Work and Energy

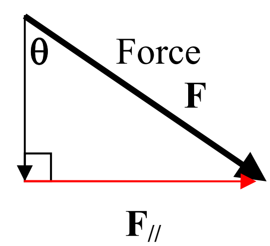
Work is done on an object that can exert a resisting force and is only accomplished if that object will move. In particular,

- we can describe *work done* by a specific object (where a force is applied) or on a specific object (where an opposing force must be overcome)
- we can also specify whether *work done* is due to one particular force or to the total net force on the object.
- *work done* is converted to other forms of energy.

Essentially, the amount of work accomplished can be determined ***two*** ways:

1) **Work = force x distance,** or **W = Fd** → units: **Joules (J)**

- Note that work is done only when a force acts *parallel* to the motion of an object, thereby affecting its motion. For any force that acts at an oblique angle to the direction of motion, only the parallel component of that force can be used to determine the work done.



→Use component $\mathbf{F}_{//}$ to find work: $\mathbf{W = F_{//}d}$

2) **Work = a change in energy,** or **W = ΔE** →the work-energy theorem

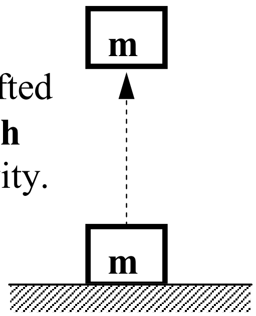
- this means that whenever one form of energy changes to another, work is done

Either method may be utilized to calculate work done, depending on the information given in the problem. What follows are examples of the *types* of work that can be done on an object.

A: Work Done Against Gravity

When an object is lifted upward, work is done on a mass against the resisting force of gravity. The energy used to do this is converted to *gravitational potential energy*, or $\mathbf{E_p}$. In fact, $\mathbf{E_p}$ increases as the mass is lifted higher and higher.

A mass is lifted a distance $\mathbf{\Delta h}$ against gravity.



→ start with $\mathbf{W = F_{App}d}$ where $\mathbf{F_{App}}$ is the applied force.

→ assuming the mass was lifted at a constant '**v**',

then $\mathbf{F_{Net} = 0}$ and $\mathbf{F_{App} = F_g}$

- therefore, the work done against gravity is $\mathbf{W = F_g d = mg\Delta h}$
- since the newly stored potential energy is $\mathbf{E_p = mgh}$, → $\mathbf{W = \Delta E_p}$

This tells us that work done against gravity = potential energy gained by the mass.

Example #1. A 6.0 kg mass is raised from 1.5 m above the ground to 6.5 m high.

a) What work is done?

b) What $\mathbf{E_p}$ does the mass now have?

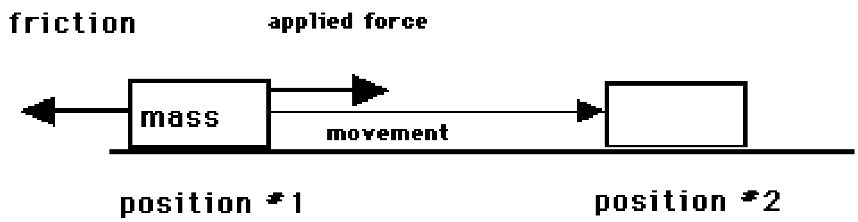
(see Work-Energy Ex 1 for answer)

Note: gravitational potential energy is a relative measurement which depends on what elevation is chosen to be **h = 0**. Usually the '**0**' location is chosen as the lowest position that an object has the potential to fall.

B: Work done against the force of Friction

As long as an object moves along a horizontal surface with constant velocity, all the work is done against friction. If acceleration occurs, then work is being done against inertia as well (we will consider this later).

Consider the following diagram of an object moved from position #1 to position #2 at constant velocity.



As with the gravity example, start with $\mathbf{W = F_{App}d}$

- at constant velocity, there is no acceleration, so $\mathbf{F_{Net} = 0}$
- this means that $\mathbf{F_{App} = F_f}$ → $\mathbf{W = F_f d}$

The work done against friction is changed to heat energy and lost to the system.

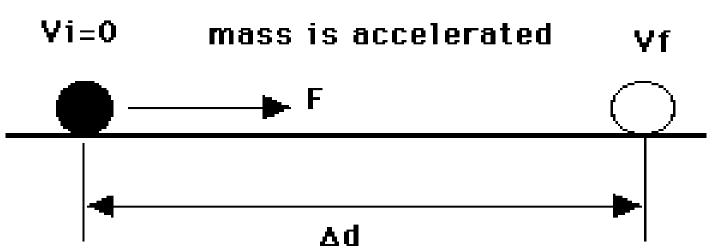
Example #2. A 150 kg object is pulled at constant velocity over a horizontal surface (μ = 0.12) for a distance of 7.0 m. How much heat energy was generated?

