Work, Energy and Momentum Notes

1 – Work and Energy

Work is defined as the **transfer** of energy from one body to another.
~~Or more rigorously:~~

$$\boxed{W = \Delta E}$$

We can calculate the ~~work done on an~~ object with:

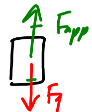
$$\boxed{W = Fd}$$

the units of work are Nm or Joules

Note that these are the same units as **torque** yet their values are used to describe *very* different quantities.

Example 1 - Work against Gravity

How much work is required to lift a 2.0 kg textbook from the floor to a height of 1.5 m at a constant velocity?



Note:
W = Fd, but what force do we need to exert to lift the book at a constant velocity?

$F_{app} = F_g$

$W = F_g d = mgd$
$= (2.0)(9.8)(1.5) = \boxed{29.4\ J}$

Since the velocity is constant what is the net force acting on the book?

$F_{net} = 0$

Example 2 - Work on an object

How much work is done on a 4.0 kg medicine ball that is held at a height of 1.8 m for 10 s?

Note:
Is energy being used to hold the ball in this position? Yes

Is work actually being done ON THE BALL?

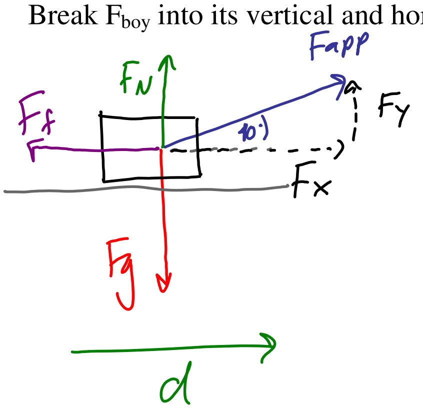
No! $W = 0\ J$

Example 3 – Forces at an angle

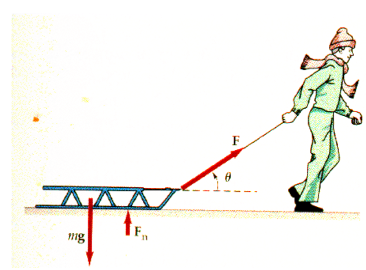
The plucky youngster pictured below is pulling his sled at a constant velocity of 1.2 m/s. He pulls the 15 kg sled with a force of 35 N at an angle of 40° to the horizontal. How much work does he do in pulling the sled 20 m?

Note:
Draw an FBD showing the forces at work on the sled.

Break F_{boy} into its vertical and horizontal components. Does the vertical component of the force **do** any work?



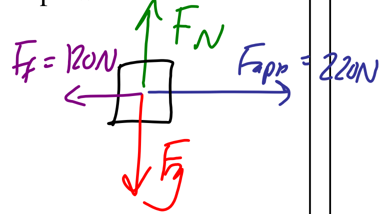
$$W = F_x d$$
$$= F\cos 40° d$$
$$= 35\cos 40° (20)$$
$$= \boxed{536\ J}$$



Rule: When finding the work done on an object we only consider… The component of the force in the direction of movement

Example 4 – F_{net} vs. F_{app}

A biology student is pushing a rope 15 m across a flat surface. The student pushes the rope with a force of 220 N while the force of friction is 120 N. How much work is the student doing?



Note: To find the amount of work done by the student should we used F_{net} or F_{app}?

$$W = F_{app} d = (220N)(15m)$$
$$= \boxed{3300 J}$$

Rule: When finding the **total** work done on an object we always use: F_{app}

Example 5 – To scalar or not to scalar?

Work is the product of a scalar and a vector, but work is a scalar. However work can be positive or negative…but how?

Imagine that you bring a 1.0 kg basketball from the floor to the top of a 1.0 m table. How much work did you do? $U = F_g d = mgd = (1.0)(9.8)(1.0)$
$= +9.8 J$

Which way did you exert the force? up

Did the energy of the ball increase or decrease?

Now suppose the ball rolls off the table and falls straight down to the floor. How much work was done on the ball?

$$W = \Delta E = -9.8 J$$

Which way is the force working on the ball now? down

Did the energy of the ball increase or decrease?

