F Math 12

5.1 Exploring Probability p. 302

Name ______________

Date ______________

Goal: Use probability to make predictions.

1. **fair game:** A game in which all the players are equally likely to win; for example, tossing a coin to get heads or tails is a fair game.

- The **theoretical probability** of event A is represented as:

$$P(A) = \frac{n(A)}{n(S)}$$

  - where $n(A)$ is the number of favourable outcomes for event A
  - $n(S)$ is the total number of outcomes in the sample space, S where all outcomes are equally likely.

- The **experimental probability** of event A is represented as:

$$P(A) = \frac{n(A)}{n(T)}$$

  - where $n(A)$ is the number of times event A occurred
  - $n(T)$ is the total number of trials, T, in the experiment

- The probability of an even can range from 0 (impossible) to 1 (certain). You can express probability as a fraction, a decimal, or a percent.

Example 1: Ross and Rachel flip a coin to see who gets to pick a movie. Rachel wins if she flips a head.

a. What is the theoretical probability of getting a head?

$P(H) = \frac{1}{2}$ or 0.5 or 50%

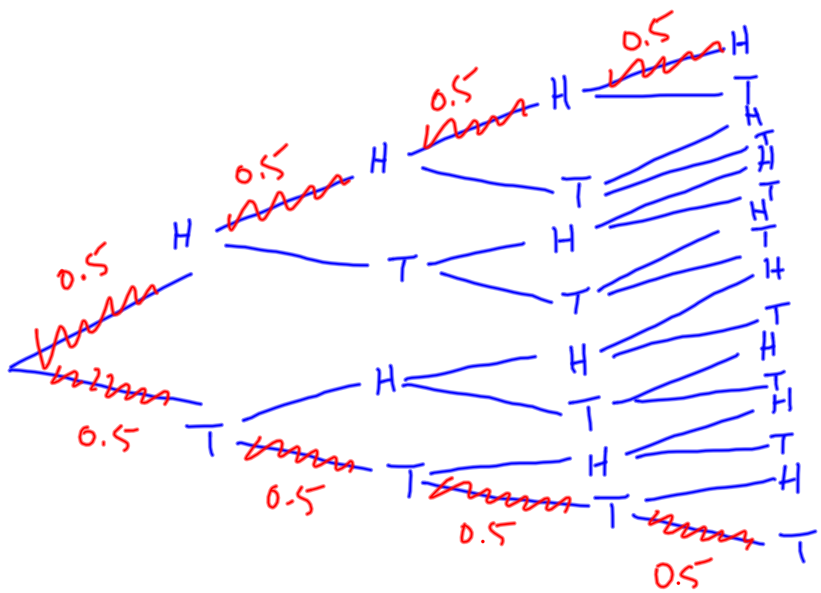
b. Simulate flipping a coin 1000 times and record the number of times a head appears. From your simulation, what is the experimental probability of getting a head?

$P(H) = \frac{484}{1001} = 0.484 = 48.4\%$

c. Is the game fair?

Yes, both outcomes are equally likely to win (if the coin is fair)

Example 2: Rachel now decides that they will toss 4 coins—a nickel, a dime, a quarter, and a loonie. If all 4 land on heads, or all 4 land on tails, Ross wins. Otherwise, Rachel wins. Create a sample space to show all possible outcomes. Determine the probability of Ross winning and of Rachel winning. Is the game fair?



$HHHH = (0.5)(0.5)(0.5)(0.5)$
$= 0.0625$
$TTTT = (0.5)(0.5)(0.5)(0.5)$
$= 0.0625$

$P(Ross) = 0.0625 + 0.0625$
$= 0.125 = 12.5\%$

$P(Rachel) = 1 - 0.125$
$= 0.875$
$= 87.5$

The game is fixed, Rachel has a better chance of winning

HW: 5.1 p. 303 #2 & 3

F Math 12

5.2 Probability and Odds p. 304

Name ____________

Date ____________

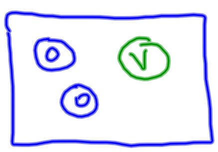
Goal: Understand and interpret odds, and relate them to probability.

1. **odds in favour**: the ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes.

2. **odds against**: The ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes.

EXPLORE...

