F Math 12

6.1 Exploring the Graphs of Polynomial Functions p. 380

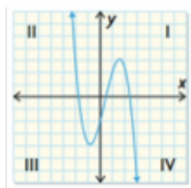
Name ____________________

Date ____________________

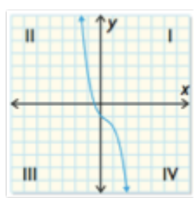
Goal: Identify characteristics of the graphs of polynomial functions.

1. **end behaviour**: The description of the shape of the graph, from left to right, on the coordinate plane.

2. **turning point**: Any point where the graph of a function changes from increasing to decreasing orfrom decreasing to increasing; for example, this curve has two turning points, since the *y*-values change from decreasing to increasing to decreasing:



This curve does not have any turning points, since the *y*-values are always decreasing:



3. **cubic functions:** a polynomial function of the third degree, whose greatest exponent is three; for example, $f(x) = 5x^3 + x^2 - 4x + 1$

4. **domain**: the set of all possible values for the $\overbrace{\text{independent variable}}^{x}$ in a relation

5. **range:** the set of all possible values for the dependent variable as the independent variable takes on all possible values of the domain

y

y-int

$y = mx + b$

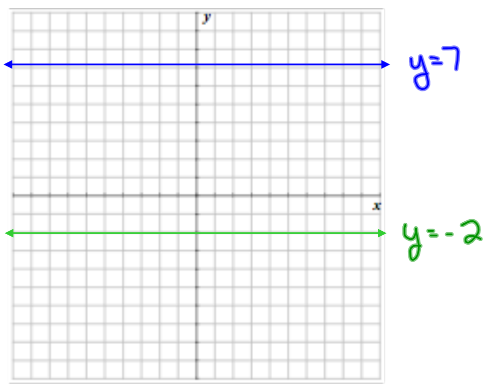
slope

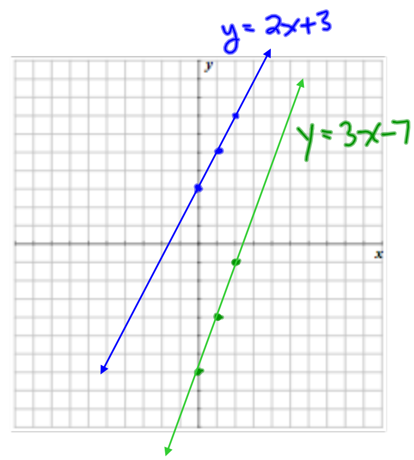
INVESTIGATE the math

Graph each function below on the same set of axes using different colours and label the graphs:

1. a) $y = 7$ b) $y = -2$

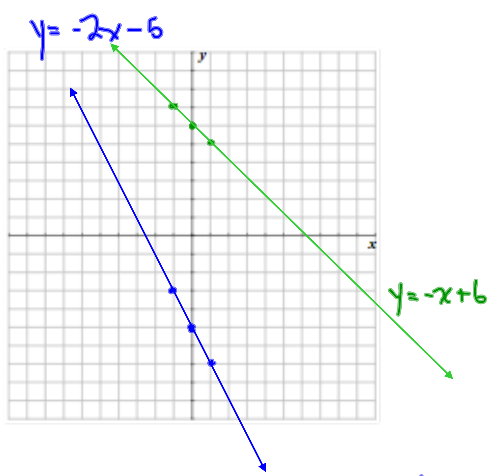
2. a) $y = 2x + 3$ b) $y = 3x - 7$

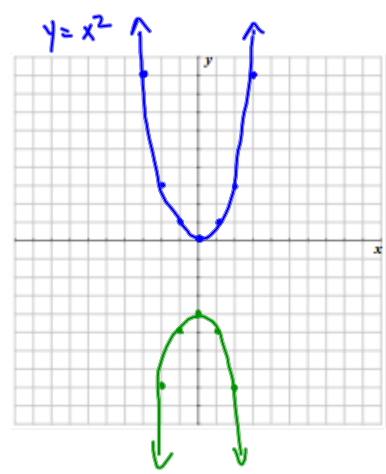




3. a) $y = -2x - 5$ b) $y = -x + 6$

4. a) $y = x^2 + 0$ b) $y = -x^2 - 4$





⊕ slope = line rises to the right

⊖ slope = line falls to the right

INVESTIGATE the math

Graph each function below on the same set of axes using different colours and label the graphs:

