F Math 12

4.1 Counting Principles p. 228

Name ______________

Date ______________

Goal: Determine the Fundamental Counting Principle and use it to solve problems.

1. **Fundamental Counting Principle (FCP):** If there are ***a*** ways to perform one task and ***b*** ways to perform another, then there are $a \cdot b$ ways of performing both.

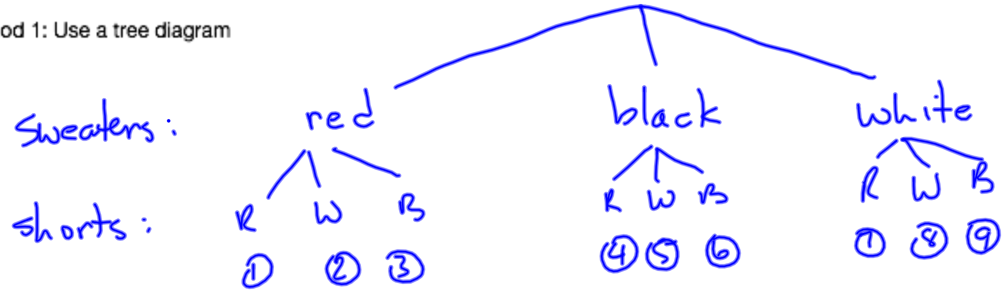
Example 1: Selecting a strategy to solve a counting problem (p. 230)

Hannah plays on her school soccer team. The soccer uniform has:

- three different sweaters: red, white, and black,
- three different shorts: red, white, and black.

How many different variations of the soccer uniform can the coach choose from for each game?

Method 1: Use a tree diagram



Method 2: Use the Fundamental Counting Principle

sweater choices
short choices

Number of uniform variations = 3 x 3 = 9

There are 9 different variations of the soccer uniform to choose from.

Example 2: A bike lock opens with the correct four-digit code. Each wheel rotates through the digits 0 to 9.

a. How many different ~~three~~ four-digit codes are possible?

Number of different codes = 10 x 10 x 10 x 10 = 10 000

There are 10 000 different four-digit codes.

b. Suppose each digit can be used only once in a code. How many different codes are possible when repetition is not allowed?

Number of different codes = 10 x 9 x 8 x 7 = 5040

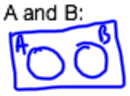
There are 5040 different four-digit codes when the digits cannot repeat.

The Fundamental Counting Principle applies when tasks are related by the word **AND**

If tasks are related by the work **OR**:

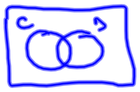
- If the tasks **are** mutually exclusive, they involve two disjoint sets A and B:

$$n(A \cup B) = n(A) + n(B)$$



- If the tasks **are not** mutually exclusive, they involve two sets that are not disjoint, C and D:

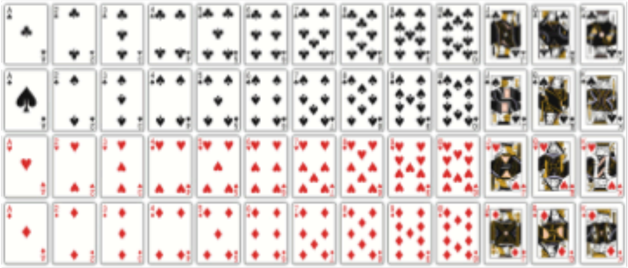
$$n(C \cup D) = n(C) + n(D) - n(C \cap D)$$



The Principle of Inclusion and Exclusion must be used to avoid counting elements in the intersection of the two sets more than once.

Example 3: Solving a counting problem when the Fundamental Counting Principle does not apply (p. 232)

A standard deck of cards contains 52 cards as shown.



Count the number of possibilities of drawing a single card and getting:

a. either a black face card or an ace

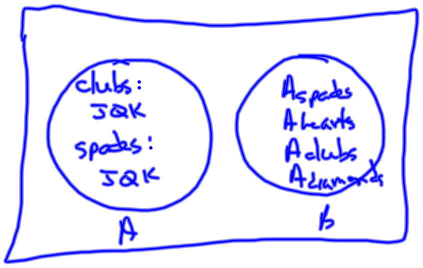
A B

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$6 + 4 - 0$$

$$= 10$$

↑ mutually exclusive



There are 10 ways to draw a single card and get either a black face card or an ace.

