{Kf}$

An alpha particle has a mass approximately 4 times larger than a proton. A proton traveling to the right at 3200 m/s strikes a stationary alpha particle it rebounds at 1920 m/s. What is the final speed of the alpha particle?

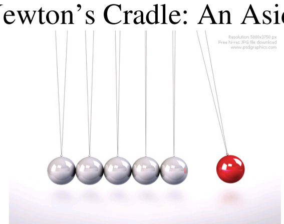
3200 m/s p+ α -1920 m/s p+ $V_f = ?$

$$m_2 = 4m_1$$

$$m_1V_{1i} + \cancel{m_2V_{2i}}^{0} = m_1V_{1f} + m_2V_{2f}$$

$$m_1V_{1i} = m_1V_{1f} + m_2V_{2f} \qquad V_{2f} = \frac{m_1V_{1i} - m_1V_{1f}}{m_2} = \frac{\cancel{m_1}V_{1i} - \cancel{m_1}V_{1f}}{4\cancel{m_1}} = \frac{3200-(-1920)}{4} = \boxed{1280\text{ m/s}}$$

Newton's Cradle: An Aside



A Useful Derivation

Elastic Collision

$$E_K: \cancel{\tfrac{1}{2}}m_1V_{1i}^2 + \cancel{\tfrac{1}{2}}m_2V_{2i}^2 = \cancel{\tfrac{1}{2}}m_1V_{1f}^2 + \cancel{\tfrac{1}{2}}m_2V_{2f}^2$$

$$m_1V_{1i} + m_2V_{2i} = m_1V_{1f} + m_2V_{2f}$$

$$m_1(V_{1i} - V_{1f}) = m_2(V_{2f} - V_{2i})$$

$$m_1V_{1i}^2 - m_1V_{1f}^2 = m_2V_{2f}^2 - m_2V_{2i}^2$$

$$\cancel{m_1}\cancel{(V_{1i} - V_{1f})}(V_{1i} + V_{1f}) = \cancel{m_2}\cancel{(V_{2f} - V_{2i})}(V_{2f} + V_{2i})$$

$$\div \quad \cancel{m_1}\cancel{(V_{1i} - V_{1f})} = \cancel{m_2}\cancel{(V_{2f} - V_{2i})}$$

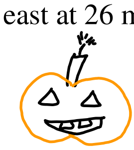
$$V_{1i} + V_{1f} = V_{2f} + V_{2i}$$

$$V_{1i} - V_{2i} = V_{2f} - V_{1f}$$

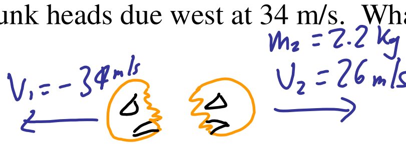
$$\boxed{V_{1i} - V_{2i} = -(V_{1f} - V_{2f})}$$

Explosions

A firecracker is placed in a pumpkin which explodes in into exactly two pieces. The first piece has a mass of 2.2 kg and flies due east at 26 m/s. The second chunk heads due west at 34 m/s. What was the initial mass of the pumpkin?



$p_i = 0$



$$0 = m_1V_{1f} + m_2V_{2f}$$

$$m_1 = \frac{-m_2V_{2f}}{V_{1f}} = \frac{-(2.2)(26)}{-34} = 1.7\text{ kg}$$

$$m_+ = m_1 + m_2$$

$$= 2.2 + 1.7$$

$$= \boxed{3.9\text{ kg}}$$