(see Work-Energy Ex 2 for answer)

<u>C: Work done against inertia (also called *net* work done)</u>

When a force acts to accelerate an object over a distance, that object is no longer "doing what it's already doing". In other words, work is done against *inertia*. This work is stored as the energy of speed, called *kinetic energy* or $\mathbf{E_k}$.

Consider the case of a ball upon which some <u>unbalanced force</u> acts. The ball is accelerated from $\mathbf{v_i=0}$ to some final speed $\mathbf{v_f}$ over a distance **d**.



Note that kinetic energy changes as the speed of the object changes. The work done against inertia to accelerate the mass a distance **d** can be determined *<u>two</u>* ways:

1) $\mathbf{W = F_{Net}d}$ 2) $\mathbf{W = \Delta E_k}$

To prove these two methods to find net work are the same:

- start with $\mathbf{F_{Net} = ma}$ → therefore $\mathbf{W = mad}$
- from kinematics, we also know that $\mathbf{a = \frac{v_f - v_i}{t}}$ and $\mathbf{d = (\frac{v_f + v_i}{2})t}$
- substituting into **W = mad,** we obtain

$$\mathbf{W = m(\frac{v_f - v_i}{t})(\frac{v_f + v_i}{2})t}$$ → **t** cancels, leaving

$$\mathbf{W = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

- and since $\mathbf{E_k = \frac{1}{2}mv^2}$, $\mathbf{W = \Delta E_k}$

In other words, work is done against inertia to change kinetic energy. If a force is exerted on a moving mass and its effect is to change the velocity of the mass, then work has been done against inertia. This is the *net* or useful work done.

Example #3: A 60.0 kg lab cart is moving at 5.00 m/s, and is accelerated to 12.0 m/s. How much work was done to cause this?

(see Work-Energy Ex 3 for answer)

Example #4: A force of 100 N is applied on a 50 kg cart that is moving with a speed of 6.0 m/s and has a force of friction of 20. N acting on it. At the end of 10. seconds, the cart is going 22 m/s.

a) **How much work was done against inertia?**
b) **How much work was done in total?**

(see Work-Energy Ex 4 for answer)

Total Work Done

The work in moving any object can be done against more than one resisting quantity. For example, when you accelerate a car up a steep hill, the car's engine is performing work against inertia, gravity and friction, all at the same time!

The total work done by the engine is

$\mathbf{W = \Delta E_k + \Delta E_p + F_f d}$ → calculate each of these quantities separately, then add them up.

However, keep in mind that when your engine applies a force to move the car up the steep hill, this total work can also be determined by

$\mathbf{W = F_{App} d}$ → where $\mathbf{F_{App}}$ is the force applied by the engine to move the car up the hill.

This second calculation can be used if you know what overall force is applied on an object, or can determine its value using vector analysis.

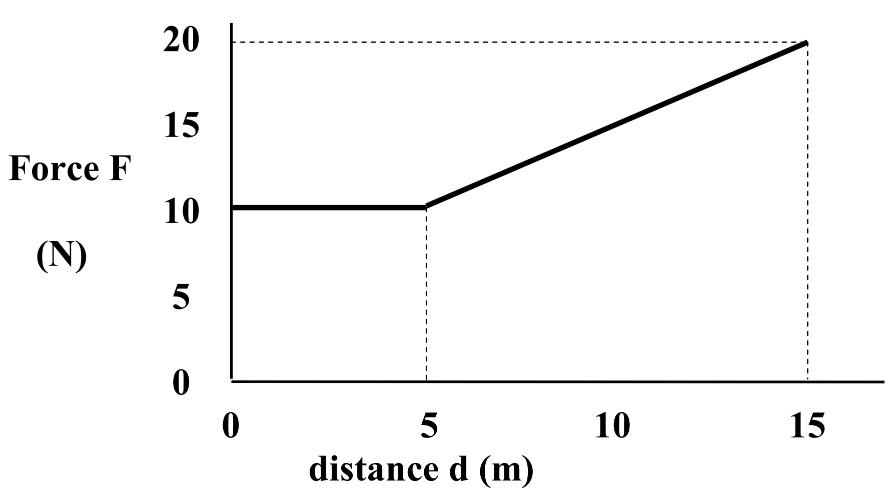
Work done when the force is not constant.

Because $\mathbf{W = Fd}$ and area $\mathbf{A = l \times w}$, work is the area under a force vs. distance graph. If an applied force is not constant, simply graph the varying force vs. distance and calculate the area; this gives total work done.

