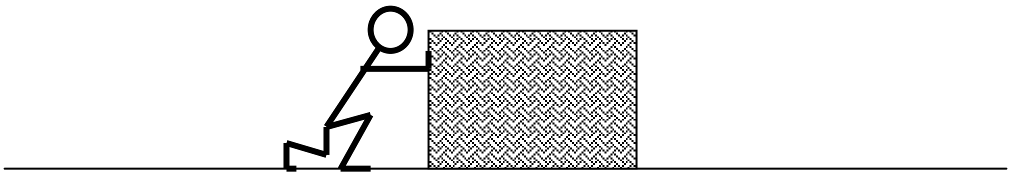
Forces and Newton's Laws

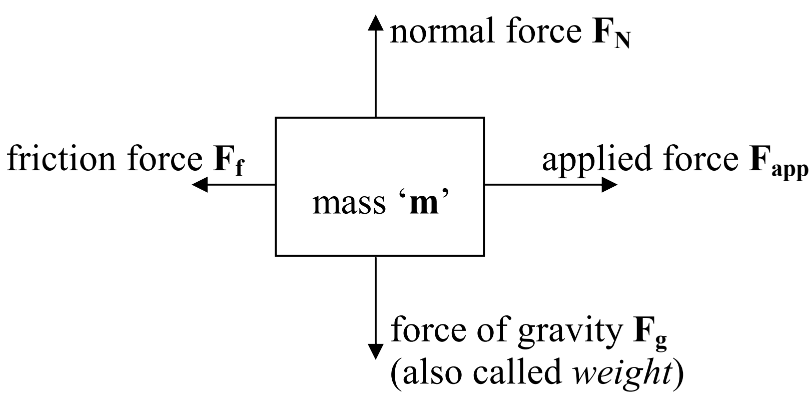
A *force* is simply a push or pull of one object on another object. Two objects are needed for a force to occur: one object applies the force, while the other object has the force applied to it. For example:



- In this system, a student (object 1) applies a force to the right, which acts on the crate (object 2).

Since a force can be directed one way or another, it is a *vector* quantity, with symbol 'F' and units being the *Newton*, or 'N'.

Typically, only the object that has forces acting on it is analyzed in a physics problem. The analysis is represented as a *free-body diagram* that isolates the object in question and shows all the force vectors acting on it. In the diagram above, there are four forces that act on the crate:



These are some typical forces that you should know from Physics 11. Note that:

- the friction force always acts to oppose motion, and may or may not be as large as the applied force. It is always *parallel* to the surface upon which the object is resting or moving.
- The normal force is a pressing force between object and surface. It always acts *perpendicular* to the surface upon which the object is resting or moving.
- The friction force and normal force vectors are always perpendicular to each other.

As a branch of physics, *dynamics* is the study of the causes of motion, and is explained by Newton's three laws of motion.

- **First Law:** Every mass has *inertia*, which is the tendency of the object to resist any change in its state of motion (i.e. resist any acceleration). Therefore, if the *net force* on an object is zero, two things can happen:
  - if the object is stationary, it will remain at rest.
  - if already in motion, the object will continue to move at a constant speed in a straight line. This is *uniform motion*.

  In either case, because $\mathbf{F_{Net}} = 0$, all forces in a free-body diagram must "cancel" out; i.e., there is no resultant vector.

- **Second Law:** This law explains an object's motion when the net force acting on it is not zero. This net force will cause the object to *accelerate*. The rate of acceleration depends on force and mass as follows:

  $$\mathbf{a} \propto \mathbf{F_{Net}} \quad \text{and} \quad \mathbf{a} \propto \frac{\mathbf{1}}{\mathbf{m}} \quad \rightarrow \quad \mathbf{F_{Net} = ma}$$

  Note that since $\mathbf{F_{Net}}$ exists, the vector-sum of all forces in a free-body diagram will reveal a resultant vector.

- **Third Law:** Whenever one object exerts a force on a second object, the second object exerts an equal but opposite force on the first object. This is the action-reaction effect: the *action* is applied on one object, but the reaction is applied back on the first object. Normal and friction forces are examples of reaction forces.

Example #1. An 8.0 kg mass is pushed along a horizontal surface at constant speed with a force of 24 N.

a) Draw a f.b.d. showing all forces that act on the mass. Include the value for each force drawn.

b) The same 8.0 kg mass is now pushed with a force of 37 N. Find the acceleration of the mass.

