

F Math 12

**2.0 Getting Started p. 78**

Name \_\_\_\_\_

Date \_\_\_\_\_

Doris works as a personal loan manager at a bank. It is her job to decide whether the bank should lend money to a customer. When she approves a loan, she thinks of it as the bank making an investment in the person who is borrowing the money. Doris is considering a loan application from Leandro, who wants to borrow \$10 000 to renovate his garage so that he can use it as a workshop. She expects the money borrowed plus interest to be repaid as a single payment at the end of 2 years. She is considering the following three loan options for Leandro:

Option A: A loan at 6% simple interest

Option B: A loan at 5.5% compound interest with annual compounding

Option C: A loan at 5% compound interest with semi-annual compounding

Which option is most beneficial for the bank, and which is most beneficial for Leandro?

	A	B	C
N =		N = 2	N = 2 × 2
I% =	use $A = P(1 + rt)$	I% = 5.5	I% = 5%
PV =		PV = 10 000	PV = 10 000
PMT =		PMT = 0	PMT = 0
FV =		FV = \$11 130.25	FV = -\$11 038.13
P/Y =		P/Y = 1	P/Y = 2
C/Y =		C/Y = 1	C/Y = 2

$$A = 10000(1 + 0.06(2))$$

$$A = 10600(1.12)$$

$$= 11200$$

A. Why do you think Doris considers a bank loan as an investment?

The bank will collect interest on the loan, making money

B. Why is it difficult to predict which option is most beneficial to the bank or to Leandro?

- small time period
- similar interest rates

C. For option A, how much would Leandro need to repay at the end of the term? How much of this amount is interest?

\$ 11 200 ; \$ 1200 is interest

$$I = A - P$$

D. For option B, how much would Leandro need to repay? How much of this amount is interest?

\$ 11 130.25 ; \$ 130.25 interest

E. For option C, how much would Leandro need to repay? How much of this amount is interest?

\$ 11 038.13 ; \$ 1038.13 is interest

F. Which of the three options is most beneficial for the bank? Which is most beneficial for Leandro? Explain.

Bank makes the most money from option A  
Leandro saves the most money from option C

G. Consider a fourth loan option: Option D: A loan at 5% interest, compounded semi-annually, with payments of \$2658.18 at the end of every 6-month period for 2 years

i) Complete the following table to show the repayment of the loan.

Payment Period	Payment (\$)	Interest Paid (\$) [Balance · $\left(\frac{0.05}{2}\right)$ ]	Principal Paid (\$) [Payment – Interest]	Balance (\$) [Balance – Principal Paid]
0				10000.00
1	2658.18	250.60	2408.18	7591.82
2	2658.18	189.80	2468.34	5123.44
3	2658.18	128.09	2530.09	2593.36
4	2658.18	64.93	2593.35	0
Total	10672.72	632.72	10000	

ii) What do you notice about the pattern in the values in each column? What other relationships do you notice in the table?

- payment stays the same
- Interest paid decrease over the life of loan
- Principal paid increase over the life of loan
- Interest plus Principal equals Payment
- Principal is applied to the outstanding balance of the loan

H. Which of the four options is most beneficial for the bank? Which is most beneficial for Leandro? Explain.

Still option A for the Bank → most interest  
Now, option D for Leandro → least interest

HW: Diagnostic Test #1-8

**2.1 Analyzing Loans p. 80**

Name \_\_\_\_\_

Date \_\_\_\_\_

**Goal:** Solve problems that involve single payment loans and regular payment loans.

1. **collateral:** An asset that is held as security against the repayment of a loan.
2. **amortization table:** A table that lists regular payments of a loan and shows how much of each payment goes toward the interest charged and the principal borrowed, as the balance of the loan is reduced to zero.
3. **mortgage:** A loan usually for the purchase of real estate, with the real estate purchased used as collateral to secure the loan.

**Investigate** the Math

Lars borrowed \$12 000 from a bank at 5%, compounded monthly, to buy a new personal watercraft. The bank will use the watercraft as **collateral** for the loan. Lars negotiated regular loan payments of \$350 at the end of each month until the loan is paid off. Lars set up an **amortization table** to follow the progress of his loan.

Payment Period (month)	Payment (\$)	Interest Paid (\$) $\left[ \text{Balance} \cdot \left( \frac{0.05}{12} \right) \right]$	Principal Paid (\$) [Payment – Interest]	Balance (\$)
0				12 000.00
1	350	50.00	300.00	11 700.00
2	350	48.75	301.25	11 398.75

How much will Lars still owe at the end of the first year?