An oil and vinegar salad dressing is made using $\overbrace{\text{2 parts oil to 1 part vinegar}}^{\text{3 parts}}$. So, the ratio of oil to vinegar is 2:1. What fraction of the dressing is oil?



$\frac{2}{3}\}$ oil ; $\frac{1}{3}\}$ vinegar

INVESTIGATE the Math

Suppose that, at the beginning of a regular CFL season, the Saskatchewan Roughriders are given a 25% chance of winning the Grey Cup.

a. The event in this situation is the Roughriders winning the Grey Cup

b. Express the probability that this event will occur as a fraction.

P(Roughriders will with the Grey Cup) = $\frac{25}{100} = \frac{1}{4}$

c. The complement of this event is the Roughriders <u>not</u> winning the Grey Cup

d. Express the probability that the complement will occur as a fraction.

$P(A') = 1 - P(A)$

P(Roughriders don't win the Grey Cup) = $1 - \frac{25}{100} = \frac{75}{100}$

$= \frac{3}{4}$

e. Write the odds in favour of the Roughriders winning the Grey Cup.

Odds in favour of event A

$\frac{P(A)}{P(A')}$ or $P(A):P(A')$

$\frac{P(win)}{P(not\ win)} = \frac{\frac{25}{100}}{\frac{75}{100}} = \frac{25}{75} = \frac{1}{3}$ or $1:3$

$\frac{25}{100} \div \frac{75}{100}$

$\frac{25}{\cancel{100}} \times \frac{\cancel{100}}{75}$

$\frac{25}{75}$

odds in favour are 1 in 3

f. Write the odds against the Roughriders winning the Grey Cup.

Odds against event A

$\frac{P(A')}{P(A)}$ or $P(A'):P(A)$

$\frac{P(not\ win)}{P(win)} = \frac{\frac{75}{100}}{\frac{25}{100}} = \frac{75}{25} = \frac{3}{1}$ or $3:1$

odds against are $3:1$

Example 1: Determining odds from probability (p. 306)

Research shows that the probability of an expectant mother, selected at random, having twins is $\frac{1}{32}$.

a. What are the odds in favour of an expectant mother having twins?

P(twins) = $\frac{1}{32}$ So, the ratio of twins to all birth combinations is: $1:32$

P(not twins) = $\frac{31}{32}$

So, the ratio of birth combinations that are not twins to all birth combinations is: $31:32$

The odds in favour of having twins,

$$P(\text{twins}) : P(\text{not twins}) = \frac{\frac{1}{32}}{\frac{31}{32}} * = 1:31$$

$$* \ \frac{1}{32} \div \frac{31}{32}$$

$$\frac{1}{\cancel{32}} \times \frac{\cancel{32}}{31}$$

$$\frac{1}{31}$$

b. W hat are the odds against an expectant mother having twins?

$$P(\text{not twins}) : P(\text{twins}) = 31:1$$

Example 2: Making a decision based on odds and probability

A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records. Who should go first?

Player	Attempts	Goals Scored	Not Scored
Ellen	13	8	5
Brittany	17	10	7

Method 1: Ellen: $P(\text{Scoring}) = \frac{8}{13} = 0.615 = 61.5\%$

Brittany: $P(\text{Scoring}) = \frac{10}{17} = 0.588 = 58.8\%$

Method 2: Brittany odds in favour $\rightarrow$ $10:7 \times 5 \rightarrow 50:35$

Ellen odds in favour $\rightarrow$ $8:5 \times 7 \rightarrow 56:35$

Ellen's probability and odds in favour are better, she should shoot first.

HW: 5.2 p. 310-312 #1, 2, 4, 5, 8, 9, 11, 12, 15 & 16

F Math 12 **5.3 Probabilities Using Counting Methods p. 313**

Name ________________

Date ________________

$${}_nP_r = \frac{n!}{(n-r)!}$$

Goal: Solve probability problems that involve counting techniques.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Example 1: Solving a probability problem using counting techniques (p. 314)

Jamaal, Ethan, and Alberto are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan, and Alberto will place in the top three, in any order.

Does order matter? ~~Somewhat, ∴ combination~~

Yes, ∴ permutation, see p. 314

Solution: There are ${}_3C_3$ ways in which three runners can place in three positions.

$${}_3C_3 = \frac{3!}{3!(3-3!)}$$
$$= 1$$

There are 1 favourable outcomes.