Example #5: A 5.0 kg cart is accelerated using a varying force. The force is a constant 10 N for 5 m, then increases at a constant rate up to 20 N for another 10 m.

a) What is the total work done on the cart?

b) If the cart was going 24 m/s when this began, what is its speed now?



(see Work-Energy Ex 5 for answer)

Power

Power is the work done per unit time, or the rate of doing work on an object. As a formula,

$$\mathbf{P = \frac{W}{t} = \frac{\Delta E}{t}}$$ where units are **J/s**, or **watts** (**W**)

If work can be done against the forces of gravity, or inertia, or friction, then power is required to do the work. For example:

- power developed when doing work against gravity is given by:

$$\mathbf{P = \frac{W}{t} = \frac{\Delta E_p}{t} = \frac{mg\Delta h}{t}}$$

- power developed when doing work against inertia (i.e. accelerating) is given by:

$$\mathbf{P = \frac{W}{t} = \frac{\Delta E_k}{t} = \frac{\frac{1}{2}m(v_f^2 - v_i^2)}{t}}$$

- power developed from work done against friction is given by:

$$\mathbf{P = \frac{W}{t} = \frac{F_f d}{t} = \frac{\mu F_N d}{t}}$$

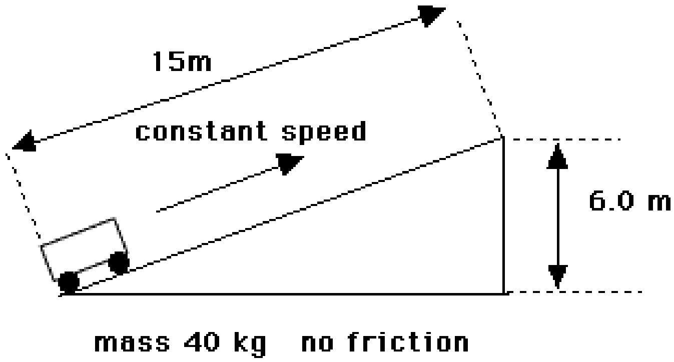
- if work is done on all of these forces at the same time, then the total work is added together and divided by time:

$$\mathbf{P = \frac{mg\Delta h + \frac{1}{2}m(v_f^2 - v_i^2) + \mu F_N d}{t}}$$

Example #6: A cart accelerates from 0 to 15 m/s in 60 sec. What power is developed if the mass of the car is 20. kg?

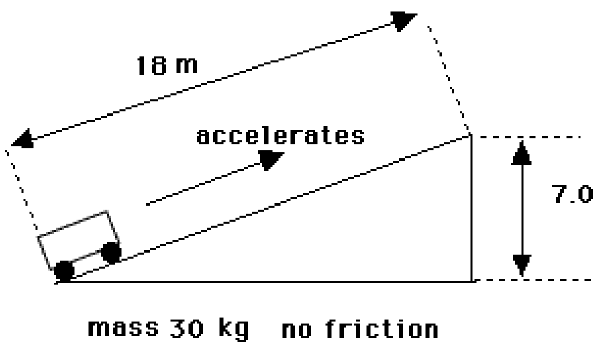
(see Work-Energy Ex 6 for answer)

Example #7: For the diagram below, if the cart goes from the bottom to the top in 16 seconds, how much power was developed?



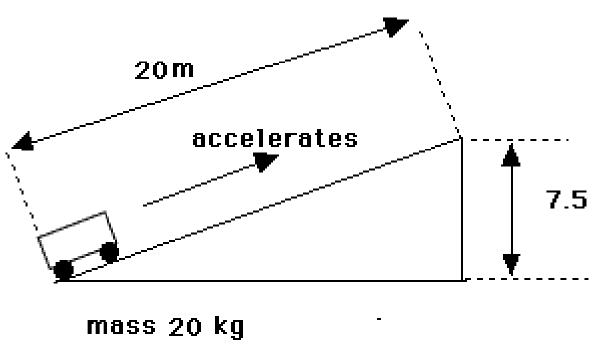
(see Work-Energy Ex 7 for answer)

Example #8: The cart below has an initial speed of 2.0 m/s and accelerates to 5.0 m/s by the time it is at the top of the ramp. How much power is developed? Hint: to find time, use kinematics.



(see Work-Energy Ex 8 for answer)

Example #9: Here there is a 22° slope. The cart starts from rest at the bottom of the ramp and accelerates to 4.0 m/s by the time it reaches the top of the ramp. With a coefficient of friction μ = 0.21, how much power was developed?