Example 6 - Work-Energy Theorem for Net Force

It is worth noting that the work done by the net force on an object is equal to the change in its kinetic energy: $W = F_{net} d$

$F_{net} = ma$

$v^2 = v_0^2 + 2ad$

$ad = \frac{1}{2}(v^2 - v_0^2)$

$m a d$

$= \frac{1}{2} m (v^2 - v_0^2)$

$= \frac{1}{2} m \Delta v^2 = \boxed{\Delta E_K = F_{net} d}$

Example:
A 1270 kg car accelerates from 15 m/s to 25 m/s over a distance of 75 m. Determine the average net force that was required to do this.

$\Delta E_K = F_{net} d$

$$F_{net} = \frac{\Delta E_K}{d} = \frac{\frac{1}{2} m \Delta v^2}{d} = \frac{\frac{1}{2} m (v^2 - v_0^2)}{d}$$

$$= \frac{\frac{1}{2}(1270)(25^2 - 15^2)}{(75)} = \boxed{3390 N}$$

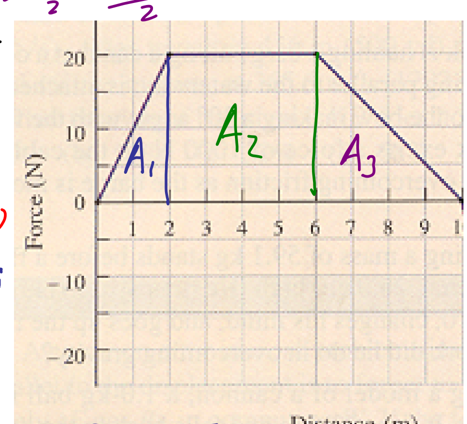
Example:
The graph shows a variable force working on a 15.0 kg mass on a level surface which is initially at rest. Find:
a. The total amount of work done.
b. The final speed of the object assuming friction is negligible.

$A_1 = \frac{b \cdot h}{2} = \frac{(2)(20)}{2} = 20 J$

$A_2 = b \cdot h = (4)(20) = 80 J$

$A_3 = \frac{b \cdot h}{2} = \frac{(4)(20)}{2} = 40 J$

a.) $W_T = 140 J$



b.) $W = \Delta E_K$

$W = E_{Kf} - \cancel{E_{Ki}}$ 0

$W = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(140)}{15}} = \boxed{4.3 m/s}$$

To summarize our work energy relationships:

$W_{Total} = \Delta E_T = F_{app} d$	$\Delta E_K = F_{net}\, d$	Heat $\Delta E_H = F_f\, d$

Types of Energy:

There are many forms of energy: mechanical, thermal, electrical, nuclear, chemical etc. One form can be converted into another by doing work.

In this chapter will be concerned mostly with potential and kinetic (and just a hint of thermal) energy.

Potential Energy (E_p): Stored energy

$$\boxed{E_p = mgh}$$

Remember:
Potential energy is always…relative to a reference point.

Cause math is fun!
Deriving the E_p formula…

$E_p = Fd$
(in this case $F = F_g = mg$)

$E_p = mgd$
(let's change letters just for fun, and call h the vertical displacement)

$E_p = mgh$

HOORAY!

Kinetic Energy (E_k): energy of motion

$$\boxed{E_K = \frac{1}{2}mv^2}$$

Cause math is fun!
Deriving the E_k formula…

$v^2 = v_o^2 + 2ad$
(take $v_o = 0$)
$v^2 = 2ad$
(but $a = F/m$)
$v^2 = \frac{2Fd}{m}$
(but $Fd=W= E_k$, when $v_o = 0$)

$v^2 = \frac{2E_k}{m}$

$E_k = ½mv^2$

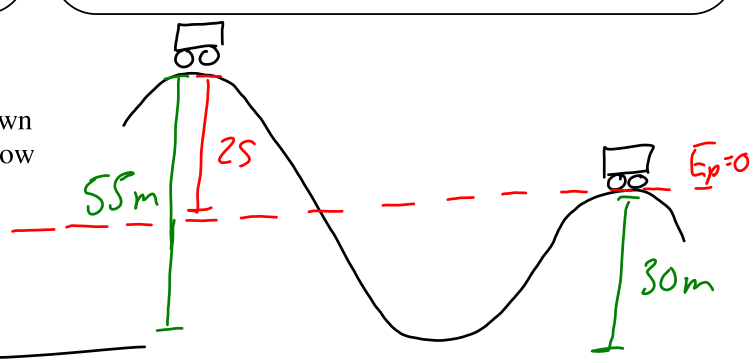
The Law of Conservation of Energy

Energy cannot be created or destroyed, only changed from one form into another.