There are ${}_{10}C_3$ ways that 10 runners can place in first, second, or third.

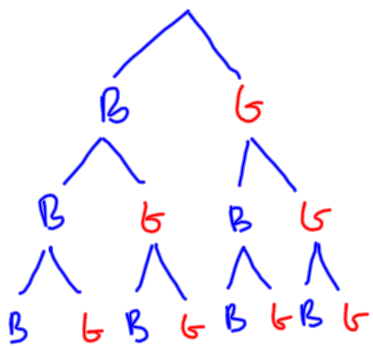
$${}_{10}C_3 = \frac{10!}{3!(10-3)!}$$
$$= \frac{10!}{7!3!}$$
$$= \frac{10 \cdot \cancel{9}^{3} \cdot 8}{\cancel{3!} \cdot 2}$$
$$= 120$$

There are 120 possible outcomes.

P(Jamaal, Ethan and Alberto place top 3) = $\frac{1}{120} = 0.0083 = 0.83\%$

Example 2: Solving a probability problem with the Fundamental Counting Principle (p. 316)

Channing has three children. Determine the probability Channing has at least one boy.



There are 8 possible outcomes.

$\therefore P(\text{at least one boy}) = \frac{7}{8} = 87.5\%$

OR There is 1 outcome where all children are girls

$\therefore P(\text{all girls}) = \frac{1}{8}$, $P(\text{all boys}) =$

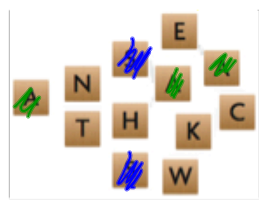
$1 - P(\text{all girls})$

$1 - \frac{1}{8}$

$7/8 = 87.5\%$

Example 3: Solving a probability problem using reasoning (p.318)

Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.



There are 12 letters in total: 2 S's, 3 A's, and 7 other letters.

The total number of ways to arrange the letters, L = $\frac{12!}{2!\,3!}$ 39 916 800

The total number of ways to spell SASKATCHEWAN, R = 1

P(winning the car) = $\frac{R}{L} = \frac{1}{39\,916\,800}$

Example 4: Solving a probability problem with conditions (p. 319)

There are 18 bikes in Marnie's spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Doug, Erica, and Franco each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all friends will be in the same row, with Allison and Franco at either end, F.

1	4	3	2	1	2
12	11	10	9	8	7
6	5	4	3	2	1

- Number of ways to seat Allison and Franco at either end: $2!$ or ${}_2P_2$
- Number of ways to seat the other 4 friends: $4!$ or ${}_4P_4$
- Number of ways to seat the other 12 people in the class: $12!$ or ${}_{12}P_{12}$
- Total number of ways to assign 18 people to 18 bikes: $18!$ ${}_{18}P_{18}$

3 rows

$$P(F) = \frac{3({}_2P_2)({}_4P_4)({}_{12}P_{12})}{{}_{18}P_{18}}$$

← # of ways to seat the 6 friends in any row

← # of ways to assign 18 bikes to 18 people

$$= \frac{3 \cdot 2! \cdot 4! \cdot \cancel{12!}}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot \cancel{12!}} \rightarrow \frac{144}{13\,366\,080} \rightarrow \frac{1}{92\,820}$$

$$P(F) = \frac{1}{92\,820}$$

HW: 5.3 p. 321-324 #1-3, 4, 8, 10, 11, 12 & 14

F Math 12

5.4 Mutually Exclusive Events p. 328

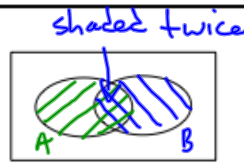
Name ______________

Date ______________

Goal: Understand and solve problems that involve mutually exclusive and non-mutually exclusive events.

1. **mutually exclusive**: Two or more events that cannot occur at the same time; for example, the Sun rising and the Sun setting are mutually exclusive events.