y-int

$y = mx + b$

slope

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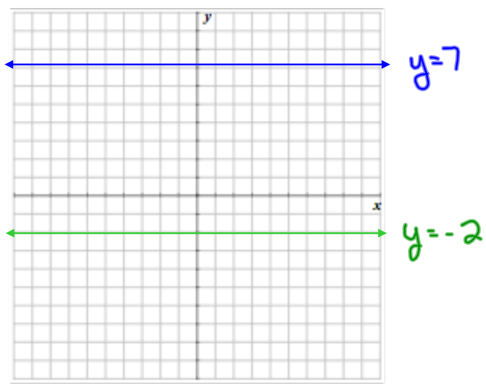

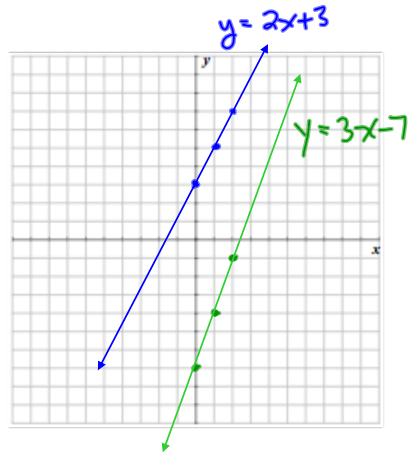

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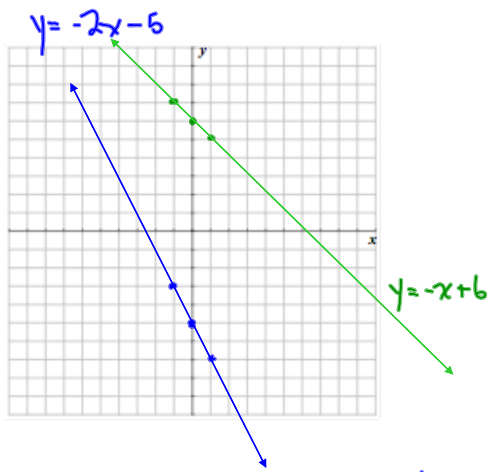
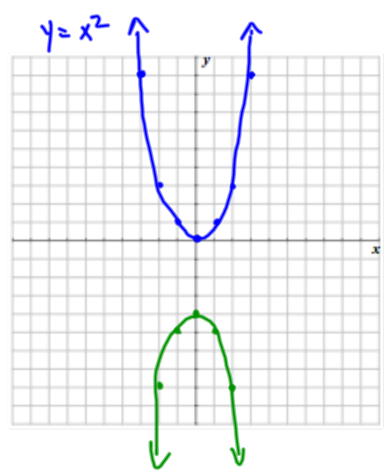