A. Complete Lars's amortization table for the first year.

Payment Period (month)	Payment (\$)	Interest Paid (\$) $[Balance \cdot (\frac{0.05}{12})]$	Principal Paid (\$) [Payment - Interest Paid]	Balance (\$)
2	350	48.75	301.25	11 398.75
3	350	47.49	302.51	11 096.24
4	350	46.23	303.77	10 792.48
5	350	44.97	305.03	10 487.45
6	350	43.70	306.30	10 181.15
7	350	42.42	307.58	9 873.57
8	350	41.14	308.86	9 564.71
9	350	39.85	310.15	9 254.56
10	350	38.56	311.44	8 943.12
11	350	37.26	312.74	8 630.38
12	350	35.96	314.04	8 316.34

$$C = A - B$$

Balance = prev. Bal. -  
Principal Paid  
(D)

A                      B                      C                      D

B. At the end of the first year,

i) how much has Lars paid altogether in loan payments?  $\$350 \times 12 = \$4200$

ii) how much interest has he paid altogether?  $\$516.34$

iii) how much of the principal has he paid back?

$$12\ 000 - 8\ 316.38 = \$3\ 683.62$$

C. At the end of the first year, what is the balance of Lars's loan?

$$\$8\ 316.38$$

**Example 1:** Solving for the term and total interest of a loan with regular payments (p.81)

As described on page 80, Lars borrowed \$12 000 at 5%, compounded monthly. After 1 year of payments, he still had a balance owing.

- a) In which month will Lars have at least half of the loan paid off?
- b) How long will it take Lars to pay off the loan?
- \* c) How much interest will Lars have paid by the time he has paid off the loan?

$N = ? \quad 37.07 \rightarrow 38 \text{ months (b)}$

$I\% = 6\%$

$PV = 12\,000$

$PMT = -350$

$FV = 0$

$P/Y = 12$

$C/Y = 12$

(a) after the 20<sup>th</sup> month

$N:$

$I\% : 5$

$PV : 12\,000$

$PMT : -350$

$FV : -6000$

$P/Y : 12$

$C/Y : 12$

b/c you are given money make PV positive

Amortization Table

$Y_1 = \text{term} - \text{Pmt}$

$Y_2 = \sum \text{Int}(x, x)$

$Y_3 = \sum \text{Pm}(x, x)$

$Y_4 = \text{bal}(x)$

c) need to use  $\sum \text{Int}$  function on calc.

$\sum \text{Int}(1, 38)$

↑                      ↑  
1<sup>st</sup> month      38<sup>th</sup> month  
int.                      int.

$\$ 975.66$

**Example #2:** Solving for the future value of a loan with a single loan payment (p.83)

Trina's employer loaned her \$10 000 at a fixed interest rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in a single payment on the maturity date, which is at the end of 5 years.

- a) How much will Trina need to pay her employer on the maturity date? What is the accumulated interest on the loan?
- b) Graph the total interest paid over 5 years. Describe and explain the shape of the graph.
- c) Suppose the interest was compounded monthly instead. Graph the total interest paid over 5 years. Compare it with your annual compounding graph from part b).

monthly      yearly  
 60 N = 5  
 6% I% = 6%  
 10 000 PV = 10 000  
 0 PMT = 0  
 +13 488.50 FV = -13 382.26  
 12 P/Y = 1  
 12 C/Y = 1

a) she will owe \$ 13 382.26

$$I = A - P$$

$$= 13\,382.26 - 10\,000$$

$$= \$3\,382.26$$

b)  $A = P(1+i)^n$

↑  
 don't need to use  
 use "Σ Int"

c)

Year	Σ Interest yearly	Σ Int monthly
1	600	616.76
2	1236	1271.60
3	1910.16	1966.81
4	2624.77	2704.89
5	3382.26	3488.50

Compounding monthly means you pay more interest

**Example #3:** Solving for the present value and interest of a loan with a single payment (p.86)

Annette wants a home improvement loan to renovate her kitchen. Her bank will charge her 3.6%, compounded quarterly. She already has a 10-year GIC that will mature in 5 years. When her GIC reaches maturity, Annette wants to use the money to repay the home improvement loan with one payment. She wants the amount of the payment to be no more than \$20 000.

- How much can she borrow?
- How much interest will she pay?