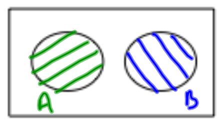
Probabilities of Events A or B



General Case:

P(A or B) =

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Mutually Exclusive events:

P(A or B) =

$P(A \cup B) = P(A) + P(B)$

Note: When the two events are mutually exclusive,

$n(A \cap B)$ __0__; therefore, $P(A \cap B)$ __0__

Example 1: One card is randomly drawn from a deck of 52 cards.

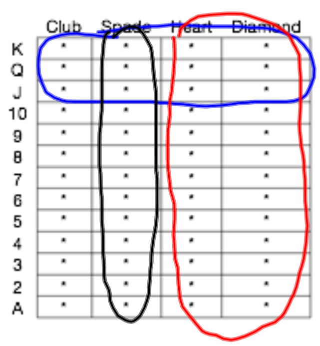
Define the following events:

S: The card is a spade
R: The card is red
F: The card is face card

Identify the events S, R, and F on the sample space.

Which of these three events are mutually exclusive?

S and R



and = intersection
or = union

a. Determine the following probabilities

P(S) = $\frac{13}{52}$ P(R) = $\frac{26}{52}$ P(S and R) = $\frac{0}{52}$ P(S or R) = $\frac{39}{52}$

P(S) = $\frac{13}{52}$ P(F) = $\frac{12}{52}$ P(S and F) = $\frac{3}{52}$ P(S or F) = $\frac{22}{52}$

b. Now determine the following probabilities using the formulas

P(S or R) = $\left.\begin{array}{l} P(S \cup R) = P(S) + P(R) - P(S \cap R) \\ \quad \frac{13}{52} + \frac{26}{52} - 0 \end{array}\right\} \frac{39}{52}$

P(S or F) = $\left.\begin{array}{l} P(S \cup F) = P(S) + P(F) - P(S \cap F) \\ \quad \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \end{array}\right\} \frac{22}{52}$

Example 2: Using a Venn diagram to solve a probability problem that involves two events (p. 332)

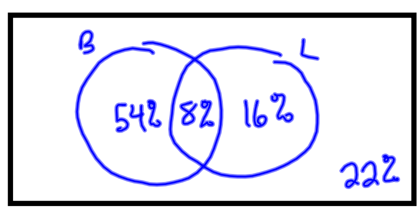
A school newspaper published the results of a recent survey.

Eating Habits:
Student Survey Results
- 62% skip breakfast B
- 24% skip lunch L
- 22% eat both breakfast and lunch

} 62 + 24 + 22 = 108%
∴ 8% counted twice

a. Are skipping breakfast and skipping lunch mutually exclusive?

no, the probabilities add up to over 100%



b. Determine the probability that a randomly selected student skips breakfast but not lunch.

$$P(B \setminus L) = P(B) - P(B \cap L)$$
$$= 62\% - 8\%$$
$$= 54\%$$

c. Determine the probability that a randomly selected student skips at least one of breakfast or lunch.

union

$$P(B \cup L) = 54\% + 8\% + 16\%$$
$$= 78\%$$

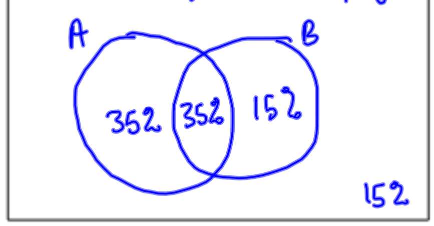
Example 3: Wilma submits bids on two web design projects. She thinks she has 70% chance of getting the first project, but just a 50% chance of getting the second. She puts only a 15% chance on getting neither of the two projects. Find the probability that she gets:

A: she gets first project
B: she gets second project

a. both projects

$70\% + 50\% + 15\% = 135\%$

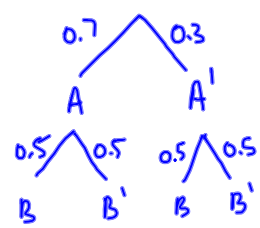
$\therefore$ 35% was counted twice



b. at least one of the two projects

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 70\% + 50\% - 35\%$$
$$= 85\%$$

can solve using a prob. tree → will discuss in detail later



c. only the first project

$$P(A \setminus B) = P(A) - P(A \cap B)$$
$$= 70\% - 35\%$$
$$= 35\%$$

d. only one of the two project

$$35\% + 15\% = 50\%$$

just A

just B

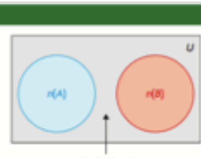
In Summary

Key Ideas

- You can represent the favourable outcomes of two **mutually exclusive events**, A and B, as two disjoint sets.