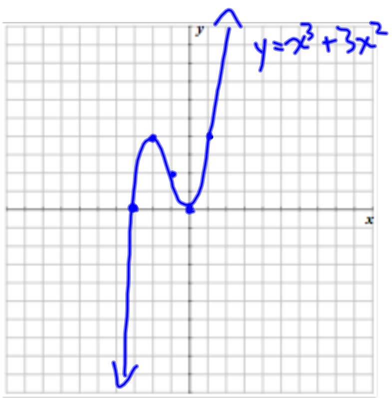


(+) slope = line rises to the right

(−) slope = line falls to the right

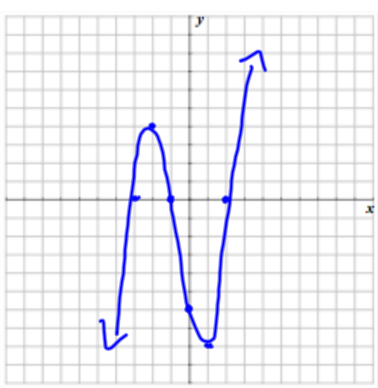
December 04, 2012

9. $y = x^3 + 3x^2$ $+\ 0$



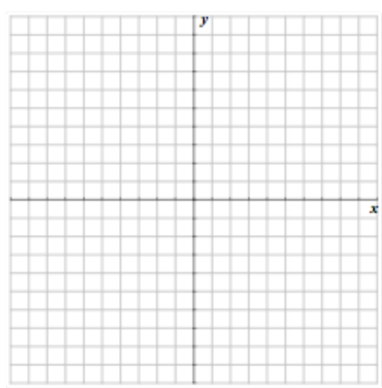
10. $y = -x^3 - 3x^2 + 4$

11. $y = x^3 + 2x^2 - 5x - 6$

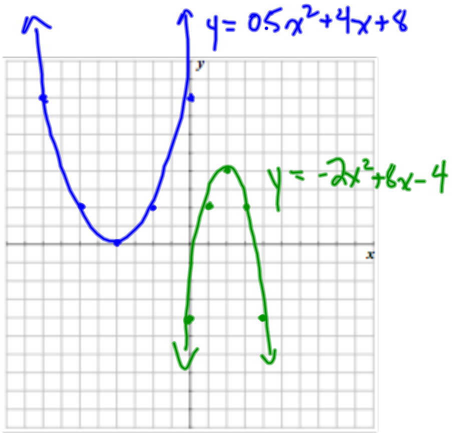


$y = x^3 + 2x^2 - 5x - 6$

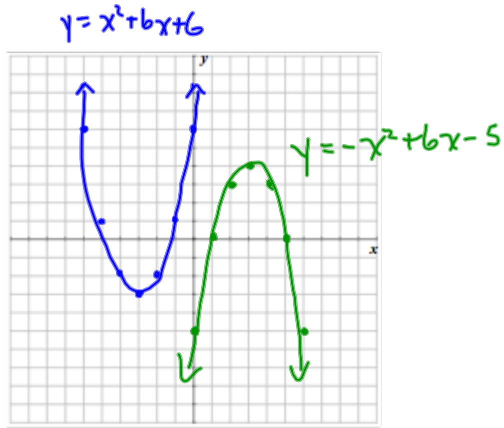
12. $y = -x^3 - x^2 + 6x$



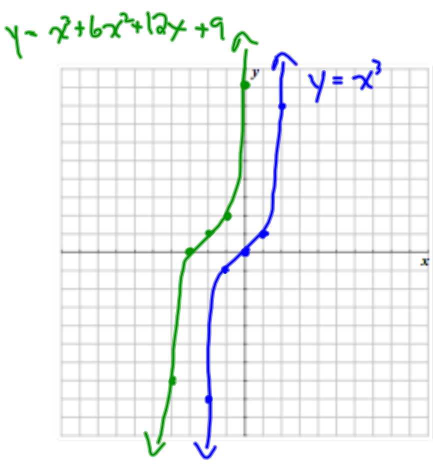
5\. a) $y = 0.5x^2 + 4x + 8$ b) $y = -2x^2 + 8x - 4$



6\. a) $y = x^2 + 6x + 6$ b) $y = -x^2 + 6x - 5$

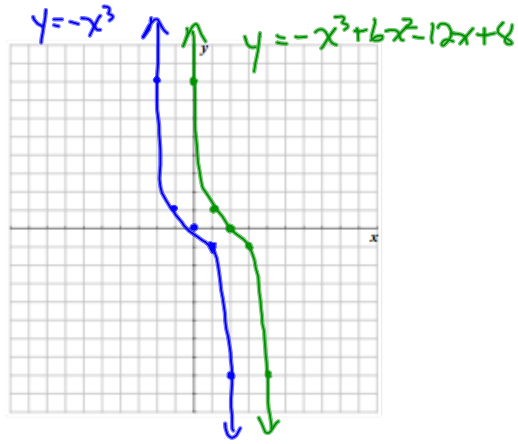


7\. a) $y = x^3 + 0$ b) $y = x^3 + 6x^2 + 12x + 9$



make negative!