(see Dynamics Ex 1 for answer)

Net Force and Vertical Motion

We'll start by examining objects where the unbalanced force acts ***vertically***.

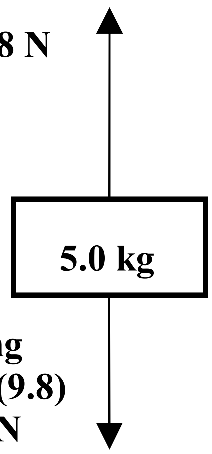
Set up by following these steps:

- draw a free-body diagram (f.b.d.) showing all relevant forces acting on the mass;
- next, write out the freebody equation, relating all forces to $\mathbf{F_{Net}}$;
- finally, substitute all given information and solve for the unknown quantity.

Note that if the force is larger than the weight, the mass accelerates *upward*; if the force is smaller that the weight, the mass accelerates *downward*.

Consider the diagram below, showing a 5.0 kg mass being pulled upward by a rope with a tension force of 68 N.

$F_T = 68$ N



$F_g = mg$
$= 5.0(9.8)$
$= 49$ N

In this case it can be seen that the net force will act *upward*, so choose **up** as positive.

→ **$F_{Net} = 68 - 49 = 19$ N** (upward)

→ then: **$F_{Net} = ma$**

$19 = 5.0a$

$a = 3.8\ m/s^2$ (upward)

Example #2. A 25.0 kg mass is pulled upward vertically by a force of 183 N. Find the acceleration of the mass.

(see Dynamics Ex 2 for answer)

Elevator Problems.

In its simplest form, an elevator moves and accelerates vertically up and down, just like the problems previously discussed. And like these problems, this motion is due to the tension force in a cable pulling up, and the force of gravity pulling down on the elevator itself. If the values of these forces are known, the acceleration of the elevator can be easily determined using the methods shown above. Or, if the acceleration is known, an unknown force can be determined in the same manner.

Example #3. What is the tension if the cable of an elevator of mass 550 kg that is accelerating upwards at the rate of 4.5 m/s^2?

(see Dynamics Ex 3 for answer)

At the same time, we can also analyze the motion of an elevator by examining the forces that act on a person *inside* the elevator as it moves up or down.

You should be able to find the apparent weight of someone in the elevator when it is *accelerating upwards*, *decelerating upwards*, *accelerating downwards* and *decelerating downwards*. It is also possible to find the tension force in the cable under these conditions.

The *apparent weight* is the reading that would appear on a bathroom scale if the person were standing on it, inside the moving elevator. This is simply the *normal force* exerted by the elevator floor (and the scale) upward on the person's feet.

The passenger feels *heavier* when accelerating upwards or decelerating downwards because the floor of the elevator must not only support her weight, but also supply an accelerating (or decelerating) force. In this instance, $\mathbf{F_N} > \mathbf{F_g}$.

Conversely, the passenger feels *lighter* when decelerating (slowing down) upwards or accelerating downwards because part of her weight is being used to supply the accelerating force, leaving less normal force on her feet. It is as if the floor is "falling" away from her feet; in this case, $\mathbf{F_N} < \mathbf{F_g}$.

To solve this type of problem, it is important to recognize that there are only two vertical forces acting on the passenger:

- the normal force of the bathroom scale (pushing upward);
- the force of gravity, i.e., her weight (pulling down).

By drawing a free-body diagram showing these forces, and recognizing the direction of the elevator's acceleration (and net force), you can determine the apparent weight (i.e., normal force) for any passenger inside.

Example. #4. A 50.0 kg student is riding an elevator while standing on a bathroom type scale. Find the scale reading when the elevator is:

a) **accelerating upwards at 0.50 m/s^2.**
b) **traveling upwards, but decelerating at 1.0 m/s^2.**
c) **accelerating downwards at 0.75m/s^2.**

(see Dynamics Ex 4 for answer)

Note that $\mathbf{F_N} < \mathbf{F_g}$ in both (b) and (c), because acceleration is ***downward***.

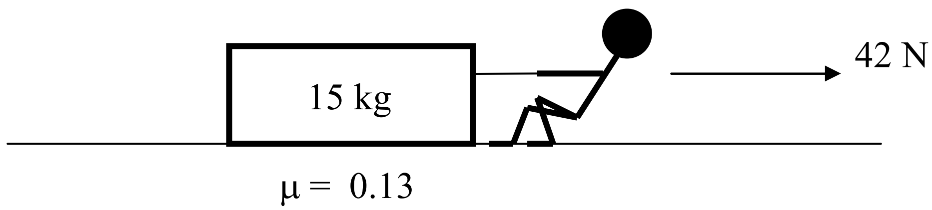
Example. #5. A 90 kg person stands on a bathroom type scale in an elevator as it accelerates downwards. If the scale reads 85 N, at what rate is the elevator accelerating?