Solve by hand and then check using the TVM Solver

$$\begin{array}{l}
 N: 5 \times 4 \\
 I\%: 3.6\% \\
 PV: ? \\
 PMT: 0 \\
 FV: -20000 \\
 P/Y: 4 \\
 C/Y: 4
 \end{array}
 \left. \vphantom{\begin{array}{l} N: 5 \times 4 \\ I\%: 3.6\% \\ PV: ? \\ PMT: 0 \\ FV: -20000 \\ P/Y: 4 \\ C/Y: 4 \end{array}} \right\} *$$

\* try to think of this as an Investment by the bank.

easier to solve by hand!

$$a) A = P(1+i)^n$$

$$20000 = P\left(1 + \frac{0.036}{4}\right)^{5 \times 4}$$

$$20000 = P(1 + 0.009)^{20}$$

$$\frac{20000}{(1.009)^{20}} = \frac{P(1.009)^{20}}{(1.009)^{20}}$$

$$\$ 16718.86 = P$$

$$b) I = A - P$$

$$= 20000 - 16718.86$$

$$= \$ 3281.14$$



**Example #5:** Relating payment and compounding frequency to interest charged (p.89)

Bill has been offered the following two loan options for borrowing \$8000. What advice would you give?

Option A: He can borrow at 4.06% interest, compounded annually, and pay off the loan in payments of \$1800.05 at the end of each year.

Option B: He can borrow at 4.06% interest, compounded weekly, and pay off the loan in payments of \$34.62 at the end of each week.

*	N =	A 5 years	B 255 weeks	$255 \text{ week} \times \frac{1 \text{ year}}{52 \text{ weeks}}$
	I% =	4.06	4.06	
	PV =	8000	8000	
	PMT =	-1800.05	-34.62	
	FV =	0	0	4.90 years
	P/Y =	1	52	
	C/Y =	1	52	4 years 47 weeks
		$5 \times \$1800.05$		$255 \times 34.64$
		$\$ 9000.25$		$\$ 8828.10$
		$I = 1000.25$		$I = 828.10$

Option B pays off quicker and pays less interest.

Paying off a loan quicker with less interest is always better!

**In Summary**

**Key Ideas**

- The large majority of commercial loans are compound interest loans, although simple interest loans are also available.
- The cost of a loan is the interest charged over the term of the loan.
- A loan can involve regular loan payments over the term of the loan or a single payment at the end of the term.
- The same formulas that are used for investment situations are also used for loans with a single payment at the end of the term:
  - For a loan that charges simple interest,  $A = P + Prt$  or  $A = P(1 + rt)$
  - For a loan that charges compound interest,  $A = P(1 + i)^n$
- Technology can be used to determine unknown variables in compound interest loan situations for both single payment loans and regular payment loans.

**Need to Know**

- The interest that is charged on a loan will be less under any or all of these conditions:
  - The interest rate is decreased.
  - The interest compounding frequency is decreased.
  - Regular payments are made.
  - The regular payment amount is increased.
  - The payment frequency is increased.
  - The term is decreased.
- An amortization table is a payment schedule for a loan with regular payments. It shows what happens in each payment period. It shows the amount of each payment, the interest and the principal portion of each payment, and the balance of the loan. An amortization table can be created with spreadsheet software.

Payment Period	Payment (\$)	Interest Paid (\$)	Principal Paid (\$)	Balance (\$)
0				
1				
2				

- With each payment period, the interest paid decreases while the principal paid increases. This occurs because each payment decreases the balance of the loan, so the interest on the remainder of the balance will be less on the next payment. Also, because the payment amount stays the same, more of the payment goes toward paying off the principal, since less is being paid toward the interest.
- Technology can be used to investigate and analyze "what if" situations that involve borrowing money.

F Math 12

**2.2 Exploring Credit Card Use p. 98**

Name \_\_\_\_\_

Date \_\_\_\_\_

Goal: Compare credit options that are available to consumers.

**EXPLORE** the Math

Jayden saw the new sound system he wanted on sale for \$2623.95, including taxes. He had to buy it on credit and had two options:

- Use his new bank credit card, which has an interest rate of 14.5%, compounded daily. (Because this credit card is new, he has no outstanding balance from the previous month.)
- Apply for the store credit card, which offers an immediate rebate of \$100 on the price but has an interest rate of 19.3%, compounded daily.

As with most credit cards, Jayden would not pay any interest if he paid off the balance before the due date on his first statement. However, Jayden cannot afford to do this. Both cards require a minimum monthly payment of 2.1% on the outstanding balance, but Jayden is confident that he can make regular monthly payments of \$110.

Which credit card is the better option for Jayden, and why?