8\. a) $y = -x^3$ b) $y = -x^3 + 6x^2 - 12x + 8$



Polynomial Functions → named according to their degree

→ normally written so that powers are in descending order e.g. $y = 2x^3 - 4x^2 + 3x - 1$

→ y can be replaced by $f(x)$
e.g. $f(x) = 2x^3 - 4x^2 + 3x - 1$

Degree → determined by the highest exponent within the polynomial function

Types of Polynomial Functions

constant	linear	quadratic	cubic
$f(x) = 3$	$f(x) = 2x+1$	$f(x) = 2x^2 - 4x + 5$	$f(x) = 5x^3 - 4x^2 + 1$
$f(x) = 3x^0$	$f(x) = 2x^1 + 1$		
degree = 0	degree = 1	degree = 2	degree = 3

Domain → all possible x-values for the polynomial

→ all possible input values eg. $D = \{x \mid x \in \mathbb{R}\}$

Range → all possible y-values for the polynomial

→ all possible output values eg. $R = \{y \mid y \geq 1, y \in \mathbb{R}\}$

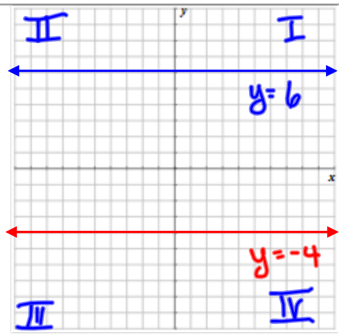
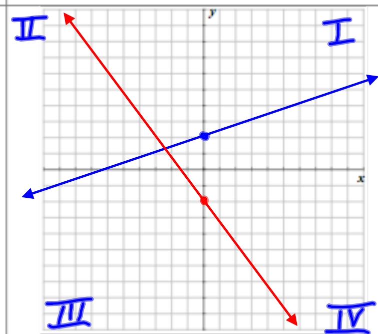
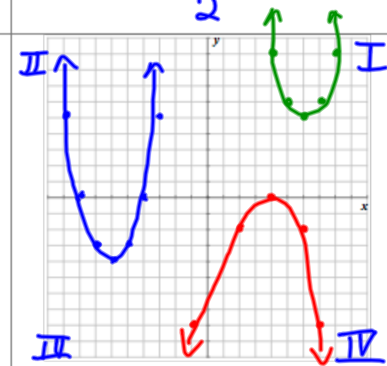
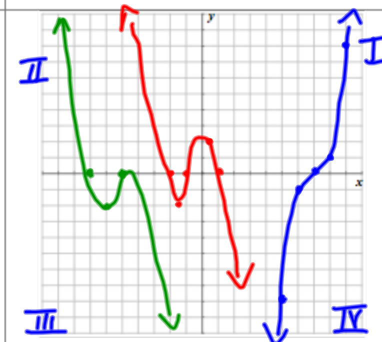
all real #s

the set of all values of y

such that

is an element of

The graphs of polynomial functions of the same degree have common characteristics: P. 382

Type of Function	Constant	Linear	Quadratic	Cubic
Degree, n	0	1	2	3
Sketch				
Number of $x-intercepts$	0, except for $y=0$, the # of x-int is infinite	1	0, 1 or 2	1, 2 or 3
Number of $y-intercepts$	1	1	1	1
End Behviour	line extends from ① quadrant II → I OR ② quadrant III → IV	line extends from ① quadrant III → I OR ② quadrant II → IV	lines extend from ① quadrant II → I OR ② quadrant III → IV	lines extend from ① quadrant III → I OR ② quadrant II → IV
Domain	$D = \{x \mid x \in \mathbb{R}\}$	$D = \{x \mid x \in \mathbb{R}\}$	$D = \{x \mid x \in \mathbb{R}\}$	$D = \{x \mid x \in \mathbb{R}\}$
Range	$R = \{y \mid y = \text{constant}, y \in \mathbb{R}\}$	$R = \{y \mid y \in \mathbb{R}\}$	$R = \{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$ OR $R = \{y \mid y \geq \text{minimum}, y \in \mathbb{R}\}$	$R = \{y \mid y \in \mathbb{R}\}$
Number of Turning Points	0	0	1	0 or 2