(see Work-Energy Ex 9 for answer)

One last point: a shortcut can be utilized to find average power developed by a moving vehicle of known velocity. To do so, examine the power equation carefully and perform these steps:

$$\mathbf{P} = \frac{W}{t} \quad \rightarrow \text{where} \quad \mathbf{W = Fd} \quad \rightarrow \quad \mathbf{P} = \frac{\mathbf{Fd}}{\mathbf{t}}$$

But recall from kinematics: $\mathbf{d = v_{av}t}$ $\rightarrow$ $\mathbf{v_{av}} == \frac{d}{t}$

By substitution, a new equation is produced: $\mathbf{P = Fv_{av}}$

This equation shows that the power developed in any moving object is directly proportional to the applied force that created it, as well as the average speed of the object.

Example #10: A motor driven sled of mass 10.0 kg moves at a constant speed of 15 m/s over a horizontal surface of coefficient of friction $\mu = 0.12$. What power would the motor have to develop to cause this to happen?

(see Work-Energy Ex 10 for answer)

Conservation of Energy Part 1

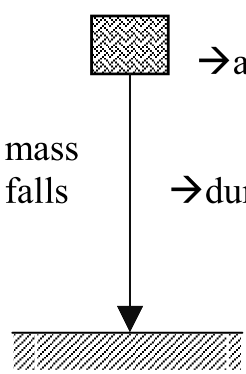
In previous grades, you learned the famous statement "energy is neither created nor destroyed, only transferred from one form to another". This really means that for any given event, the total energy contained in a system is constant, regardless of how different types of energy change during the event. This is similar to the conservation of momentum theorem that was dealt with in the last section, except that since energy is a *scalar* quantity, no vector diagrams are required!

However, unlike momentum (which has only one form and one equation), there are many forms of energy in nature. In Physics 12 however, we are given only two equations for two types of energy: $\mathbf{E_p}$ and $\mathbf{E_k}$. This limits our ability to utilize the conservation of energy theorem.

Essentially, we can only examine systems or events where gravitational and kinetic energies are involved, as well as the heat energy produced when friction occurs. Such systems include: falling objects, roller coasters/ramps, pendulums and slides.

To simplify things, in this section we will only examine conservation of energy problems in situations where friction can be ignored.

Consider an object dropped from a height '**h**' above the ground. When this mass falls from rest and loses vertical height, the loss of gravitational potential energy ($\mathbf{E_p}$) is converted entirely to kinetic energy ($\mathbf{E_k}$).



→at rest, $\mathbf{E_k = 0} \;\therefore\; \mathbf{E_{total} = E_p = mgh}$

→during the fall, $\mathbf{E_p}$ is converted to $\mathbf{E_k}$

→at impact, $\mathbf{E_p = 0} \;\therefore\; \mathbf{E_{total} = E_k = \frac{1}{2} m v_f^2}$

- Since total energy remains the same from start to finish, in this example:

$$\mathbf{mgh = \frac{1}{2} m v_f^2}$$

Another way of looking at conservation of energy is to consider how energy is gained or lost. In the above example:

$$\Delta \mathbf{E_p} \text{ lost} = \Delta \mathbf{E_k} \text{ gained} \quad \rightarrow \quad \mathbf{mg\Delta h = \frac{1}{2} m v_f^2}$$

→Note that there is no initial speed $\mathbf{v_i}$ in this situation.

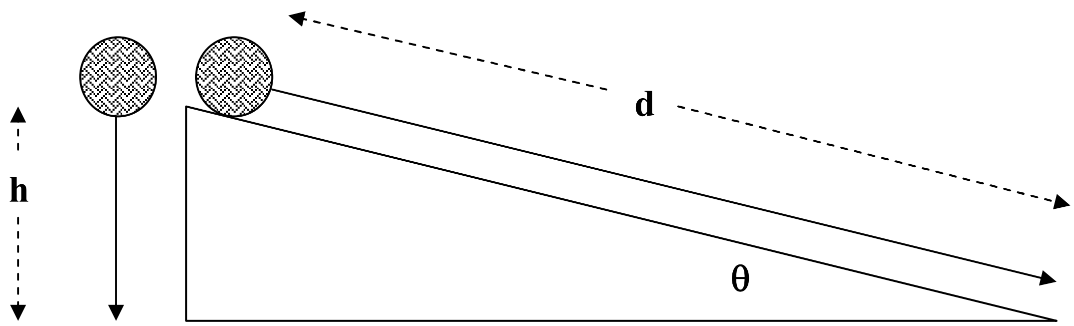
Example #11: A 35 kg mass falls 4.0 m to the ground.

a) How much kinetic energy does it have when it strikes the ground?

b) With what speed does it strike the ground?

(see Work-Energy Ex 11 for answer)

Now consider two identical balls: one dropped from a height ‘**h**’, the other rolling from rest at the same height down a frictionless incline of length ‘**d**’.



It can be proven algebraically that with no friction, the final speed of each ball will be the same!