b. either a red card or a 10

C D

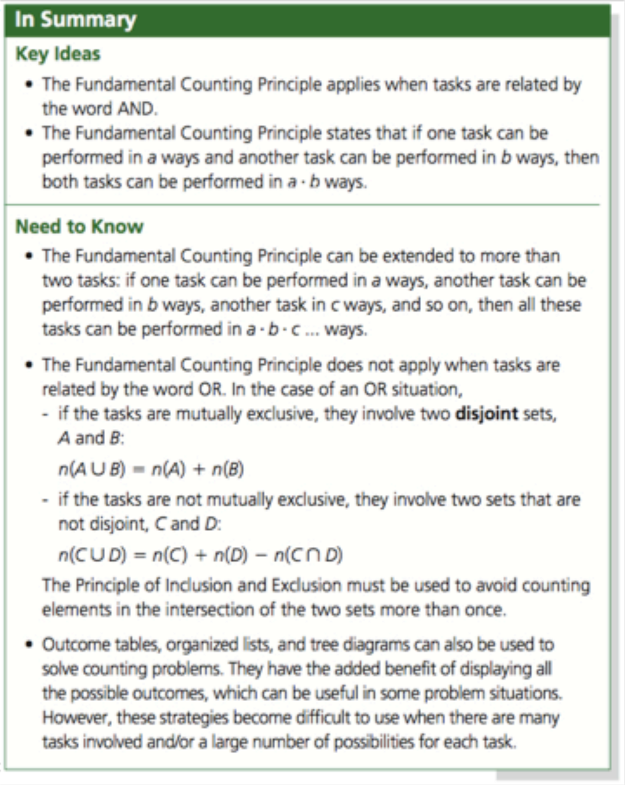
$$n(C \cup D) = n(C) + n(D) - n(C \cap D)$$

$$26 + 4 - 2$$

$$= 28$$

overlap ∴ not mutually exclusive

There are 28 ways to draw a single card and get either a red card or a 10.



In Summary

Key Ideas

- The Fundamental Counting Principle applies when tasks are related by the word AND.
- The Fundamental Counting Principle states that if one task can be performed in *a* ways and another task can be performed in *b* ways, then both tasks can be performed in $a \cdot b$ ways.

Need to Know

- The Fundamental Counting Principle can be extended to more than two tasks: if one task can be performed in *a* ways, another task can be performed in *b* ways, another task in *c* ways, and so on, then all these tasks can be performed in $a \cdot b \cdot c \ldots$ ways.
- The Fundamental Counting Principle does not apply when tasks are related by the word OR. In the case of an OR situation,
  - if the tasks are mutually exclusive, they involve two **disjoint** sets, *A* and *B*:

    $n(A \cup B) = n(A) + n(B)$
  - if the tasks are not mutually exclusive, they involve two sets that are not disjoint, *C* and *D*:

    $n(C \cup D) = n(C) + n(D) - n(C \cap D)$

  The Principle of Inclusion and Exclusion must be used to avoid counting elements in the intersection of the two sets more than once.
- Outcome tables, organized lists, and tree diagrams can also be used to solve counting problems. They have the added benefit of displaying all the possible outcomes, which can be useful in some problem situations. However, these strategies become difficult to use when there are many tasks involved and/or a large number of possibilities for each task.

HW: 4.1 p. 235-237 #4-12, 14 & 16

April 01, 2013

F Math 12

4.2 Introducing Permutations and Factorial Notation p. 238

Name ______________

Date ______________

Goal: Use factorial notation to solve simple permutation problems.

1. **permutation**: An arrangement of distinguishable objects in a definite order. For example, the objects *a* and *b* have two permutations, ab and ba.
2. **factorial notation**: A concise representation of the product of consecutive descending natural numbers:

   $1! = 1$

   $2! = 2 \cdot 1$

   $3! = 3 \cdot 2 \cdot 1$

   $4! = 4 \cdot 3 \cdot 2 \cdot 1$

   $n! = n(n-1)(n-2)\ldots(3)(2)(1)$

LEARN ABOUT the Math

Naomi volunteers after school at a daycare centre in Whitehorse, Yukon. Each afternoon, around 4 p.m., she lines up her group of children at the fountain to get a drink of water.

How many different arrangements of children can Naomi create for the lineup for the water fountain if there are six children in her group?

ABCDEF

ACDEFB

ADEFBC

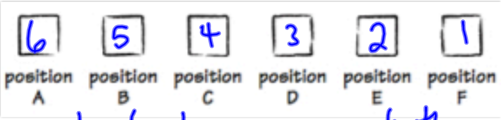
AEFBCD

AFBCDE

too much work to solve by listing the different orders

Example 1: Solving a counting problem where order matters (p. 238)

Determine the number of arrangements that six children can form while lining up to drink.



position A	position B	position C	position D	position E	position F
6	5	4	3	2	1

Let L represent the total # of permutations:

$L = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$L = 6!$

$L = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 30 \cdot 4 \cdot 6$

$= 120 \cdot 6 \rightarrow 720$

There are 720 permutations of the 6 children @ the fountain.

Example 2: Evaluating numerical expressions involving factorial notation (p. 240)

Evaluate the following:

a) 10!

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

3 628 800

$\frac{12!}{9!3!}$

$$\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$\frac{{}^{4}\cancel{12} \cdot 11 \cdot \overset{5}{\cancel{10}} \cdot \cancel{9!}}{{}^{1}\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{9!}}$$

$4 \cdot 11 \cdot 5$

220

* restrictions are only necessary when solving for n

- 0! is defined to be equal to 1

* Restrictions on n if $n!$ is defined:

For example: State the values of n for which each expression is defined, where $n \in I$

a) $(n-3)!$

$n-3 \geq 0$

$n \geq 3$

b) $\frac{n!}{(n-2)!}$

← $n \geq 0$

← $n-2 \geq 0$

$n \geq 2$

choose the biggest as your restriction

$\therefore n \geq 2$

Example 3: Simplifying an algebraic expression involving factorial notation (p. 241)