HW: 6.1 p. 383 #1-4

F Math 12

6.3 Modelling Data With A Line of Best Fit p. 401

Name ______________

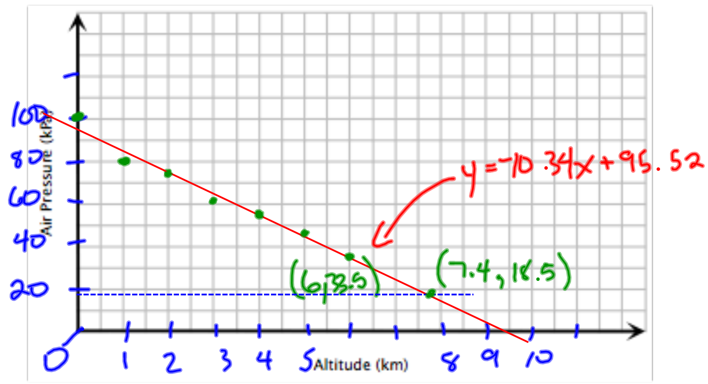
Date ______________

Goal: Determine the linear function that best fits a set of data, and use the function to solve a problem.

1. **line of best fit:** A straight line that best approximates the trend in a scatter plot.
2. **regression function**: A line or curve of best fit, developed through a statistical analysis of data.
3. **interpolation**: The process used to estimate a value within the domain of a set of data, based on a trend.
4. **extrapolation**: The process used to estimate a value outside the domain of a set of data, based on a trend.

Example 1: The table shows how the outside air pressure changes as an airplane rises after takeoff.

a. Create a scatter plot on the graphing calculator
b. Plot the points on a graph



x y

Altitude (km)	Air Pressure (kPa)
0	101
1	80
2	74
3	62
4	55
5	46

c. What ***term*** best describes the trend?

linear

d. Write the linear regression equation of the data $y = ax + b$

$y = -10.34x + 95.52$

e. What will the air pressure be at an altitude of 6 km?

33.5 kPa @ 6 km extrapolation

f. At what altitude would the airplane be if the pressure was 18.5 kPa?

7.4 km extrapolation

Example 2: The one-hour record is the farthest distance travelled by bicycle in 1 hour. The table below shows the world-record distances and the years after 1990.

(Use Window Settings: Xmin = –1, Xmax = 30, Ymin = –10, Ymax = 110)

Years after 1990 (yr)	6	8	9	12	13	14	17	18	19
Distance (km)	78.04	79.14	81.16	82.60	83.72	84.22	86.77	87.12	90.60

a. Create a scatter plot on the graphing calculator
b. Plot the points on a graph

One-Hour Bike Distance Record

c. Write the linear regression equation of the data.

$y = 0.858x + 72.643$

d. Based on this data, what was the world-record distance in 2000?

81.23 km

e. Based on this data, when might the world-record distance be 95 km?

26 years after 1990 → 2016

HW: 6.3 pp.407-412 #4, 5, 6, 7, 11 & 13

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6.4 Modelling Data With A Curve of Best Fit p. 413

Name ______________

Date ______________

Goal: Determine the quadratic or cubic function that best fits a set of data, and use the function to solve a problem.

1. **curve of best fit:** A curve that best approximates the trend on a scatter plot.

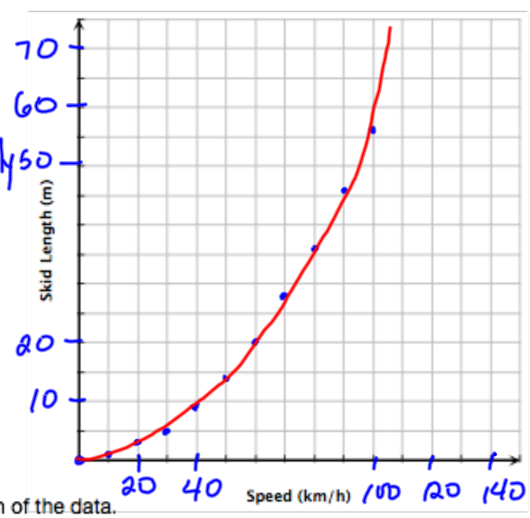
Example 1: Shannon is a police officer who investigates accidents. Shannon can estimate the speed of a car before a collision based on skid length.

Speed on Dry Pavement (km/h)	0	10	20	30	40	50	60	70	80	90	100
Skid Length (m)	0	0.6	2.3	5.3	9.1	14.0	20.4	27.6	35.9	45.5	56.3

a. Create a scatter plot on the graphing calculator
(Use Window Settings: Xmin = –10, Xmax = 140, Xscl = 10, Ymin = –10, Ymax = 120, Yscl = 10, Xres = 1)

Skid Length vs. Speed

b. Plot the points on a graph



c. How can you describe the **trend** in the data?