**Bank CC**

N = 29  
 I% = 14.5  
 PV = 2623.95  
 PMT = -110  
 FV = 0  
 P/Y = 12  
 C/Y = 365

**Store CC**

N = 29  
 I% = 19.3  
 PV = 2523.95  
 PMT = -110  
 FV = 0  
 P/Y = 12  
 C/Y = 365

Interest: \$493.73

Interest: \$657.83

- A. Jayden could make smaller payments each month or he could pay a different amount each month, as long as each payment is at least 2.1% of the outstanding balance. Why would he choose to make regular payments of \$110 instead?

$$2.1\% \text{ of } \$2623.95 = \$55.10$$

He would choose to make payments of \$110 to pay off sooner with **less interest.**

- B. With a partner, decide which credit card, his new bank card or the store card, would be the better option if the conditions were changed as described below. Provide your reasoning.

- I. The store credit card offers an immediate rebate of \$200, instead of \$100.
- II. The store credit card offers an immediate rebate of \$200, instead of \$100, and has an interest rate of 20.3%, compounded daily.
- III. ~~The store credit card offers an immediate rebate of \$200 and has an interest rate of 20.3%, compounded daily.~~ Jayden's new bank credit card has an interest rate of 13.5%, instead of 14.5%, compounded daily.

I.	II.	III.
N = 28	N = 28	N = 28
I% = 15.3	I% = 20.3	I% = 13.5
PV = 2423.95	PV = 2423.95	PV = 2623.95
PMT = -110	PMT = -110	PMT = -110
FV = 0	FV = 0	FV = 0
P/Y = 12	P/Y = 12	P/Y = 12
C/Y = 365	C/Y = 365	C/Y = 365

$$\Sigma \text{Int} = \$599.34$$

$$\Sigma \text{Int} = \$641.93$$

$$\Sigma \text{Int} = \$451.35$$

Option III is the best  $\rightarrow$  bank CC @ 13.5%

- A. Jayden could make smaller payments each month or he could pay a different amount each month, as long as each payment is at least 2.1% of the outstanding balance. Why would he choose to make regular payments of \$110 instead?

$$2.1\% \text{ of } \$2623.95 = \$55.10$$

He would choose to make payments of \$110 to pay off sooner with **less interest.**

- B. With a partner, decide which credit card, his new bank card or the store card, would be the better option if the conditions were changed as described below. Provide your reasoning.

- I. The store credit card offers an immediate rebate of \$200, instead of \$100.
- II. The store credit card offers an immediate rebate of \$200, instead of \$100, and has an interest rate of 20.3%, compounded daily.
- III. ~~The store credit card offers an immediate rebate of \$200 and has an interest rate of 20.3%, compounded daily.~~ Jayden's new bank credit card has an interest rate of 13.5%, instead of 14.5%, compounded daily.

I.	II.	III.
N = 28	N = 28	N = 28
I% = 15.3	I% = 20.3	I% = 13.5
PV = 2423.95	PV = 2423.95	PV = 2623.95
PMT = -110	PMT = -110	PMT = -110
FV = 0	FV = 0	FV = 0
P/Y = 12	P/Y = 12	P/Y = 12
C/Y = 365	C/Y = 365	C/Y = 365

$$\Sigma \text{Int} = \$599.34$$

$$\Sigma \text{Int} = \$641.93$$

$$\Sigma \text{Int} = \$451.35$$

In Summary
<b>Key Ideas</b> <ul style="list-style-type: none"><li>• Incentives or promotions are sometimes offered to entice people to use credit cards. For example, an immediate cash rebate may be offered on the first purchase using a credit card. Low interest rates, rewards, or no annual fees may also be offered.</li><li>• The full cost of borrowing should be considered before making a decision about using a credit card. This includes the total interest charged, as well as the total payments and the time it will take to pay off the balance.</li></ul>
<b>Need to Know</b> <ul style="list-style-type: none"><li>• Credit cards usually have a minimum amount that must be paid each month, based on a percent of the outstanding balance. If there is no outstanding balance from the previous month and the new balance is paid off in full by the payment due date, no interest is charged.</li><li>• If a credit card does not have an outstanding balance and it is used for a single purchase, it can be treated as a loan. The purchase price is the principal borrowed, and regular payments can be made until the balance is paid off.</li><li>• The cost of using credit is not just the amount of interest charged. There are incentives, such as cash rebates, that reduce the principal. This may end up costing more in interest but result in a lower total loan payment amount.</li></ul>

**2.3 Solving Problems Involving Credit p. 104**

Name \_\_\_\_\_

Date \_\_\_\_\_

**Goal:** Solve problems that involve credit.

- line of credit:** A pre-approved loan that offers immediate access to funds, upto a pre-defined limit, with a minimum monthly payment based on accumulated interest; a **secure line of credit** has a lower interest rate because it is guaranteed against the client's assets, usually property.
- Bank of Canada prime rate:** A value set by Canada's central bank, which other financial institutions use to set their interest rates.