(see Dynamics Ex 5 for answer)

Applications of Friction Force

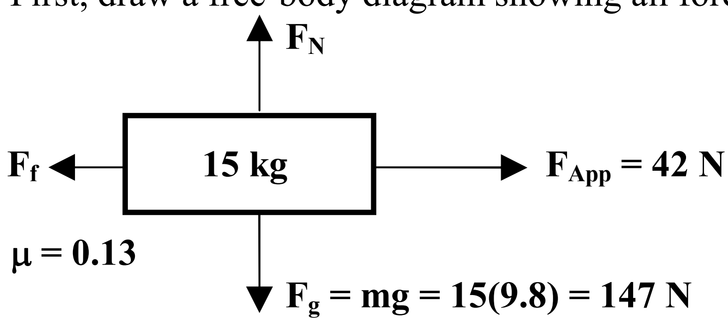
To start, think about objects that are moved along a surface, and the net force acts parallel to that surface. This is similar to problems you encountered in Physics 11.

Consider the following: a 15 kg mass is pulled along a horizontal surface with a force of 42 N. A coefficient of friction of $\mu = 0.13$ exists between the mass and the surface.



To find the acceleration of the mass, follow these procedures:

- First, draw a free-body diagram showing all forces acting on the mass:



- From inspection, $\mathbf{F_{Net}}$ acts horizontally to the right: $\mathbf{F_{Net} = 42 - F_f}$

  → to find $\mathbf{F_f}$, use $\mathbf{F_f = \mu F_N}$

  where $\mathbf{F_N = F_g = 15(9.8) = 147\ N}$

  → $\mathbf{F_f = .13(147) = 19.1\ N}$

- Now solve for $\mathbf{F_{Net}}$ and **a:**

  → $\mathbf{F_{Net} = 42 - 19.1 = 23\ N}$

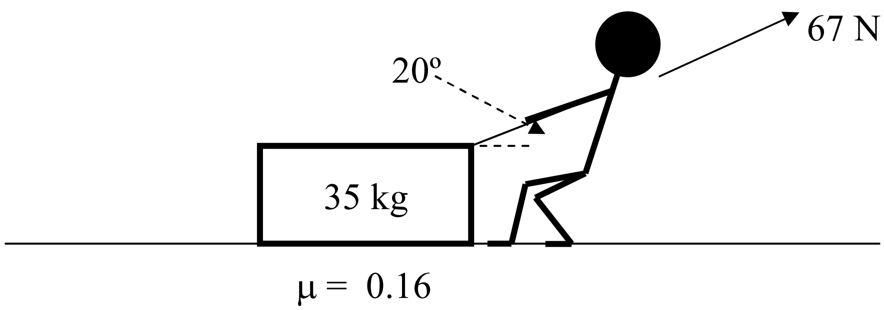
  → $\mathbf{F_{Net} = ma}$ →$\mathbf{23 = 15a}$ →$\mathbf{a = 1.5\ m/s^2}$

Example #6. Find the acceleration of a 4.0 kg mass when a 20 N force is applied and the coefficient of friction between mass and surface is 0.10.

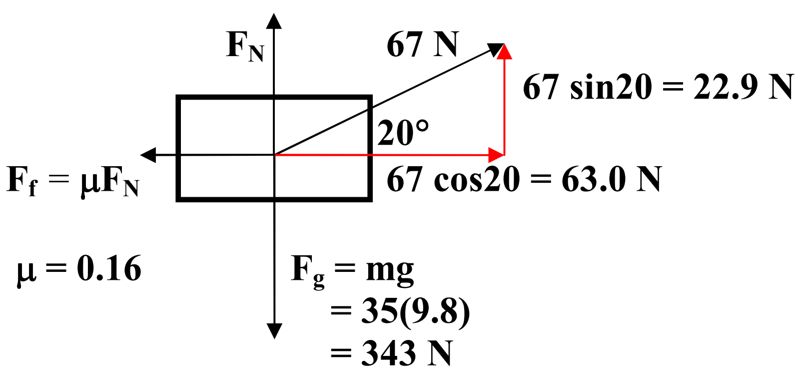
(see Dynamics Ex 6 for answer)

Next, let's consider an angled pull, involving friction. An angled applied force affects the normal force and therefore <u>also affects friction</u>. An angled force upwards *reduces* the magnitude of both forces, while an angled force downwards *increases* those same quantities.