- Using kinematics, $\mathbf{v_f^2 = v_i^2 + 2ad}$ → where $\mathbf{v_i = 0}$
- Therefore, $\mathbf{v_f = \sqrt{2ad}}$
- For the left ball, $\mathbf{a = g}$ and $\mathbf{h = d\sin\theta}$ so $\mathbf{v_f = \sqrt{2gd\sin\theta}}$
- For the right ball, $\mathbf{a = g\sin\theta}$ so once again, $\mathbf{v_f = \sqrt{2gd\sin\theta}}$

This means that, so long as there is no friction, the speed of an object travelling on any path depends only on its change in height. Conservation of energy can be used to solve for unknown values based on this knowledge.

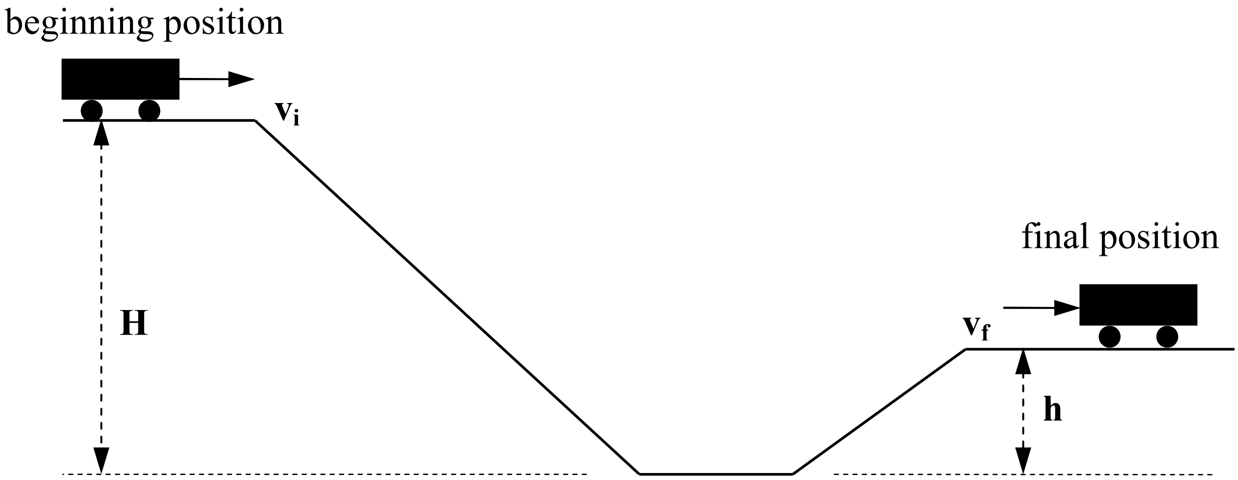
Keep in mind though, these two points:

- We are only referring to speed, and not velocity, which is a vector quantity.
- If there *is* significant friction, this shortcut for finding speed will not work.

Example #12: In the diagram above, if the right ball has a mass of 5.2 kg and an initial speed of 1.4 m/s at the top of the 2.8-m high ramp, what will its speed be at the bottom of the ramp?

(see Work-Energy Ex 12 for answer)

Here's a slightly more complex problem: energy conservation on a roller coaster. Once again, assume friction is negligible, as well as wind resistance, etc.



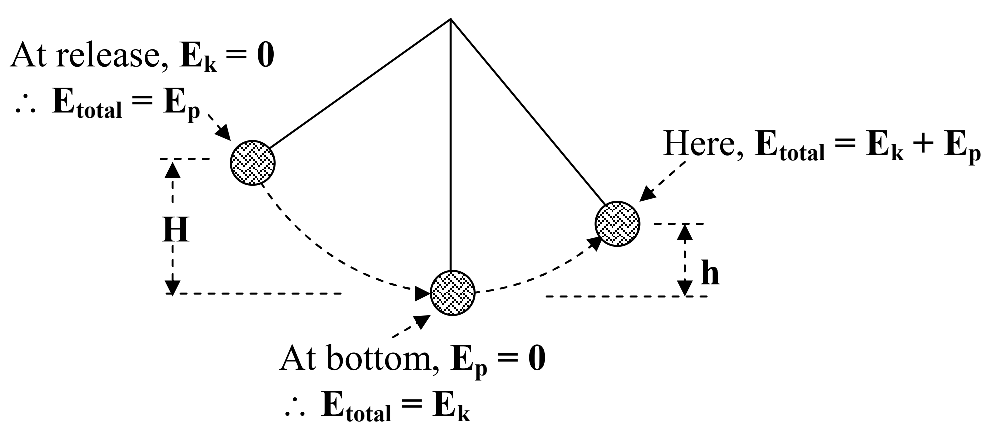
Since the total energy is constant, and since friction is negligible,

the sum of $\mathbf{E_p} + \mathbf{E_k}$ before = sum of $\mathbf{E_p} + \mathbf{E_k}$ after

Example #13: If a cart of mass 10 kg and with an initial speed of 3.5 m/s rolls down a 50 m high frictionless incline and then proceeds to roll up another similar incline to a height of 20 m, what is the speed of the cart at this point?