Simplify where $n \in N$. not solving ∴ no restrictions!

a) $(n+3)(n+2)!$

$$(n+3)\overbrace{(n+2)(n+1)(n)(n-1)\ldots(3)(2)(1)}$$

$$(n+3)!$$

b) $\frac{(n+1)!}{(n-1)!}$

$$\frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$$(n+1)(n)$$

$$n^2+n$$

Example 4: Solving an equation involving factorial notation (p.242)

Solve $\frac{n!}{(n-2)!} = 90$, where $n \in I$

need to determine restrictions

$$\frac{(n)(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$$

$$\left.\begin{array}{r} n \geq 0 \\ n-2 \geq 0 \\ n \geq 2 \end{array}\right\} n \geq 2$$

$$(n)(n-1) = 90$$

$$n^2 - n = 90$$

$$n^2 - n - 90 = 0$$

Method 1: Factoring $ax^2 + bx + c = 0$

$$n^2 - n - 90 = 0$$

to factor a trinomial when $a = 1$, we need 2 numbers that multiply to "c" and add to "b"

$$(n-10)(n+9) = 0 \begin{cases} c = -90 \\ b = -1 \end{cases}$$

take each binomial and set them equal to zero and solve

$$n - 10 = 0$$

$$n = 10$$

$$\cancel{n + 9 = 0}$$

$$\cancel{n = -9}$$

extraneous answer

$$\underline{\underline{n = 10}}$$

Method 2: Quadratic Formula

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$n^2 - n - 90 = 0$;

$a = 1$
$b = -1$
$c = -90$

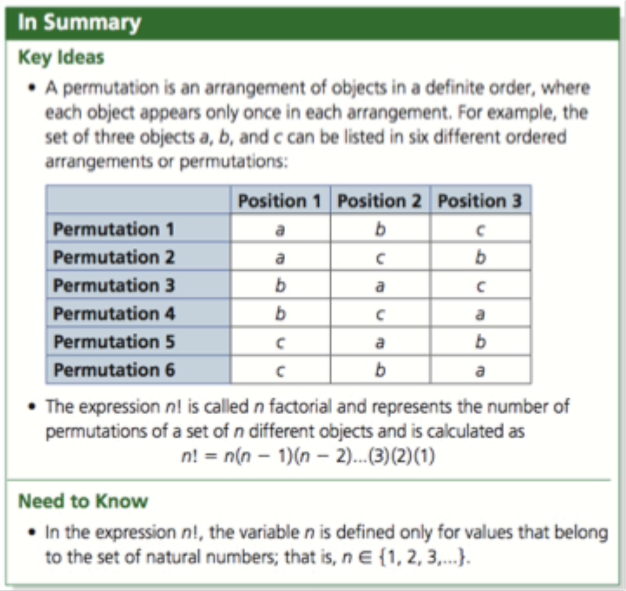
$$n = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-90)}}{2(1)}$$

$$n = \frac{1 \pm \sqrt{361}}{2}$$

$$n = \frac{1 \pm 19}{2}$$

$$n = \frac{20}{2} = 10$$

$$n = \frac{-18}{2} = \cancel{-9}$$



In Summary

Key Ideas

- A permutation is an arrangement of objects in a definite order, where each object appears only once in each arrangement. For example, the set of three objects *a*, *b*, and *c* can be listed in six different ordered arrangements or permutations:

	Position 1	Position 2	Position 3
Permutation 1	*a*	*b*	*c*
Permutation 2	*a*	*c*	*b*
Permutation 3	*b*	*a*	*c*
Permutation 4	*b*	*c*	*a*
Permutation 5	*c*	*a*	*b*
Permutation 6	*c*	*b*	*a*

- The expression $n!$ is called *n* factorial and represents the number of permutations of a set of *n* different objects and is calculated as

$$n! = n(n-1)(n-2)...(3)(2)(1)$$

Need to Know

- In the expression $n!$, the variable n is defined only for values that belong to the set of natural numbers; that is, $n \in \{1, 2, 3,...\}$.

HW: 4.2 p. 243-243 #2, 3, 5, 6, 9, 12, 14 & 15

F Math 12

4.3 Permutations When All Objects Are Distinguishable p. 246

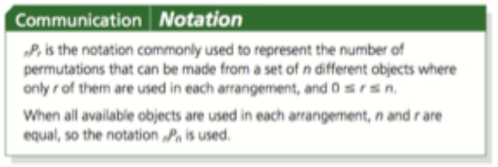
Name ________________

Date ________________

Goal: Determine the number of permutations of n objects taken r at a time, where $0 \leq r \leq n$.

The number of permutations from a set of n different objects, where r of them are used in each arrangement, can be calculated using the formula:

$$ {}_nP_r = \frac{n!}{(n-r)!}, \text{ where } 0 \leq r \leq n $$



Communication | *Notation*

${}_nP_r$ is the notation commonly used to represent the number of permutations that can be made from a set of n different objects where only r of them are used in each arrangement, and $0 \leq r \leq n$.

When all available objects are used in each arrangement, n and r are equal, so the notation ${}_nP_n$ is used.