Suppose you have a force of 67 N pulling at an angle of 20º up on a mass of 35 kg in order to move it along a horizontal floor. If the coefficient of friction is 0.16, find the acceleration.



- Start with a f.b. diagram, and break the applied force of **67 N** into components: (note that vectors are <u>not</u> drawn to scale)



- As usual, to find '**a**' we must first determine $\mathbf{F_{Net}}$, which is to the right, and horizontal.

- By examining the horizontal vectors in the diagram, $\mathbf{F_{Net} = 63.0 - F_f}$

- We must find $\mathbf{F_f}$, where $\mathbf{F_f = \mu F_N}$
  → so we need to first find $\mathbf{F_N}$ by analyzing the *vertical* force vectors, which are perpendicular to the surface:

$$\mathbf{F_N + 22.9 = F_g \quad \rightarrow \quad F_N = 343 - 22.9 = 320\ N}$$

→ now calculate $\mathbf{F_f}$: $\quad \mathbf{F_f = \mu F_N = 0.16(320) = 51.2\ N}$

- At this point, we can find both $\mathbf{F_{Net}}$ and **a**:

  →F_{Net} = 63.0 – F_f = 63.0 – 51.2 = 11.8 N

  →F_{Net} = ma →11.8 = 35a →a = 0.34 m/s^2

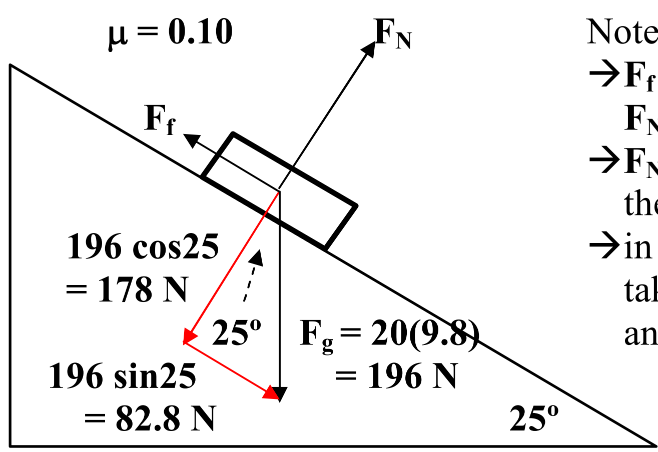
Keep in mind that for any problems involving Newton's Laws, you may have to deal with some or all of the following: force *components*, $\mathbf{F_N}$, $\mathbf{F_f}$, $\mathbf{F_{Net}}$, and/or **a.**

Example #7. A force of 75 N pushes <u>down</u> at an angle of 15º on a mass of 25 kg. If the coefficient of friction is 0.15, find the acceleration.

(see Dynamics Ex 7 for answer)

Slightly more difficult problems involve objects placed on a sloped surface. For this situation, consider a 20.0 kg mass, set on a smooth inclined ramp of μ = 0.10. What is its acceleration if the angle of the slope is 25º?

- First, review how pulling and pressing forces are found on an incline:



Note the following:
→$\mathbf{F_f}$ is parallel to the surface, while $\mathbf{F_N}$ is perpendicular
→$\mathbf{F_{Net}}$ will act downslope, parallel to the surface
→in order to find $\mathbf{F_N}$, $\mathbf{F_f}$ and $\mathbf{F_{Net}}$ we must take components of $\mathbf{F_g}$ that are parallel and perpendicular to the surface

Note the following items shown on the diagram:
a) Calculation of weight $\mathbf{F_g}$
b) The parallel and perpendicular vector components of $\mathbf{F_g}$, along with the angle within the components triangle
c) Calculation of the parallel component of $\mathbf{F_g}$, pulling the mass down-slope; this is **mg sinθ**
d) Calculation of the perpendicular component of $\mathbf{F_g}$, pressing the mass into the ramp; this is **mg cosθ**

- Second, examine *perpendicular* vectors to find $\mathbf{F_N}$:

  → by looking at the diagram, $\mathbf{F_N = 178\ N}$

- Finally, examine *parallel* vectors to find $\mathbf{F_f}$, $\mathbf{F_{Net}}$ and finally **a**:

  → since acceleration and $\mathbf{F_{Net}}$ are down-slope,

  $\mathbf{F_{Net} = 82.8 - F_f}$ where $\mathbf{F_f = \mu F_N = 0.10(178) = 17.8\ N}$

  → so $\mathbf{F_{Net} = 82.8 - 17.8 = 65.0\ N}$

  → $\mathbf{F_{Net} = ma}$ → $\mathbf{65.0 = 20.0a}$ → $\mathbf{a = 3.25\ m/s^2}$

Remember to draw free-body diagrams as you work through <u>any</u> multiple force-related problem; they help in keeping your thoughts organized, particularly as you work through the following examples.