**INVESTIGATE** the Math

Liam wants to buy a carving by Inuvialuit artist Eli Nasogaluak. He thinks it will cost \$3900 and is considering these two credit options:

- A **line of credit**, which has a limit of \$10 000 and an interest rate of 2%, compounded daily, above the **Bank of Canada prime rate** (which is currently 0.5%), to be repaid in 16 monthly payments
- A bank loan at 4%, compounded monthly, to be repaid in one payment at the end of the term

Liam chose the bank loan when he found out that the interest amount would be the same as he would pay if he used the line of credit.

What is the term for Liam's bank loan?

**LOC**  
 N = 16  
 I% = 2 + 0.5  
 PV = 3900  
 \* PMT = -248.09  
 FV = 0  
 P/Y = 12  
 C/Y = 365

$\leq \text{Int}(1, 16)$   
 \$ 69.49

**Bank Loan**  
 N = 5.367 ...  
 I% = 4%  
 PV = -3900  
 PMT = 0  
 FV = 3969.49  
 \* P/Y = 12  
 † C/Y = 12

5.307 months or 0.442 years

monthly payments  $\therefore$  6 months of payments

‡ with a lump sum, P/Y and C/Y need to be the same.

$\rightarrow 2.5\%$

lump sum

A. How much interest would Liam pay if he used the line of credit?

\$ 69.49

B. Predict whether the term for Liam's bank loan will be more or less than 16 months. Explain.

The term has to be less  $\because$  the interest rate is higher on the bank loan

C. What term for the bank loan will accumulate the same amount of interest as the line of credit?

6 months

D. Why do you think Liam chose the bank loan over the line of credit?

- ① can't make monthly payments initially
- ② needs to use cash for something else right now

**Example 1:** Solving a Credit Problem that Involves overall cost and number of payments (p.105)

Meryl and Kyle are buying furniture worth \$1075 on credit. They can make monthly payments of \$75 and have two credit options. Which option should they choose? Explain.

Option A: The furniture store credit card, which is offering a \$100 rebate off the purchase price and an interest rate of 18.7%, compounded daily

Option B: A new bank credit card, which has an interest rate of 15.4%, compounded daily, but no interest for the first year

**Option A**

\* N = 4.654...

I% = 18.7

PV = 1075 - 100

PMT = -75

FV = 0

P/Y = 12

C/Y = 365

15 payments

$\sum \ln(1, 15)$

\$ 124.20

Total Cost: 975 + 124.20

\$ 1099.20

**Option B**

\* N = 2.384...

I% = 15.4

PV = 1075 - (\$75 x 12) = 775

PMT = -75

FV = 0

P/Y = 12

C/Y = 365

$\sum \ln(1, 3)$

\$ 3.95

Total Cost: 1075 + 3.95

\$ 1078.95

need to subtract the non-interest payments

They should use the bank CC

**Example 2:** Solving a credit problem that involves payment amount and overall cost (p.106)

Ed wants to buy a car and needs to use credit to finance it. The cost, with taxes and shipping, is \$24 738. Ed wants to repay his loan in 4 years using monthly payments and has two credit options:

- His secured line of credit at 1.7%, compounded monthly, above the Bank of Canada rate, which is currently 0.5%
- The dealership's financing plan at 2.5%, compounded daily

a) Which option should he choose? Why?

LOC	Dealership Financing
N = $4 \times 12$	N = $4 \times 12$
i% = $1.7 + 0.5$	i% = 2.5
PV = 24 738	PV = 24 738
* PMT = -538.86	* PMT = -542.14
FV = 0	FV = 0
P/Y = 12	P/Y = 12
C/Y = 12	C/Y = 365
$\Sigma \text{Int} (1, 48)$	$\Sigma \text{Int} (1, 48)$
\$1 127.09	\$1 284.56

The line of credit is the better deal  
(less interest paid)

- b) Suppose that the Bank of Canada rate changed to 1.1% after 2 years. How would this affect his line of credit payments if he still wanted to pay off the loan in 4 years?

Years 1 & 2	Years 3 & 4
N = 2 x 12	N = 2 x 12
i% = 2.2 (1.7 + 0.5)	i% = 2.8 (1.7 + 1.1)
PV = 24 734	PV = 12 640.83
PMT = - 538.86	* PMT = 542.20
* FV = -12 640.83	FV = 0
P/Y = 12	P/Y = 12
C/Y = 12	C/Y = 12

total payments

$$24 \times \$538.86 + 24 \times \$542.20 = \$25\,945.44 \text{ (LOC)}$$

$$48 \times \$542.14 = \$26\,022.72 \text{ (Dealership)}$$

- c) If the Bank of Canada rate changed as described in part b), does your answer to part a) change? Explain.