As speed increase constantly skid length increase more and more (exponentially)

d. What ***term*** best describes the trend?

quadratic

e. Write the regression equation of the data.

$$y = 0.0056x^2 + 0.0022x + 0.0371$$

$$y = ax^2 + bx + c$$

f. Based on these data, how long might a skid length be if a car had been travelling

i) 65 km/h? 23.8 m Interpolation

ii) 84 km/h? 39.7m Interpolation

iii) 120 km/h? 80.9 m Extrapolation

g. Based on these data, at what speed would a car have been travelling if there was a skid length measuring

i) 7.2 metres? 35.6 km/h

ii) 32.5 metres? 76 km/h

iii) 95 metres? 130.1 km/h

Example 2: The following data shows that average retail price of gasoline, per litre, for a selection of years in a 30-year period beginning in 1979.

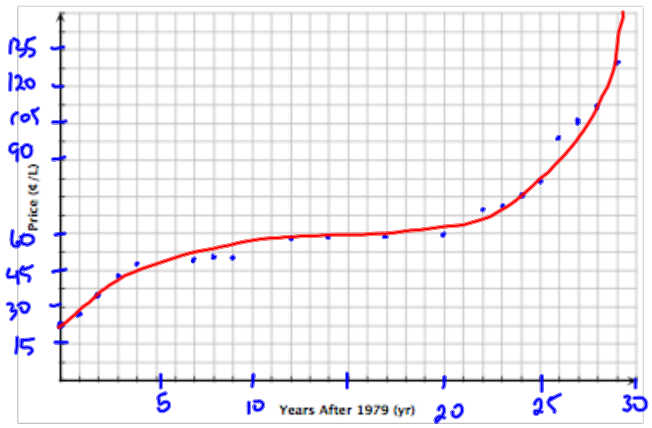
Years after 1990 (yr)	6	8	9	12	13	14	17	18	19
Distance (km)	78.04	79.14	81.16	82.60	83.72	84.22	86.77	87.12	90.60

a. Create a scatter plot on the graphing calculator
(Use Window Settings: Xmin = –10, Xmax = 40, Xscl = 10, Ymin = –10, Ymax = 130, Yscl = 10, Xres = 1)

b. Plot the points on a graph

Gasoline Retail Prices

Year after 1979 (yr)	Price of Gas (¢/L)
0	21.98
1	26.18
2	35.63
3	43.26
4	45.92
7	45.78
8	47.95
9	47.53
12	57.05
14	54.18
17	58.52
20	59.43
22	70.56
23	70.00
24	74.48
25	82.32
26	92.82
27	97.86
28	102.27
29	115.29



c. What ***term*** best describes the trend?

cubic

d. Write the regression equation of the data.

$$y = 0.0123x^3 - 0.465x^2 + 6.295x + 23.454$$

e. Based on these data, what would the average gas price have been (or will be) in

i) 1984? (5 years) 44.85 ¢/L

ii) 2000? (21 years) 64.55 ¢/L

iii) 2015? (36 years) 221.16 ¢/L

f. Based on these data, when would the average gas price have been (or will be)

i) 50 ¢/L? ii) 100 ¢/L? iii) 120 ¢/L?

7.7 years 27.4 years 29.5 years

1987 2006 2009

In Summary

Key Idea

- If the points on a scatter plot seem to follow a predictable curved pattern, then there may be a quadratic or cubic relationship between the independent variable and the dependent variable.

Need to Know

- If the points on a scatter plot follow a quadratic or cubic trend, then graphing technology can be used to determine and graph the equation of the curve of best fit.
- To solve an equation, you can graph the corresponding function of each side of the equation. The x-coordinate of the point of intersection is the solution to the equation.
- Technology uses polynomial regression to determine the curve of best fit. Polynomial regression results in an equation of a curve that balances the points on both sides of the curve.
- A curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the curve of best fit on a scatter plot or by using the equation of the curve of best fit.

HW: 6.4 pp.419-422 #2-4, 7, 8 & 10