  You can represent the probability that either A or B will occur by the following formula:

  $$P(A \cup B) = P(A) + P(B)$$



$n(A \cap B) = 0$
(no common elements)

- You can represent the favourable outcomes of two **non-mutually exclusive events**, A and B, as two intersecting sets.

  You can represent the probability that either A or B will occur by this formula:

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

  An alternative formula is

  $$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

  When the two events are mutually exclusive, both formulas are equivalent,

  $$n(A \cap B) = 0$$

  which results in

  $$P(A \cap B) = 0$$

$n(A \cap B)$ has been shaded twice

Need to Know

- You can use the Principle of Inclusion and Exclusion, which is used to count the elements in the union of two sets, to determine the probability of non-mutually exclusive events.

HW: 5.4 p. 338-342 #1-5, 8 & 12-14

F Math 12

5.5 Conditional Probability p. 328

Name ________________

Date _________________

Goal: Understand and solve problems that involve dependent events.

1. **dependent events:** Events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn).
2. **conditional probability**: The probability of an event occurring given that another event has already occurred.

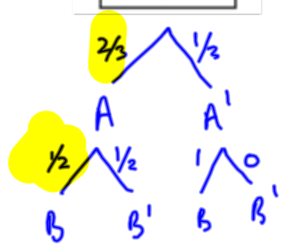
INVESTIGATE

Situation #1: Drawing two balls from the pot, ***without*** replacement.

A ball is randomly selected from the pot and **is not** replaced. Then a second ball is drawn.

Define the flowing events:

A: The first ball is white
B: The second ball is white



Find: P(A) = $\frac{2}{3}$

P(B given that the first ball drawn is white) = $\frac{1}{2}$

P(A and B) = $\frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

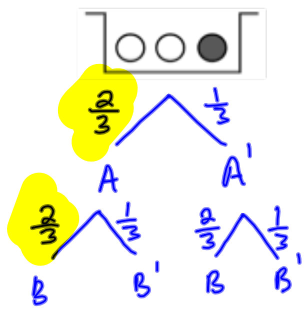
Events A and B are ***dependent*** because the outcome of the second event depends on the outcome of the first event.

Situation #2: Drawing two balls from the pot, ***with*** replacement.

A ball is randomly selected from the pot and **is** replaced. Then a second ball is drawn.

Define the flowing events:

A: The first ball is white
B: The second ball is white



Find: P(A) = $\frac{2}{3}$

P(B given that the first ball drawn is white) = $\frac{2}{3}$

P(A and B) = $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

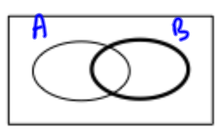
Events A and B are ***independent*** because the outcome of the second event does not depend on the first

Probabilities of Events A and B

General Case:

$$P(A \cap \boldsymbol{B}) = P(A) \cdot P(B|A)$$

given



"probability of A and B"

Equivalently:

$$P(\boldsymbol{B}|A) = \frac{P(A \cap B)}{P(A)}$$

"probability of B given A"

Note: Events A and B are independent if $P(B|A) = P(B)$

For independent events: $P(A \cap B) = P(A) \cdot P(B)$

Example 1: Two cards are drawn without replacement from a shuffled deck of 52 cards.

Define the following events:

A: The first card is a face card.
B: The second card is a face card.

Determine:

a) $P(A \cap B) = P(A) \cdot P(B|A) = \frac{\cancel{12}^{\cancel{3}\,1}}{\cancel{52}_{13}} \cdot \frac{11}{\cancel{51}_{17}} = \frac{11}{221} = 0.0498 = 4.98\%$

b) $P(A' \cap B) = P(A') \cdot P(B|A') = \frac{40}{\cancel{52}_{13}} \cdot \frac{\cancel{12}^{\cancel{3}\,1}}{\cancel{51}_{17}} = \frac{40}{221} = 0.1810 = 18.10\%$