No, an increase in the BoC prime to 1.1% doesn't make the dealership financing a better choice.

**Example 3:** Solving a problem that involves interest amount and rate (p. 109)

Jon's \$475 car insurance payment is due. He does not have enough cash to make the payment, so he is considering these two credit options:

- Borrow the money from a payday loan company for a \$100 fee if it is paid back in full within 2 months.
- Get a cash advance on his credit card, which is carrying a zero balance. The interest charged for cash advances is 19.99%, compounded daily, and takes effect immediately. He can afford to pay the required \$5 minimum payment after the first month and then plans to pay off the balance in full at the end of the second month.

as soon as there is a payment, do the question by hand

- Which is the better option for Jon? Explain.
- What annual interest rate would equate to the fee charged by the payday loan company?

a) Cost of payday loan: \$100

Cost of CC loan:  $A = P(1+i)^n$

$$A = 475 \left(1 + \frac{0.1999}{365}\right)^{\frac{365}{12}}$$

$$= 482.976\dots$$

\$5 payment @ end of 1 month

$$A = (482.976\dots - 5) \left(1 + \frac{0.1999}{365}\right)^{\frac{365}{12}}$$

$$= \$486.00$$

Cost of CC advance:  $486 - 475$   
\$11.00

b)  $A = P(1+i)^n$

$$\frac{575}{475} = \frac{475(1+i)^2}{475}$$

monthly interest rate  
∴ compounding period is a month

$$\sqrt{1.210\dots} = \sqrt{(1+i)^2}$$

$$1.100 = 1+i$$

$$0.10 = i$$

$$i = 10\% \text{ (monthly interest rate)}$$

$$10\% \times 12 = 120\% / \text{a}$$

**Example 4:** Solving a debt consolidation problem that involves an interest amount (p.110)

Nicki wants to be debt-free in 5 years. She has two credit cards on which she makes monthly payments:

- Card A has a balance of \$2436.98 and an interest rate of 18.5%, compounded daily.
- Card B has a balance of \$3043.26 and an interest rate of 19%, compounded daily.

Nicki has qualified for a line of credit at her bank with an interest rate of 9.6%, compounded monthly, and a credit limit of \$6000. She plans to pay off both credit card balances by borrowing the money from her line of credit. How much interest will she save?

Consolidated	Card A	Card B
N = $5 \times 12$	N = $5 \times 12$	N = $5 \times 12$
I% = 9.6	I% = 18.5	I% = 19
PV = $2436.98 + 3043.26$	PV = 2436.98	PV = 3043.26
* PMT = -115.36	* PMT = -62.73	* PMT = -79.19
FV = 0	FV = 0	FV = 0
P/Y = 12	P/Y = 12	P/Y = 12
C/Y = 12	C/Y = 365	C/Y = 365
$\sum \text{Int}(1,60)$	$\sum \text{Int}(1,60)$	$\sum \text{Int}(1,60)$
\$ 1441.55	\$ 1327.00	\$ 1708.07

Interest saved: CC - LOC

$$\$1327 + 1708.07 - 1441.55$$

**\$ 1593.52 saved by consolidating**



In Summary
<b>Key Ideas</b> <ul style="list-style-type: none"><li>• Forms of credit that can be used to make purchases or acquire cash include bank loans, lines of credit, credit cards, payday loans, and dealership or in-store financing.</li><li>• There are many factors that determine the best credit option, such as the interest charged, the total payment, the amount of each payment, and the length of time it takes to pay off the loan. All of these factors must be considered carefully before making a decision.</li></ul>
<b>Need to Know</b> <ul style="list-style-type: none"><li>• Credit cards have a credit limit, which is the maximum amount you can borrow. The credit limit varies from person to person, based on credit history.</li><li>• Cash advances on credit cards have no period in which no interest is charged and sometimes have a greater interest rate than purchases.</li><li>• A line of credit has a lower interest rate than most loans and credit cards. Because of this, a line of credit can be useful for consolidating debt.</li><li>• As with a credit card, a line of credit allows for flexibility in how the loan is paid back, as long as the minimum payment is made. The minimum payment is often based on the accumulated interest each month.</li><li>• Credit that is offered in conjunction with a special offer or promotion must be considered very carefully. There may be conditions for how the loan is paid back, which may result in unexpected costs or penalties.</li><li>• Payday loans must also be considered carefully, since the fee for borrowing is often high.</li><li>• An amortization table is particularly useful when you need to know interim values and when payment amounts or interest rates vary throughout the term of a loan.</li></ul>