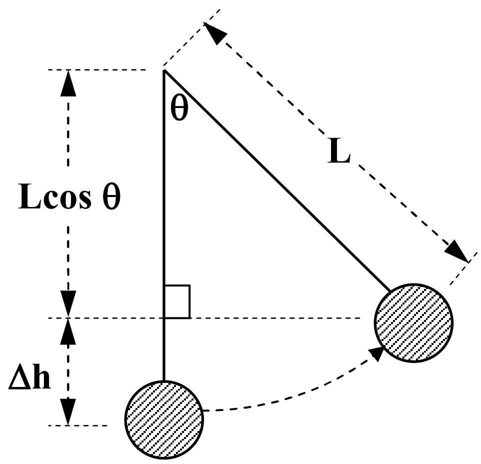
(see Work-Energy Ex 13 for answer)

Finally, we'll look at energy conservation in a frictionless pendulum.



Note the following:

- Height is measured from the bottom of the pendulum's swing.
- At any point, the sum of $\mathbf{E_k + E_p}$ = **total energy**.
- Since **total energy** is constant (cons. of energy):
  $\mathbf{E_p}$ at highest point = $\mathbf{E_k}$ at lowest point
- The change in height Δ**h** of a pendulum can be determined if the pendulum's length '**L**' is known as well as the angle **θ** (from vertical) to which it was raised.



$$\mathbf{\Delta h = L - L\cos\theta}$$

$$\mathbf{\Delta h = L(1 - \cos\theta)}$$

Example #14: A pendulum bob of mass 5.0 kg falls through a height of 25 cm as it swings from maximum height to lowest position.

a) How fast is it going at the bottom?
b) What is the energy of the bob at the bottom of the swing?
c) What is the speed of the bob as it swings up past the bottom of its arc and rises 10 cm from the bottom position?
d) What is the total energy at this position?
e) What is the potential energy at this position?

(see Work-Energy Ex 14 for answer)

Finally, be clear on this: these frictionless systems do not exist, except at the sub-atomic level. If they did, they would be described as *perpetual motion* systems that would continue to move without any additional energy required.

Conservation of Energy Part 2

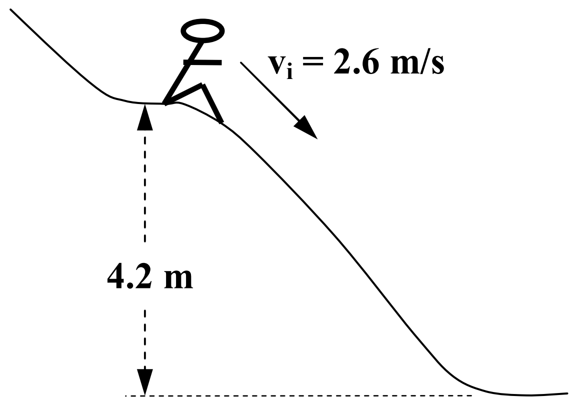
Where friction exists (i.e. everywhere on earth), heat and other forms of energy are produced, and must be considered when utilizing the conservation of energy theorem to solve problems.

→ **total energy before = total energy after**

→ the sum of $\mathbf{E_p} + \mathbf{E_k}$ before = the sum of $\mathbf{E_p} + \mathbf{E_k}$ + **Heat etc.** after

Example #15: Consider the diagram to the right showing a 60 kg student on a large slide.

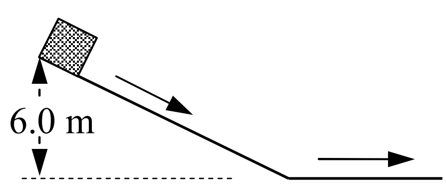
a) In the absence of friction, what would her speed be at the bottom?
b) If her actual speed at the bottom is 6.0 m/s, how much heat was generated on the section shown?



(see Work-Energy Ex 15 for answer)

Example #16: An object of mass 12 kg starts from rest and slides down a ramp that has a vertical drop of 6.0 m. Heat generated as the object moves down the ramp is 310 J.

a) How fast will the object be going at the bottom of the ramp?
b) If the object *then* slides along a horizontal surface of $\mu = 0.25$, how far will it travel before coming to a rest?



(see Work-Energy Ex 16 for answer)

Note that the total energy of the object at the start of the run is **mgh**, equal to **706 J**. At the end of the run, the entire **706 J** of energy has gone up in heat, lost to the atmosphere.