Therefore in a closed system the total change in energy is always ZERO.

Total Initial Energy = Total Final Energy	Total Change in Energy = 0
$E_{Ki} + E_{pi} = E_{Kf} + E_{pf}$	$\Delta E_K + \Delta E_p = 0$
$\boxed{\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f}$	$\boxed{\Delta E_K = -\Delta E_p}$

Example
The first peak of a roller coaster is 55 m above the ground. The 1200kg car starts from rest and goes down the hill and up the second hill which is 30 m high. How fast is the car traveling at the top of the second hill?



$\cancel{E_{Ki}}^{0} + E_{pi} = E_{Kf} + \cancel{E_{pf}}^{0}$

$E_{pi} = E_{Kf}$

$\cancel{m}gh_i = \frac{1}{2}\cancel{m}v_f^2$

$v_f = \sqrt{2gh_i}$

$= \sqrt{2(9.8)(25)} = \boxed{22.1\ m/s}$

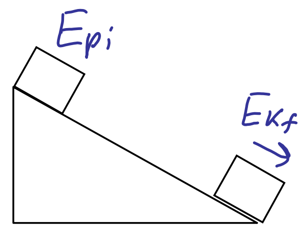
Back in grade 11 it really was that easy…

When **non-conservative** forces (such as *friction*) act on an object, not all energy is transferred between kinetic and potential. This is what physicists have termed *REALITY*. Deal with it.

The “work” done by friction does produce another form of energy known as HEAT (aka. Thermal)

This energy is quickly conducted or radiated in all directions and effectively dispersed.

Consider a block of wood sliding down a ramp with a small amount of friction.



How would the block’s kinetic energy at the bottom compare to its potential energy at the top? Why? $E_{pi} > E_{kf}$ some $E_p \rightarrow E_H$

The fact that the amount of energy in the block decreases as it slides down the ramp doesn’t change the fact that the total energy in the system is **CONSTANT**. We need modify our earlier equation for The Law of Conservation of Energy only slightly:

Total Energy Initial = Total Energy Final

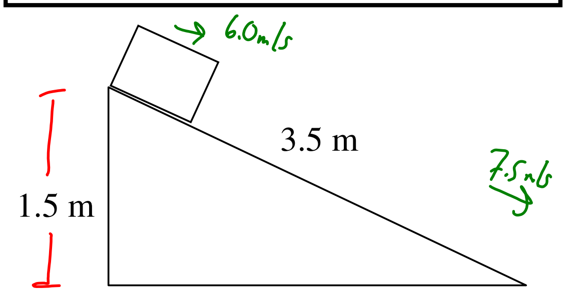
$\Delta E_H \geq 0$

$$\boxed{E_{ki} + E_{pi} = E_{kf} + E_{pf} + E_H}$$

$$\boxed{\Delta E_K + \Delta E_p + \Delta E_H = 0}$$

Example

A 5.0 kg block of wood is now pushed down a ramp with a velocity of 6.0 m/s. At the bottom of the ramp it is traveling at 7.5 m/s.



a. How much thermal energy is generated due to friction?

$$E_{ki} + E_{pi} = E_{kf} + \cancel{E_{pf}}^{\,0} + E_H$$

$$E_H = E_{ki} + E_{pi} - E_{kf}$$

$$E_H = \frac{1}{2}mv_i^2 + mgh_i - \frac{1}{2}mv_f^2$$

$$= \frac{1}{2}(5.0)(6.0)^2 + (5.0)(9.8)(1.5) - \frac{1}{2}(5.0)(7.5)^2 = \boxed{22.9\ J}$$

b. Determine the force of friction.

$$\Delta E_H = F_f d$$

$$F_f = \frac{\Delta E_H}{d} = \frac{22.9\ J}{3.5\ m}$$

$$= \boxed{6.5\ N}$$