Example 2: According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

C : own a cellphone

S : own a smartphone

$P(C) = 0.91$

$* P(S|C) = 0.42$

$*$ smartphones are a subset of cellphones $\therefore P(S) = P(S \cap C) = P(C \cap S)$

$P(C \cap S) = P(C) \cdot P(S|C)$
$= (0.91)(0.42)$
$= 0.3822 = 38.22\%$

$P(S) = 38.22\%$

Example 3: Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probability that

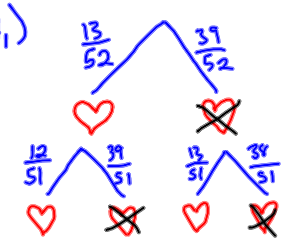
a) both cards are hearts?

$$P(H_1 \text{ and } H_2) = P(H_1) \cdot P(H_2 | H_1)$$

$$= \frac{\cancel{13}^{1}}{{}_{4}\cancel{52}} \cdot \frac{\cancel{12}^{1}}{\cancel{51}_{17}}$$

$$= \frac{1}{17}$$

$$= 0.588 = 5.88\,\%$$



b) neither card is a heart?

$$P(H_1' \text{ and } H_2') = P(H_1') \cdot P(H_2' | H_1')$$

$$= \frac{\cancel{39}^{13}}{{}_{2}\cancel{4}\cancel{52}} \cdot \frac{\cancel{38}^{19}}{\cancel{51}_{17}} = \frac{19}{34} = 0.5588 = 55.88\,\%$$

c) exactly one of the two cards is a heart?

$$P(\text{exactly one heart}) = \frac{\cancel{13}^{1}}{{}_{4}\cancel{52}} \cdot \frac{\cancel{39}^{13}}{\cancel{51}_{17}} + \frac{\cancel{39}^{13}}{{}_{4}\cancel{52}} \cdot \frac{\cancel{13}^{1}}{\cancel{51}_{17}}$$

$$= \frac{13}{68} + \frac{13}{68}$$

$$= \frac{13}{34} = 0.3824 = 38.24\,\%$$

d) both cards are aces?

$$P(A_1 \text{ and } A_2) = P(A_1) \cdot P(A_2 | A_1)$$

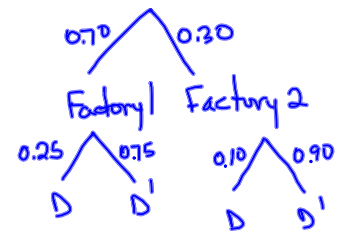
$$= \frac{\cancel{4}^{1}}{{}_{13}\cancel{52}} \cdot \frac{\cancel{3}^{1}}{\cancel{51}_{17}}$$

$$= \frac{1}{221}$$

$$= 0.0045$$

$$= 0.45\,\%$$

Example 4: A company has two factories that make computer chips. Suppose 70% of the chips come from Factory 1 and 30% come from Factory 2. In Factory 1, 25% of the chips are defective; in Factory 2, 10% of the chips are defective.



a) Suppose it is not known from which factory a chip came. What is the probability that the chip is defective?

$$P(D) = (0.70)(0.25) + (0.30)(0.10)$$
$$= (0.175) + (0.03)$$
$$= 0.205 \rightarrow 20.5\%$$

b) Suppose a defective chip is discovered. What is the probability that the chip came from Factory 1?

$$P(F_1|D) = \frac{P(D \cap F_1)}{P(D)}$$
$$= \frac{(0.70)(0.25)}{0.205} = 0.8537 \rightarrow 85.37\%$$

In Summary

Key Ideas

- If the probability of one event depends on the probability of another event, then these events are called **dependent events**. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events.
- If event B depends on event A occurring, then the **conditional probability** that event B will occur, given that event A has occurred, can be represented as follows:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Need to Know

- If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

- A tree diagram is often useful for modelling problems that involve dependent events.
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events.

HW: 5.5 p. 350-353 #2-5 & 7

F Math 12

5.6 Independent Events p. 354

Name ______________

Date ______________

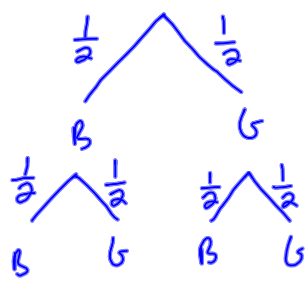
Goal: Understand and solve problems that involve independent events.