Efficiency

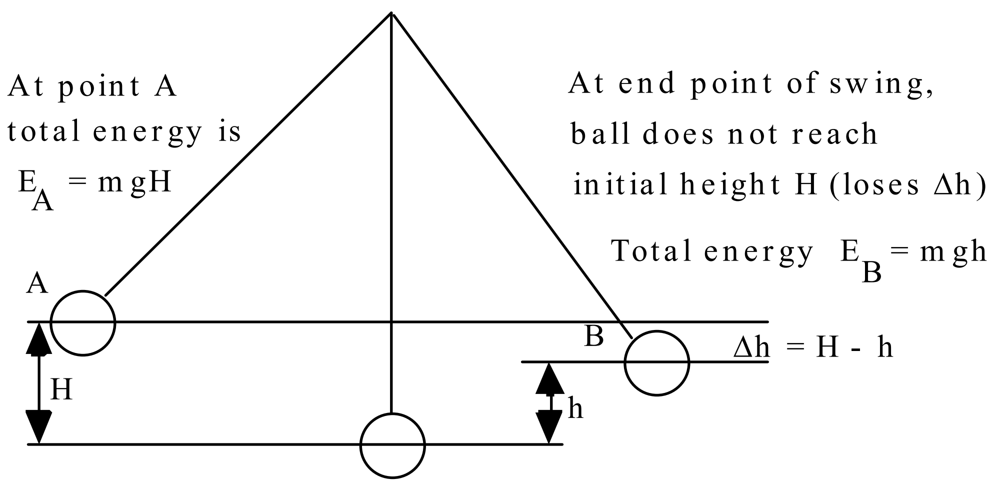
Because of heat generated (and essentially *lost*) due to friction, the energy left over as $\mathbf{E_k}$ and/or $\mathbf{E_p}$ is described as useful energy. *Efficiency* is a way of comparing the total energy a system started with to the useful energy retained or converted after the event is finished:

$$\textbf{Efficiency} = \frac{\textbf{useful energy transferred}}{\textbf{energy put in}} \times 100\%$$

Example #17: Consider the slide from Example #15. Using the information from part (b) only, what is the efficiency of this section of the slide?

(see Work-Energy Ex 17 for answer)

Example #18: In the following system, if H = 25 cm and h = 23 cm, what is the efficiency?



(see Work-Energy Ex 18 for answer)

Example #19: Find the % efficiency of a long hit baseball of mass 200 g; the ball leaves the bat at 18 m/s and is caught in the field (same height as when it was hit) at a speed of 14 m/s.

(see Work-Energy Ex 19 for answer)

Remember that any frictionless system will always have an efficiency of 100%. In essence, it is a perpetual motion machine which would ***never*** require any additional energy to maintain its motion.

Now consider the efficiency of a collision between two masses. In most cases, when two (or more) moving objects collide, some of their kinetic energy is lost to heat, sound, etc. as a result of the impact. Whatever kinetic energy exists after the collision is less than the kinetic energy between the objects before the collision took place.

In other words, kinetic energy is NOT conserved in most collisions. However, there are a few exceptions to this rule – e.g., collisions between molecular and nuclear particles, repelling magnetic objects, as well as very hard materials such as ball bearings. Collisions of this type – where kinetic energy IS conserved – are described as *perfectly elastic*.

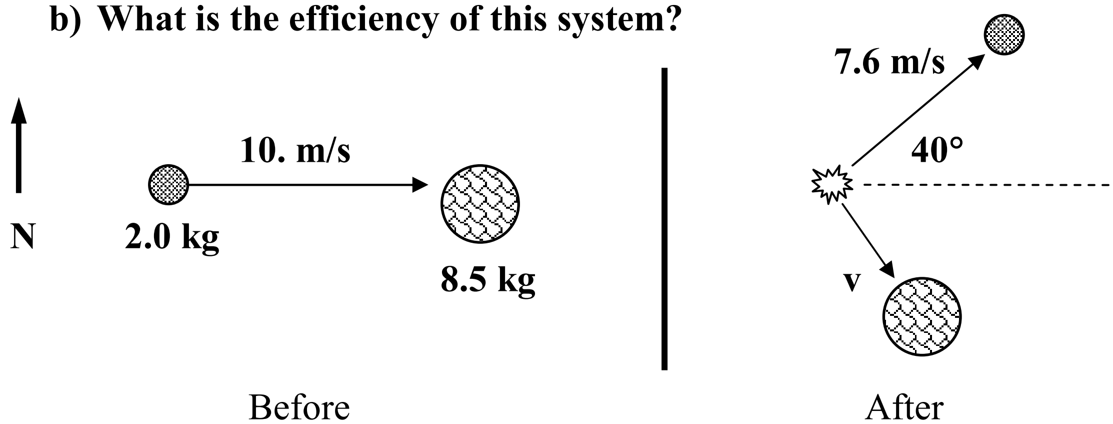
Some points you need to remember from this:

- problems involving perfectly elastic collisions can be analyzed using either conservation of energy or momentum. If the collision is not 100% elastic, ONLY conservation of momentum can be used to solve for unknowns.
- if two equal masses in an oblique collision (as above) show a 90° angle after the collision, that collision is perfectly elastic.