Example #8. A 35 kg mass is set on a smooth inclined surface of μ=0.20. What is its acceleration if θ = 34°?

(see Dynamics Ex 8 for answer)

Example #9. A 75 kg skier starts down a 30° slope from rest. If the coefficient of friction is 0.10, what is the acceleration <u>and</u> speed 6.0 seconds after starting?

(see Dynamics Ex 9 for answer)

A final note: in these examples where an object is on an incline, the free-body equation always comes down to

$$\mathbf{F_{Net} = mg\sin\theta - \mu F_N}$$

→ where $\mathbf{F_N = mg\cos\theta}$ and $\mathbf{F_{Net} = ma}$

→ so $\mathbf{ma = mg\sin\theta - \mu mg\cos\theta}$ → masses cancel!

→ this leaves $\mathbf{a = g\sin\theta - \mu g\cos\theta}$ → a shortcut for finding **a**.

There are two last aspects to consider in terms of friction: *static* and *kinetic* (sliding).

- **Static friction** is that force used to keep a stationary object from moving; because there is more contact between object and surface, μ_s is large.
- **Kinetic friction** is that force which opposes the motion of an object; since the sliding of object over surface causes less contact between the two, μ_k is small.

It requires more force, therefore, to overcome static friction and *begin* to move a stationary object than it does to keep that object moving against kinetic friction.

Example #10. A muscular physics student needs to move a 145 kg crate across the room over a floor where $\mu_s = 0.370$ and $\mu_k = 0.210$.

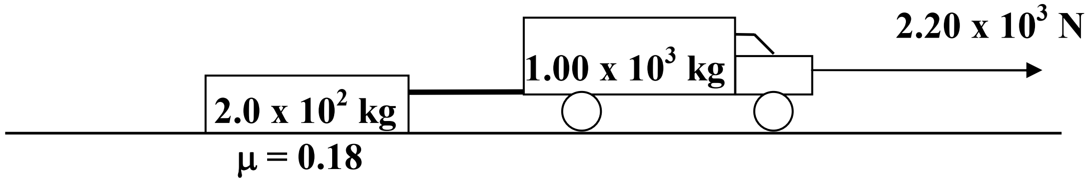
a) What minimum horizontal force is required to *just* start the crate sliding?
b) If this force continues to be applied, what will be the rate of acceleration?

(see Dynamics Ex 10 for answer)

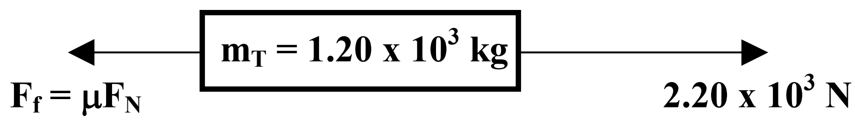
Connected Mass Problems

Consider this problem: a truck pulls a log with a force of 2.2×10^3 N. Sliding friction exists between log and road, and friction between truck's tires and road can be ignored. The mass of the truck is 1.0×10^3 kg, while the mass of the log is 2.0×10^2 kg. Find the acceleration of the system, and the tension in the rope connecting the two masses.

Start with a sketch of the system: (choose $\longrightarrow$ as positive)



➢ To find acceleration, consider a f.b. diagram of the system:



→first, find $\mathbf{F_f = \mu F_N}$ where $\mathbf{F_N}$ = the weight of the log

$$\mathbf{F_f = 0.18(2.0 \times 10^2)(9.8) = 353\ N}$$

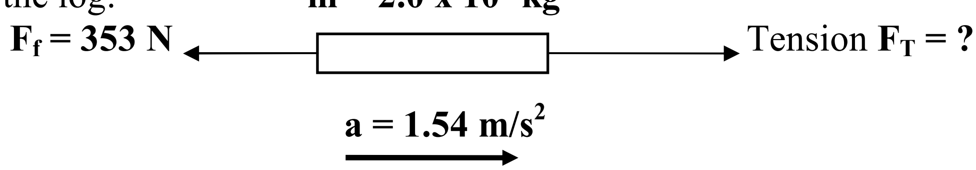
→so $\mathbf{F_{Net} = 2200 - 353 = 1847\ N}$

→finally, $\mathbf{F_{Net} = m_T a}$ →$\mathbf{1847 = 1200a}$ →$\mathbf{a = 1.54\ m/s^2}$ **(right)**

→ this acceleration is the same for any part of the system.