HW: 2.3 p. 114-118 #4, 7, 9, 11 & 15

F Math 12

**2.4 Buy, Rent or Lease p. 120**

Name \_\_\_\_\_

Date \_\_\_\_\_

**Goal:** Solve problems by analyzing renting, leasing, and buying options.

1. **lease:** A contract for purchasing the use of property, such as a building or vehicle, from another, the lessor, for a specified period.
2. **equity:** The difference between the value of an item and the amount still owing on it; can be thought of as the portion owned. For example, if a \$25 000 down payment is made on a \$230 000 home, \$205 000 is still owing and \$25 000 is the equity or portion owned.
3. **asset:** An item or a portion of an item owned; also known as property. Assets include such items as real estate, investment portfolios, vehicles, art, and gems.
4. **appreciation:** increase in the value of an asset over time.
5. **depreciation:** Decrease in the value of an asset over time.
6. **disposable income:** The amount of income that someone has available to spend after all regular expenses and taxes have been deducted.

$$A = P(1 - R)^n$$

Depreciation  
Formula

where  $A$  = future value

$P$  = present value (principal)

$R$  = depreciation rate

$n$  = number of depreciating periods

**LEARN ABOUT** the Math

Amanda is a civil engineer. She needs a vehicle for work, on average, 12 days each month. She has been renting a vehicle when she needs it. The advantage to renting is that she simply fills the gas tank and drops off the vehicle when she is done with it. The disadvantage is that she has to spend time arranging for the rental, picking up the vehicle, and getting home after dropping it off. She is wondering if renting is the most economical choice and is considering her options:

- She could **lease** a vehicle, which requires a down payment of \$4000 and lease payments of \$380 per month plus tax. She would need insurance at \$1220 each year (which could be paid monthly) and would have to pay for repairs and some maintenance, which would average \$50 each month. For the 4-year lease she is looking at, she would have no **equity** in the vehicle at the end of the term, since the car would belong to the leasing company.
- She could buy a vehicle for \$32 800 and finance it for a 4-year term at 4.5% interest, compounded monthly. She would have the same insurance, repair, and maintenance costs that she would have with leasing. However, the equity of the vehicle would be considered an **asset**.
- She could continue to rent at \$49.99 per day, plus tax, with unlimited kilometres.

Which option would you recommend for Amanda, and why?

**Example 1:** Solving a problem that involves leasing, buying, or renting a vehicle (p.121)

Figure out the monthly cost for the three options listed above.

Lease

$$\underbrace{\$4000}_{\text{down payment}} + \underbrace{\$380 \times 12 \times 4}_{\text{monthly payment}} + \underbrace{\$1220 \times 4}_{\text{insurance}} + \underbrace{\$50 \times 12 \times 4}_{\text{maintenance}}$$

\$29 520 → total cost of leasing  
for 4 years

$$\frac{\$29\,520}{12 \cdot 4} = \$615 \text{ per month}$$

## Buying

$$N = 4 \times 12$$

$$I\% = 4.5$$

$$PV = 32800$$

$$* PMT = -747.95 \rightarrow \text{monthly payment}$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

$$\underbrace{\$ 747.95}_{\text{payment}} + \underbrace{\frac{\$1220}{12}}_{\text{insurance}} + \underbrace{\$50}_{\text{repairs}}$$

$$\underline{\$ 899.62}$$

## Renting

$$\$49.99 \times 12 = \underline{\$ 599.88}$$

$$\text{Leasing: } \$615 \quad \text{Buying: } \$899.62 \quad \text{Renting: } \$599.88$$

Recommendation would vary based on need

**Example 2:** Solving a problem that involves vehicle depreciation (p.122)

A luxury vehicle rental company depreciates the value of its vehicles each year over 5 years. At the end of the fifth year, the company writes off a vehicle for its scrap value. The company uses a depreciation rate of 40% a year.