Example 1: Solving a permutation problem where only some of the objects are used in each arrangement (p. 247)

Matt has downloaded 10 new songs from an online music store. He wants to create a playlist using 6 of these songs arranged in any order. How many different 6-song playlists can be created from his new downloaded songs?

Use ${}_nP_r$ formula

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_{10}P_6 = \frac{10!}{(10-6)!} $$

$$ = \frac{10!}{4!} $$

$$ = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} $$

$$ = 151\ 200 $$

Use FCP (Fundamental counting pr.)

Choices:

$$ \text{song} \quad \frac{10}{1} \cdot \frac{9}{2} \cdot \frac{8}{3} \cdot \frac{7}{4} \cdot \frac{6}{5} \cdot \frac{5}{6} $$

= 151 200 different playlists

Example 2: Solving a permutation problem involving cases (p. 250)

FB

Tania needs to create a password for a social networking website she registered with. The password can use any digits from 0 to 9 and/or any letters of the alphabet. The password is case sensitive, so she can use both lower- and upper-case letters. A password must be at least 5 characters to a maximum of 7 characters, and each character can be used only once in the password. How many different passwords are possible?

2 523 690 780 000 diff. passwords

number of characters: $10 + 2(26) = 62$

case 1: 5 character passwords : $_{62}P_5 = 776\ 520\ 240$

$n = 62$ and $r = 5$

case 2: 6 character passwords : $_{62}P_6 = 44\ 261\ 653\ 680$

$n = 62$ and $r = 6$

case 3: 7 character passwords : $_{62}P_7 = 2\ 478\ 652\ 606\ 000$

$n = 62$ $r = 7$

total

Example 3: Solving a permutation problem with conditions (p. 251)

At a used car lot, seven different car models are to be parked close to the street for easy viewing.

a. The three red cars must be parked so that there is a red car at each end and the third red car is exactly in the middle. How many ways can the seven cars be parked?

ways to place red cars →

$\underline{3}\ \ \underline{4}\ \ \underline{3}\ \ \underline{2}\ \ \underline{2}\ \ \underline{1}\ \ \underline{1}$

ways to place other cars

number of arrangements = # of permutations of red AND # of permutations of other cars

$A = {_3P_3} \cdot {_4P_4} = 3! \cdot 4!$

$6 \cdot 24 = 144$

b. The three red cars must be parked side by side. How many ways can the seven cars be parked?

→ need to consider as 1 object

$\underbrace{\boxed{\underline{3}\ \ \underline{2}\ \ \underline{1}}\ \ \underline{\ \ }\ \ \underline{\ \ }\ \ \underline{\ \ }\ \ \underline{\ \ }}_{_5P_5}$

$_3P_3$

number of arrangements:

$_3P_3 \cdot {_5P_5}$

$3! \cdot 5!$

$6 \cdot 120$

720

Example 4: Comparing arrangements created with and without repetition (p. 252)

A social insurance number (SIN) in Canada consists of a nine-digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if each digit can be repeated? ← need to use fundamental counting principle

$$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$$

$$10^9$$

1 000 000 000 SIN #s

How many SINs can be created if no repetition is allowed? → use $_nP_r$

$$_{10}P_9 = \frac{10!}{(10-9)!} = \frac{10!}{1!}$$

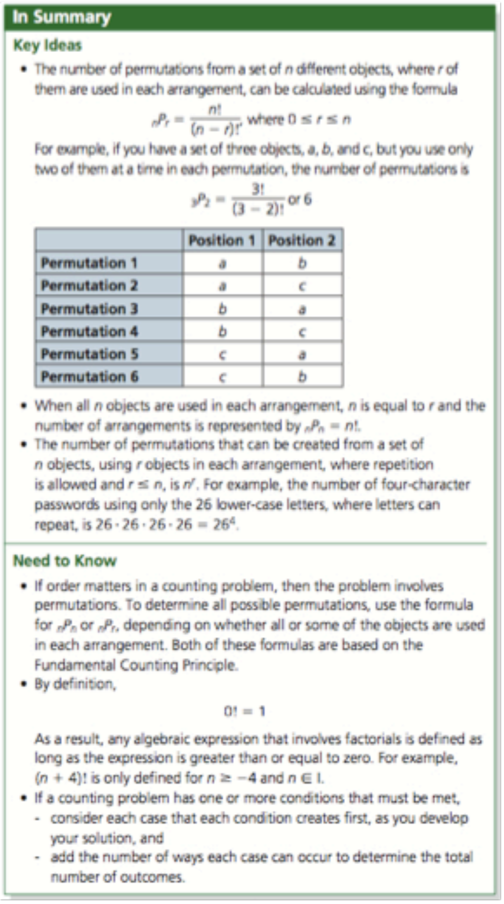
$$10! = 3\ 628\ 800$$

In reality, the Canadian government does not use 0, 8, or 9 as the first digit when assigning SINs to citizens and permanent residents, and repetition of digits is allowed. How many nine-digit SINs do not start with 0, 8, or 9?