➢ To find tension, consider a f.b. diagram of only the log OR only the truck:

→ for the log:

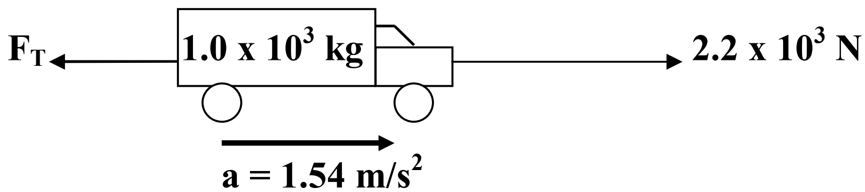


→First, find $\mathbf{F_{Net} = ma = 200(1.54) = 308\ N}$

→Now use the f.b.d. to make an equation:

$$\mathbf{F_{Net} = F_T - 400} \quad \rightarrow \quad \mathbf{308 = F_T - 353} \quad \rightarrow \quad \mathbf{F_T = 6.5 \times 10^2\ N}$$

The same answer would result if only the truck was analyzed. The f.b.d. for the truck would appear as:



(these are the only horizontal forces acting on the truck; with no friction force here, the vertical forces have no effect on the net force)

→First, find $\mathbf{F_{Net} = ma = 1000(1.54) = 1540\ N}$

→Now use the f.b.d. to make an equation:

$$F_{Net} = 2200 - F_T \quad \rightarrow \quad 1540 = 2200 - F_T \quad \rightarrow \quad F_T = 6.6 \times 10^2\ N$$

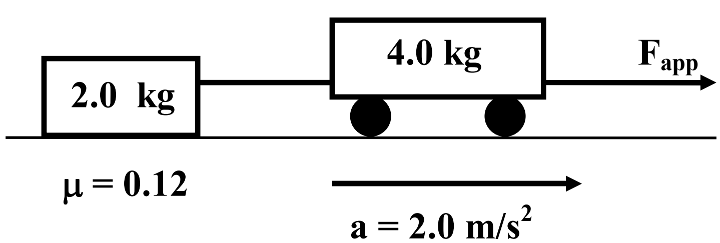
The difference between the two values for $\mathbf{F_T}$ results from rounding off answers as the problem is worked out. To avoid this, carry extra sig. figs. as you proceed through your calculations.

Example #11. A Truck pulls a log with a force of 2500 N. The log drags back with a 800 N force of friction. The mass of the truck is 2500 kg, the mass of the log is 600 kg. Find:

a) the acceleration of the truck & log system.
b) the tension in the rope.

(see Dynamics Ex 11 for answer)

Example #12. Two masses shown below are connected together and pulled by an applied force to the right, causing an acceleration of 2.0 m/s^2. There is a coefficient of friction between the 2.0 kg mass and the floor, while the friction between the cart and the floor is negligible. Find:



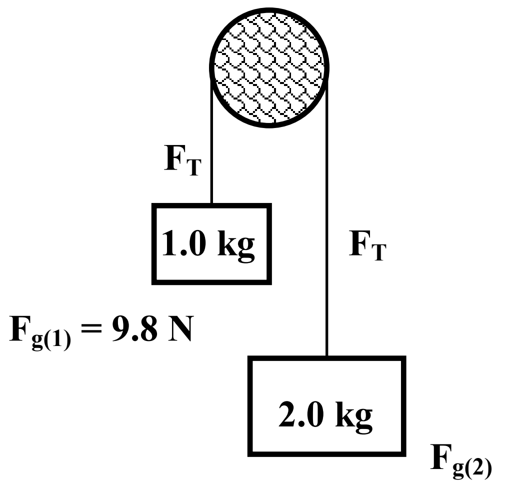
a) the tension in the string attaching the two masses.
b) the applied force used to pull the system.

(see Dynamics Ex 12 for answer)

Connected Masses and Pulleys

For these problems we need a sign convention; let the direction of movement (in this case, the direction of the **net force**) be positive.