EXPLORE...

The Fortin family has two children. Cam determines the probability that the family has two girls. Rushanna determines the probability that the family has two girls, given that the first child is girl. How are these probabilities similar, and how are they different?

Cam: $P(G \text{ and } G) = \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{4}$



Rushanna: $P(G_2 | G_1) = \dfrac{P(G_1 \text{ and } G_2)}{P(G_1)}$

$= \dfrac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1}$

$= \frac{2}{4} = \frac{1}{2}$

Probabilities of Events A and B

General Case: intersection

$P(A \cap B) = P(A) \cdot P(B|A)$

Note: Events A and B are independent if $P(B|A) = P(B)$

For independent events: $P(A \cap B) = P(A) \cdot P(B)$

Example 1: A die is rolled 3 times. What is the probability of getting

a) a 5 on all three rolls?

$$P(FFF) = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$$
$$= \frac{1}{216}$$
$$= 0.46\%$$

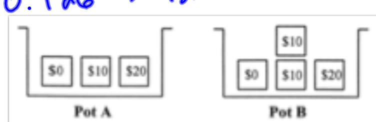
b) at least one 5 on the three rolls?

$$P(\text{at least one } 5) = 1 - P(\text{no } 5)$$
$$= 1 - \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}$$
$$= 1 - 0.579 \longrightarrow 0.421 \longrightarrow 42.1\%$$

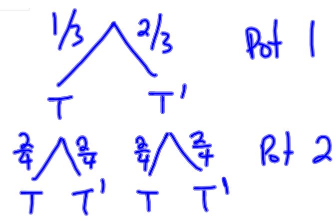
c) at most one 5 on the three rolls?

$$P(\text{at most one } 5) = P(\text{no } 5) + P(\text{one } 5)$$
$$= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} + \left[3\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\right]$$
$$= 0.579 + 3[0.347]$$
$$= 0.926 \rightarrow 92.6\%$$

Example 2: Randomly select one bill from Pot A and one from Pot B.



What is the probability of getting



a) a $10 bill on each draw?

$$P(\text{two } \$10) = \left(\frac{1}{3}\right)\left(\frac{2}{4}\right)$$
$$= \frac{1}{6} = 16.7\%$$

b) a $10 bill on only one of the two draws?

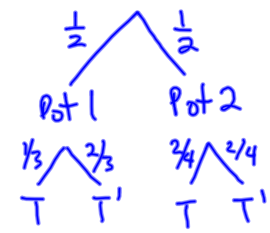
$$P(\text{only one } \$10) = \left(\frac{1}{3}\right)\left(\frac{2}{4}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{4}\right)$$
$$= \frac{1}{6} + \frac{1}{3}$$
$$= \frac{3}{6} = \frac{1}{2} \rightarrow 50\%$$

Example 3: A pot is randomly selected, then a bill is randomly chosen from that pot.

What is the probability that

a) a $10 bill is chosen?

$$P(\$10) = \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left(\frac{2}{4}\right)$$
$$= \frac{1}{6} + \frac{1}{4}$$
$$= \frac{2}{12} + \frac{3}{12}$$
$$= \frac{5}{12} = 41.7\%$$



b) if a $10 bill is chosen, what is the probability that it came from Pot 1?

$$P(\text{Pot } 1 \mid T) = \frac{P(\text{Pot } 1 \text{ and } T)}{P(T)}$$
$$= \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{2}{4}}$$

$$\frac{\frac{1}{6}}{\frac{5}{12}}$$
$$\frac{1}{\cancel{6}_1} \times \frac{\cancel{12}^2}{5} = \frac{2}{5}$$
$$= 40\%$$

In Summary

Key Ideas

- If the probability of event *B* does not depend on the probability of event *A* occurring, then these events are called independent events. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events.
- The probability that two independent events, *A* and *B*, will both occur is the product of their individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

Need to Know

- A tree diagram is often useful for modelling problems that involve independent events.
- Drawing an item and then drawing another item, after replacing the first item, results in a pair of independent events.

HW: 5.5 p. 350-353 #6, 8, 9, 13 & 18; 5.6 p. 360-363 #5, 7, 8 & 12