$$\#\text{ of SINs} = \underline{7} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$$

$$= 7 \times 10^8$$

$$= 700\ 000\ 000$$



In Summary

Key Ideas

- The number of permutations from a set of n different objects, where r of them are used in each arrangement, can be calculated using the formula

  $${}_nP_r = \frac{n!}{(n-r)!}, \text{ where } 0 \leq r \leq n$$

  For example, if you have a set of three objects, a, b, and c, but you use only two of them at a time in each permutation, the number of permutations is

  $${}_3P_2 = \frac{3!}{(3-2)!} \text{ or } 6$$

	Position 1	Position 2
Permutation 1	*a*	*b*
Permutation 2	*a*	*c*
Permutation 3	*b*	*a*
Permutation 4	*b*	*c*
Permutation 5	*c*	*a*
Permutation 6	*c*	*b*

- When all n objects are used in each arrangement, n is equal to r and the number of arrangements is represented by ${}_nP_n = n!$.
- The number of permutations that can be created from a set of n objects, using r objects in each arrangement, where repetition is allowed and $r \leq n$, is n^r. For example, the number of four-character passwords using only the 26 lower-case letters, where letters can repeat, is $26 \cdot 26 \cdot 26 \cdot 26 = 26^4$.

Need to Know

- If order matters in a counting problem, then the problem involves permutations. To determine all possible permutations, use the formula for ${}_nP_n$ or ${}_nP_r$, depending on whether all or some of the objects are used in each arrangement. Both of these formulas are based on the Fundamental Counting Principle.
- By definition,

  $$0! = 1$$

  As a result, any algebraic expression that involves factorials is defined as long as the expression is greater than or equal to zero. For example, $(n + 4)!$ is only defined for $n \geq -4$ and $n \in \mathrm{I}$.
- If a counting problem has one or more conditions that must be met,
  - consider each case that each condition creates first, as you develop your solution, and
  - add the number of ways each case can occur to determine the total number of outcomes.

HW: 4.3 p. 255-257 # 1, 2, 5, 7, 9, 11 & 14

F Math 12

4.4 Permutations When Objects Are Identical p. 260

Name ________________

Date ________________

Goal: Determine the number of permutations when some objects are identical.

INVESTIGATE the Math

1. The permutations of the 4 different letters A, B, E, and F are:

ABEF	ABFE	AEBF	AFBE	AEFB	AFEB
BAEF	BAFE	EABF	FABE	EAFB	FAEB
BEAF	BFAE	EFAB	FEAB	EBAF	FBAE
BEFA	BFEA	EFBA	FEBA	EBFA	FBEA

How many permutations are there? $4! = 24$

$(4\cdot3\cdot2\cdot1)$

2. a) ***What happens if two of the letters are the same?*** Investigate this by converting each F to an E in the list below. Then count the number of permutations of the letters A, B, E, and E.

ABEE	ABEE	AEBE	AEBE	AEEB	AEEB
BAEE	BAEE	EABE	EABE	EAEB	EAEB
BEAE	BEAE	EEAB	EEAB	EBAE	EBAE
BEEA	BEEA	EEBA	EEBA	EBEA	EBEA

There are 12 permutations of the letters A, B, E, and E.

b) How does this number compare with step 1? half as many $\frac{24}{2} = 12$

$\left(\frac{4\cdot3\cdot2\cdot1}{2}\right)$

3. a) ***What happens if three of the letters are the same?*** Investigate this by converting each F and E to a B. Then count the number of permutations of the letters A, B, B, and B.

ABEF	ABFE	AEBF	AFBE	AEFB	AFEB
BAEF	BAFE	EABF	FABE	EAFB	FAEB
BEAF	BFAE	EFAB	FEAB	EBAF	FBAE
BEFA	BFEA	EFBA	FEBA	EBFA	FBEA

There are 4 permutations of the letters A, B, B, and B.

b) How does this number compare with step 1? $1/6$ $\frac{24}{6} = 4$ $\frac{4\cdot3\cdot2\cdot1}{3\cdot2}$

4. Generalize the pattern from the investigation to determine the number of permutations of:

a. A, B, C, D, D

$$\frac{5!}{2!} = \frac{5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 60$$

b. A, B, D, D, D

$$\frac{5!}{3!} = 20$$

c. A, D, D, D, D

$$\frac{5!}{4!} = 5$$

d. A, B, B, C, C

$$\frac{5!}{2!2!} = 30$$

e. A, A, A, B, B

$$\frac{5!}{3!2!} = \frac{5\cdot\cancel{4}\cdot\cancel{3}\cdot2\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{2}\cdot\cancel{1}}$$

$$= 10$$

Generalization

The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on, is:

$$\frac{n!}{a!b!c!\ldots}$$

Example 1: Determine the number of permutations of all the letters in the following the words.

a. STATISTICIAN

S - 2
T - 3
I - 2
A = 3

$$P = \frac{12!}{2!\,3!\,2!\,3!}$$

$$= \frac{12!}{2\cdot 6\cdot 2\cdot 6}$$

$$= \frac{12!}{144}$$

$$= 3326400$$

b. CANADA

CANADA

$$P = \frac{6!}{3!}$$

$$= \frac{6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}}$$

$$= 120$$

Example 2: Solving a conditional permutation problem involving identical objects (p. 263)

How many ways can the letters of the word CANADA be arranged, if the first letter must be N and the last letter must be C?