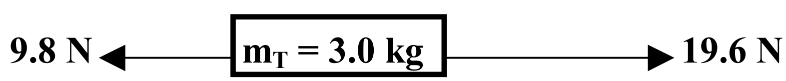
Here is a standard Physics 11 problem (with a wrinkle): two connected masses hang over a pulley, as shown below. Determine the tension in the rope.



Recall: a frictionless pulley can only change the direction of a force; the tension F_T in the rope connecting the masses is the same on either side of the pulley.

- Ignore tension for now; first, find what unbalanced force and acceleration acts on the whole system.

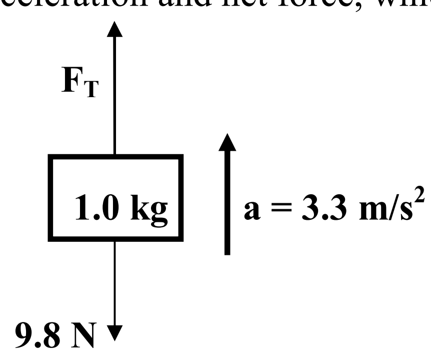
  →examine the f.b.d. of the system:



→$F_{Net} = 19.6 - 9.8 = 9.8$ N

→ $F_{Net} = m_T a$ →$9.8 = 3.0a$ →$a = 3.3\ m/s^2$

- Now find the tension in the string by examining a f.b. diagram of only one of the two masses. If we choose the 1.0 kg mass, we must recognize its upward acceleration and net force, which means:



→$F_{Net} = ma = 1.0(3.3) = 3.3$ N

→$F_{Net} = F_T - 9.8$

→$3.3 = F_T - 9.8$ → $F_T = 13$ N

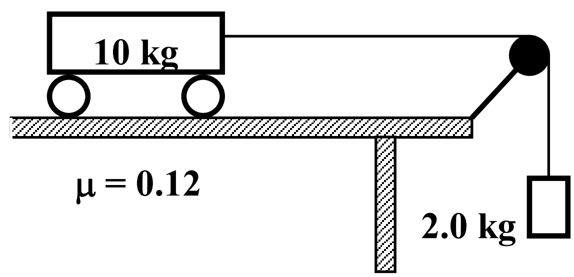
Note that the same answer would result from analyzing only the 2.0 kg mass. In this case, the acceleration is downward, so $\mathbf{F_{Net} = 19.6 - F_T}$. I'll leave this one for you to prove.

Example #13. Two masses are suspended by a single pulley, and hang on each side of it. One mass is 4.0 kg and the other is 6.0 kg. Find:

a) the acceleration of the system.
b) the tension in the rope.

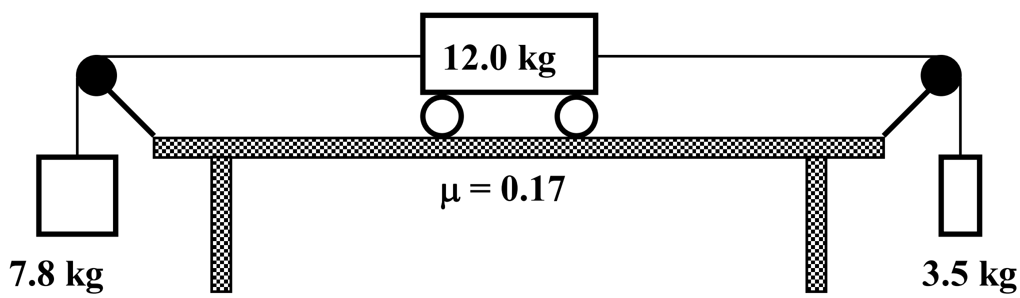
(see Dynamics Ex 13 for answer)

Example #14. In the diagram to the right, the weight of the 2.0 kg mass exerts a force on the system causing both masses to move. Given the information listed, find: (a) the acceleration of the system and (b) the tension in the string.



(see Dynamics Ex 14 for answer)

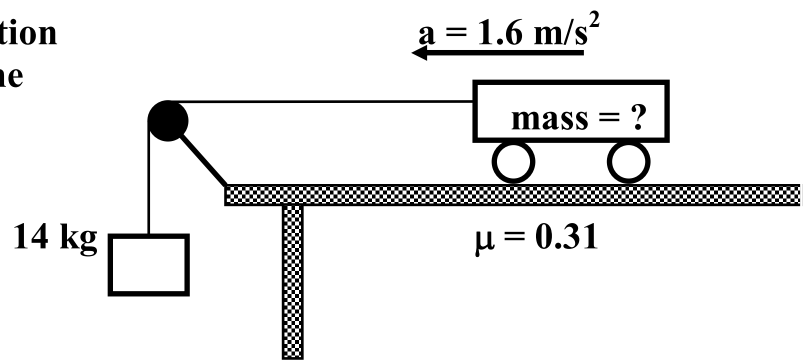
Example #15. Two hanging masses are attached to one horizontal mass. Note that the two tensions are not the same.



a) What is the unbalanced or net force?
b) What is the acceleration of the system?
c) What is the tension in each rope?