- a) What is the scrap value of each car below?  
 i) Car A, which is currently 2 years old and has a value of \$43 200  
 ii) Car B, which is currently 1 year old and has a value of \$75 600
- b) What was the original purchase price of each car?

$$a) \ i) \ A = 43200(1-0.40)^3$$

$$= \$9331.20$$

$$ii) \ A = 75600(1-0.40)^4$$

$$= \$9797.76$$

$$A = P(1-R)^n$$

5-2  
 ↑    ↑  
 scrap current  
 age    age

Car A

$$b) \ \frac{43200}{(1-0.40)^2} = \frac{P(1-0.40)^2}{(1-0.40)^2}$$

$$\frac{43200}{0.60^2} = P$$

$$\$120000 = P$$

Car B

$$\frac{75600}{1-0.40} = \frac{P(1-0.40)^1}{1-0.40}$$

$$\frac{75600}{0.60} = P$$

$$\$126000 = P$$

**Example 3:** Solving a problem that involves leasing or buying a water heater (p. 124)

The 10-year-old hot water heater in Tom's home stopped working, so he needs a new one. Tom works for minimum wage. After paying his monthly expenses, he has \$35 **disposable income** left. He has an unused credit card that charges 18.7%, compounded daily. He has two options:

- Tom could lease from his utility company for \$17.25 per month. This would include parts and service.
  - He could buy a water heater for \$712.99, plus an installation fee of \$250, using his credit card. He could afford to pay no more than \$35 each month.
- a) What costs are associated with buying and leasing?
  - b) What do you recommend for Tom? Justify your recommendation.
  - c) Suppose that the life expectancy of a water heater is 8 years. Would this change your recommendation?

Buying on CC	Lease
* $N: 36.307... \rightarrow 37$	$\$17.25 \times 37$
$i\%: 18.7$	<b><math>\\$638.25</math></b>
$PV: 712.99 + 250$	
$PMT: -35$	the lease company
$FV: 0$	would cover repairs
$P/Y: 12$	
$C/Y: 365$	
total cost: $N \cdot PMT$	
<b><math>\\$1270.76</math></b>	

b) It makes more sense to lease - lower cost, no repair cost

c)  $\$17.25 \times 12 \times 8$   
 $\$1656$

total lease cost exceeds purchase price,  $\therefore$  he should buy

**Example 5:** Solving a problem that involves renting or buying a house (p. 127)

Two couples made different decisions about whether to rent or buy:

- a) Helen and Tim bought a house for \$249 900. They have negotiated a mortgage of 95% of the purchase price, so they will need a 5% down payment. The mortgage is compounded semi-annually at 5.5%, has a 20-year term, and requires monthly payments.
- b) Don and Pat are renting a house for \$1600 per month. They plan to renew the lease yearly. After 3 years, both couples decide to move. Helen and Tim discover that the value of their house has depreciated by 10% over the 3 years.

Compare each couple's situation after 3 years.

$$a) \$249\,900 - \underbrace{\$12\,495}_{\text{down payment}} = \$237\,405$$

Renting

$$b) \$1600 \times 12 \times 3 = \underline{\$57\,600}$$

↑  
Cost of renting

N: 20 x 12	}	monthly mortgage payment is \$1624.78
i%: 5.5		
PV: 237 405		
* PMT: -1624.78		
FV: 0		
P/Y: 12		
C/Y: 2		

N: 3 x 12	}	to find out how much is left on the mortgage
i%: 5.5		
PV: 237 405		
* PMT: -1624.78		
FV: -215 992.56		
P/Y: 12		
C/Y: 2		

depreciation  
↓  
1-R

new value of the house:  $\$249\,900(0.90) = \$224\,910$

paying back the bank:  $224\,910 - 215\,992.56 = \underline{\$8\,917.44}$  profit from selling the house

Cost of owning:  $\$12\,495 + 3 \times 12 \times 1624.78 - 8\,917.44$

↑  
down payment      mortgage payments      profit from selling

$\underline{\$62\,069.64}$

The renters spent \$57 600, the owners spent \$62 069.64

### In Summary

#### Key Ideas

- When deciding whether to rent, buy (with or without financing), or lease, each situation is unique. A cost and benefit analysis should take everything into account.
  - Costs include initial costs and fees, short-term costs, long-term costs, disposable income, the cost of financing, depreciation and appreciation, penalties for breaking contracts, and equity.
  - Benefits include convenience, commitments, flexibility, and personal needs or wants, such as how often you want to buy a new car.
- Since each situation is unique, it is impossible to generalize about whether renting, leasing, or buying is best.

#### Need to Know

- When renting, leasing, and buying, you often need to make payments up front. Some payments go toward the overall cost, such as a down payment on a house or a lease deposit and the first and last month's rent. Other deposits, such as a rental damage deposit, are refunded at a later date.
- Appreciation and depreciation affect the value of a piece of property and should be considered when making decisions about renting, buying, or leasing, based on the particular situation. They are usually expressed as a rate per annum.
- Equity can make buying a house a form of investment.

HW: 2.4 p. 129-1133 #4, 6, 9, 10, 11 & 14