$$\underset{N}{\boxed{1}}\quad \underbrace{\boxed{4}\quad \boxed{3}\quad \boxed{2}\quad \boxed{1}}\quad \underset{C}{\boxed{1}}$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_1P_1 = 1$$

$${}_4P_1 = \frac{4!}{(4-1)!}$$

$$= \frac{4!}{3!}$$

← 4 letters choices

← 3 are letter "As"

$${}_1P_1 = 1$$

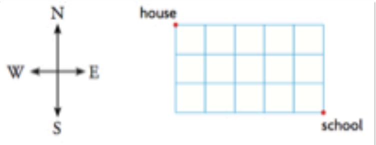
multiply all three permutations together to find the total number of arrangements

$$\text{Arrangements} = 1\cdot \frac{4!}{3!}\cdot 1$$

$$= 4$$

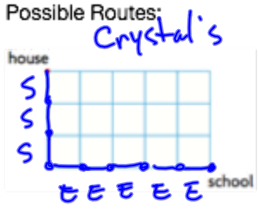
Example 3: Solving a permutation problem involving routes (p. 264)

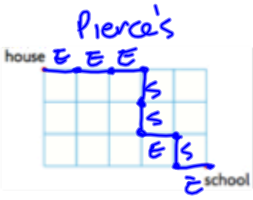
Julie's home is three blocks north and five blocks west of her school. How many routes can Julie take from home to school if she always travels either south or east?

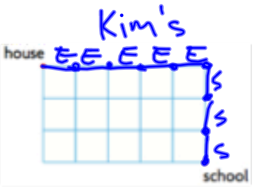


Method 1: Using permutations

Possible Routes:







SSSEEEEE

EEESSESE

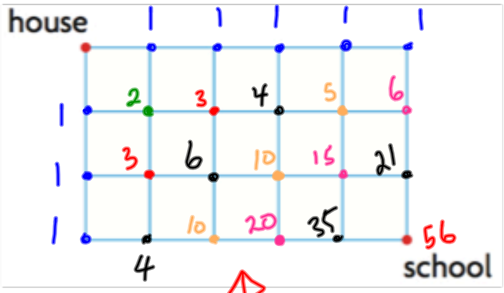
EEEEESSS

Permutations of 8 letters: 5 E's and 3 S's

$$\# \text{ Routes} = \frac{8!}{5!\,3!} = \frac{8\cdot7\cdot\cancel{6}\cdot\cancel{5!}}{\cancel{5!}\cdot\cancel{3!}} = 56$$

There are 56 routes

Method 2: Use a diagram



1

1 1

1 2 1

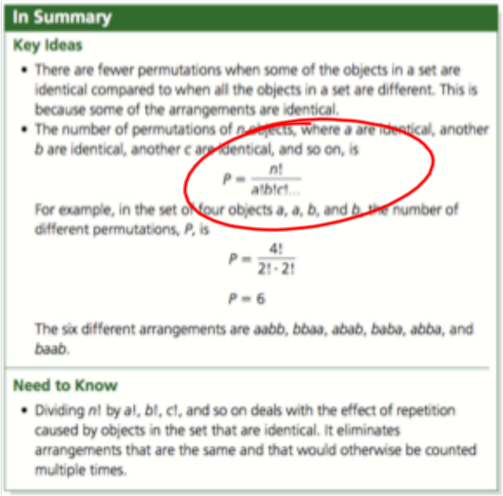
1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Follow the diagonal, the pattern is Pascal's Triangle



In Summary

Key Ideas

- There are fewer permutations when some of the objects in a set are identical compared to when all the objects in a set are different. This is because some of the arrangements are identical.
- The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on, is

$$P = \frac{n!}{a!b!c!\dots}$$

For example, in the set of four objects *a*, *a*, *b*, and *b*, the number of different permutations, *P*, is

$$P = \frac{4!}{2! \cdot 2!}$$

$$P = 6$$

The six different arrangements are *aabb*, *bbaa*, *abab*, *baba*, *abba*, and *baab*.

Need to Know

- Dividing *n*! by *a*!, *b*!, *c*!, and so on deals with the effect of repetition caused by objects in the set that are identical. It eliminates arrangements that are the same and that would otherwise be counted multiple times.

HW: 4.4 p. 266-269 # 5, 6, 7, 9, 11 & 15

F Math 12

4.5 & 4.6 Combinations p. 270, 273

Name ______________

Date ______________

Goal: Explore how counting combinations differs from counting permutations.

INVESTIGATE the Math

1. If 5 sprinters compete in a race, how many different ways can the medals for first, second and third place, be awarded?

$$\underset{1^{st}}{\underline{5}} \cdot \underset{2^{nd}}{\underline{4}} \cdot \underset{3^{rd}}{\underline{3}} = 60 \text{ ways} \rightarrow \quad {}_5P_3$$

Does order matter here? Yes

This is an example of a permutation of 5 objects, taken 3 at a time.

2. If 5 sprinters complete in a race and the fastest 3 qualify for the relay team, how many different relay teams can be formed?

Visualize the 5 sprinters below. Since 3 will qualify for the relay team and 2 will not, consider the number of ways of arranging 3 Y's and 2 N's.

$$YYYNN \quad \frac{5!}{3!\,2!} = \frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}\cdot 2!} = 10 \text{ ways} \quad {}_5C_3$$

Does order matter here? No

This is an example of a combination of 5 objects, taken 3 at a time.