(see Dynamics Ex 15 for answer)

Example #16. Given the information in the diagram to the right, find the unknown mass of the cart.

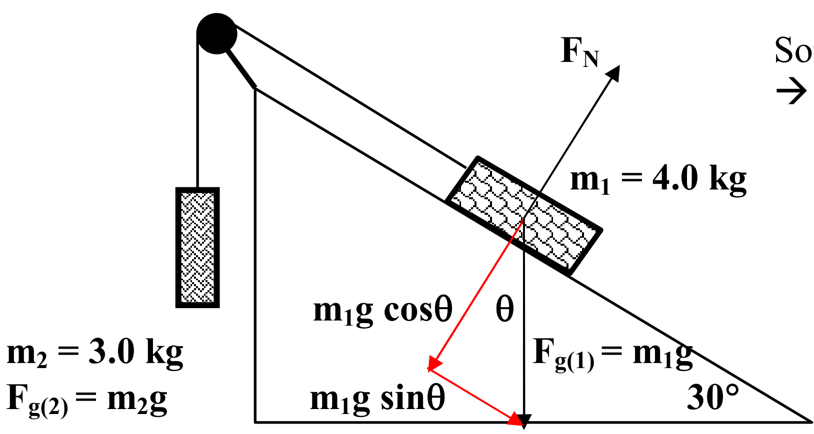


Hint: start by isolating the known mass to find tension $\mathbf{F_T}$ in the string.

(see Dynamics Ex 16 for answer)

Finally, consider a coupled mass system, one on an incline and one hanging.

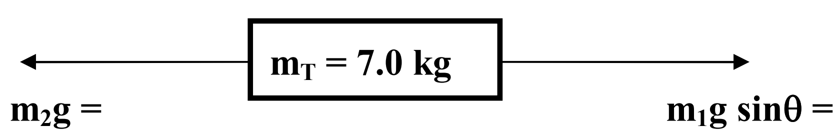
Example #17. In the diagram below, a 4.0 kg mass rests on a 30º <u>frictionless</u> slope and is pulled by a 3.0 kg mass connected to it over a pulley by a cord. What is the acceleration of the system and the tension in the cord?



Some helpful hints:

→ first, calculate all forces that affect motion; this includes taking components of $\mathbf{F_g}$ on the right mass, as shown.

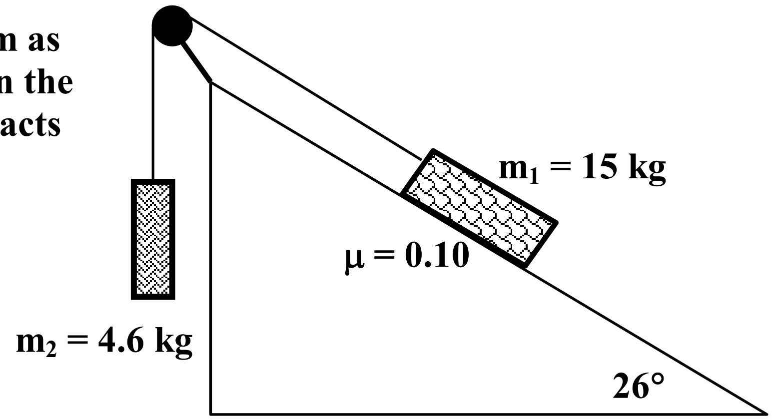
→Next step: to find the acceleration, consider the ***whole system***. Note which force is greater; that will be the direction of the net force and the acceleration.



→Finally, to find the **tension**, analyze just one mass. Try **mass 2** (3.0 kg); it has fewer forces acting on it so is probably easier to work with.

(see Dynamics Ex 17 for answer)

Example #18. Similar problem as #17, but with friction acting on the 15 kg mass. Note that friction acts in the *opposite* direction to the largest force. Find the acceleration by <u>first determining the direction of motion</u>.



(see Dynamics Ex 18 for answer)