Combinations

A grouping of objects where order does not matter.

For example, the two objects *a* and *b* have one combination because ab is the same as ba.

The number of combinations from a set of *n* different objects, where only *r* of them are used in each combination, can be denoted by ${}_nC_r$ or $\binom{n}{r}$ (read "n choose r"), and is calculated using the formula:

$$ {}_nC_r = \frac{n!}{r!(n-r)!} \text{ where } 0 \le r \le n $$

Example 1: A group of 7 people consists of 3 males and 4 females.

a. How many different committees of 3 people can be formed from 7 people?

order doesn't matter

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

$$ {}_7C_3 = \frac{7!}{3!(7-3)!} $$

$$ = \frac{7\cdot\cancel{6}\cdot5\cdot\cancel{4!}}{\cancel{3!}\cdot\cancel{4!}} $$

$= 35$ differents committees

$\boxed{7}\ \boxed{\text{Math}}\ \boxed{\text{PRB}}\ \boxed{3}\ \boxed{3}\ \boxed{=}$

↑

nCr

b. How many different committees of 3 people can be formed if the first person selected serves as the chairperson, the second as the treasurer, and the third as the secretary?

order matters

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_7P_3 = \frac{7!}{(7-3)!} $$

$$ = \frac{7\cdot6\cdot5\cdot\cancel{4!}}{\cancel{4!}} $$

$= 210$ different committees

c. How many different committees of 3 people can be formed with 1 male and 2 females?

Think: you must choose 1 male out of the group of 3 males and 2 females out of the group of 4 females

order doesn't matter

$$_3C_1 \cdot {}_4C_2 = \frac{3!}{1!(3-1)!} \cdot \frac{4!}{2!(4-2)!}$$

$$= \frac{3!}{2!} \cdot \frac{4!}{2!2!}$$

$$= 3 \cdot 6$$

18 different committees

d. How many different committees of 3 people can be formed with at least one male on the committee?

Method 1 : Direct Reasoning

Case 1 : 1 male, 2 females

$$_3C_1 \cdot {}_4C_2 = 18$$

Case 2 : 2 males, 1 female

$$_3C_2 \cdot {}_4C_1 = \frac{3!}{2!(3-2)!} \cdot \frac{4!}{1!(4-1)!}$$

$$= 3 \cdot 4 = 12$$

Case 3 : 3 males, 0 females

$$_3C_3 \cdot {}_4C_0 = \frac{3!}{3!(3-3)!} \cdot \frac{4!}{0!(4-0)!}$$

$$1 \cdot 1 = 1$$

# of committees : 18 + 12 + 1

31

Method: Indirect Reasoning

total # of committees : $_7C_3 = 35$

# of committees with no males, 3 females : $_3C_0 \cdot {}_4C_3$

$$\frac{3!}{0!(3-0)!} \cdot \frac{4!}{3!(4-3)!}$$

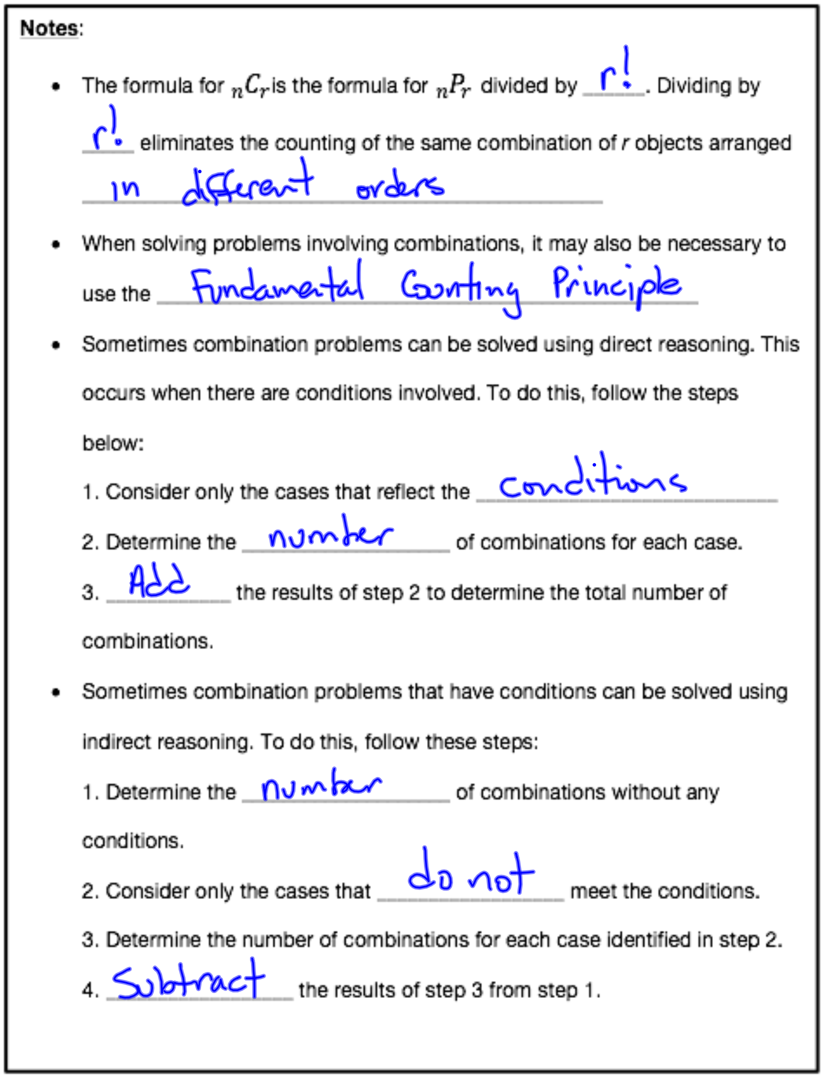
$$1 \cdot 4$$

4

# of committees with at least 1 male :

35 − 4 = 31

November 14, 2012



Notes:

- The formula for $_nC_r$ is the formula for $_nP_r$ divided by r!. Dividing by r! eliminates the counting of the same combination of *r* objects arranged in different orders
- When solving problems involving combinations, it may also be necessary to use the Fundamental Counting Principle
- Sometimes combination problems can be solved using direct reasoning. This occurs when there are conditions involved. To do this, follow the steps below:

  1. Consider only the cases that reflect the conditions
  2. Determine the number of combinations for each case.
  3. Add the results of step 2 to determine the total number of combinations.

- Sometimes combination problems that have conditions can be solved using indirect reasoning. To do this, follow these steps:

  1. Determine the number of combinations without any conditions.
  2. Consider only the cases that do not meet the conditions.
  3. Determine the number of combinations for each case identified in step 2.
  4. Subtract the results of step 3 from step 1.

HW: 4.4 p. 280-282 # 1, 4-8 & 10

F Math 12

4.7 Solving Counting Problems p. 283

Name ______________

Date ______________

Goal: Solve counting problems that involve permutations and combinations.

Example 1: Solving a permutation problem with conditions (p. 284)

Mr. Rice and some of his favourite students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

$$\underbrace{\overset{\text{Rice}}{\boxed{1}}\quad \overset{\text{Boys}}{\boxed{3!}}\quad \overset{\text{Girls}}{\boxed{5!}}}$$

R B G , B R G . . .

3! arrangements

there are 3! ways the boys can be arranged

there are 5! ways the girls can be arranged

number of seating arrangements

$(1\cdot 3!\cdot 5!)\,3! = 4320$

Example 2: Solving a combination problem involving cases (p.286)

A standard deck of 52 playing cards consists of 4 suits (spades, hearts, diamonds, and clubs) of 13 cards each.

a) How many different 5-card hands can be formed? order not important

52 cards, choose 5

$$_{52}C_5 = \frac{52!}{5!(52-5)!}$$

$= 2\ 598\ 960$ 5 card hands

b) How many different 5-card hands can be formed that consist of all hearts?

13 hearts

∴ 39 non-heart

$_{13}C_5 \cdot {}_{39}C_0$

$\frac{13!}{5!\,8!} \cdot \frac{39!}{0!\,39!} = 1287$ 5 card hand consist of all hearts.

c) How many different 5-card hands can be formed that consist of all face cards?

12 face cards

40 non-face cards

$_{12}C_5 \cdot {}_{40}C_0 \longrightarrow$ any time you choose zero there is only 1 combination

$\frac{12!}{5!\,7!} = 792$

5 card hands consist of all face cards

d) How many different 5-card hands can be formed that consist of 3 hearts and 2 spades?

$_{13}C_3 \cdot {}_{13}C_2$

$\frac{13!}{3!\,10!} \cdot \frac{13!}{2!\,11!} = 22\ 308$

e) How many different 5-card hands can be formed that consist of at least 3 hearts?

Case 1: 3 hearts, 2 non-hearts

$_{13}C_3 \cdot {}_{39}C_2$

Case 2: 4 hearts, 1 non-heart

$_{13}C_4 \cdot {}_{39}C_1$

Case 3: 5 hearts. 0 non-hearts

$_{13}C_5 \cdot {}_{39}C_0$

add all the cases together

241 098 diff. hands consist of at least 3 hearts

f) How many different 5-card hands can be formed that consist of at most 1 black card? * from text

Case 1: 1 black, four reds

$_{26}C_1 \cdot {}_{26}C_4$

case 2: 0 black, five reds

$_{26}C_0 \cdot {}_{26}C_5$

add cases

454 480 hands with at most 1 black card

When solving counting problems, you need to determine if order plays a role in the situation. Once this is established, you can use the appropriate permutation or combination formula. You can also use these strategies:

- Look for conditions. Consider these first as you develop your solution.
- If there is a repetition of *r* of the *n* objects to be eliminated, it is usually done by dividing by r! (combinations)
- If a problem involves multiple tasks that are connected by the word AND, then the Fundamental Counting Principle can be applied: multiply the number of ways that each task can occur.
- If a problem involves multiple tasks that are connected by the word OR, the Fundamental Counting Principle does not apply; Add the number of ways that each task can occur. This typically is found in counting problems that involve several cases.

HW: 4.7 p. 281 #11; p. 288-290 # 1, 4-6 & 10