Example #20: A 2.0 kg ball collides at 10. m/s with a much larger stationary 8.5 kg ball as shown to the right. After the collision, the 2.0 kg ball changes its speed to 7.6 m/s @ 40° N of E.

a) At what speed 'v' does the 8.5 kg ball move after the collision?

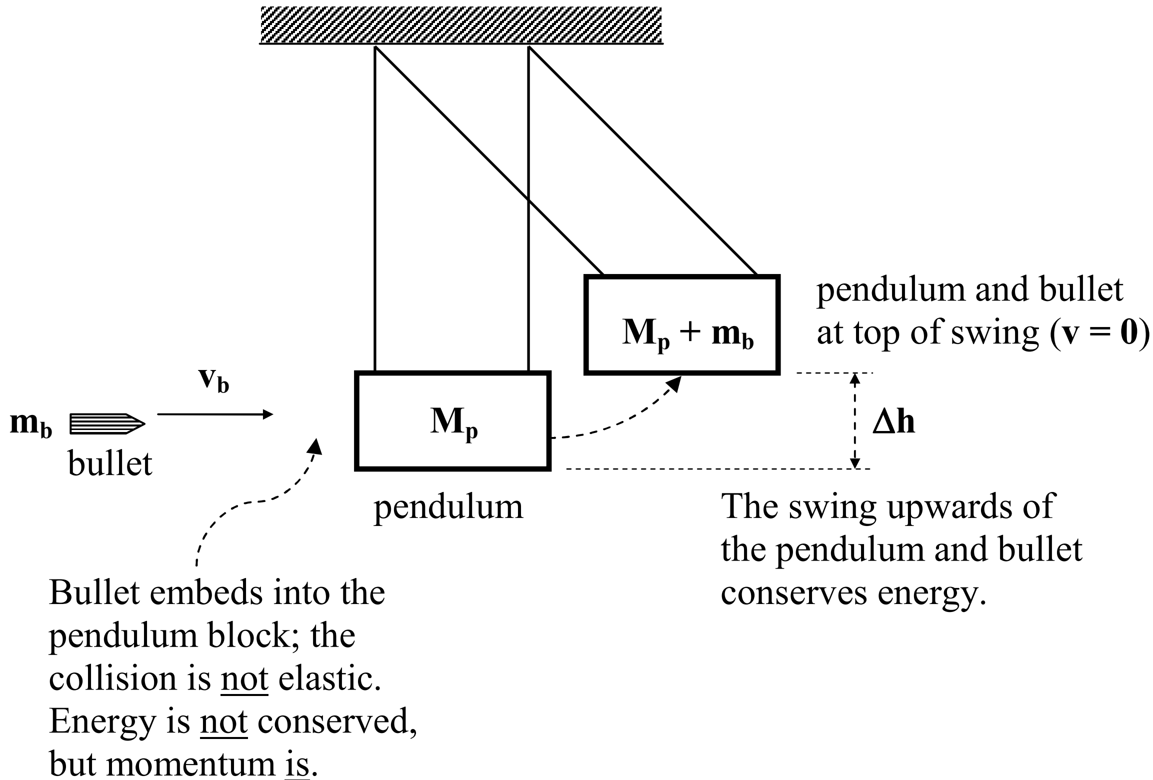
b) What is the efficiency of this system?



(see Work-Energy Ex 20 for answer)

The next and final problem nicely summarizes both aspects of conservation of momentum and conservation of energy. It is described as a *ballistic pendulum* problem.

In a ballistic pendulum problem, a bullet or arrow is shot into a stationary soft pendulum, which then swings upwards. The object is to find the speed of the bullet or the height reached by the pendulum.



Use these steps (not necessarily in this order):

- Use conservation of momentum to deal with the collision between the bullet and the pendulum, where:

  total momentum of bullet before collision
  = total momentum of block & bullet after collision

- Use conservation of energy to deal with the swing of the bullet and pendulum *after* the collision, where:

  total energy at bottom of swing (E_k) = total energy at top of swing (E_p)

Example #21: A 0.015 kg bullet is fired horizontally into a 3.0 kg block of wood suspended by a long cord. The bullet sticks in the block. Compute the original velocity of the bullet if the impact causes the block to swing 10 cm above its initial level.

(see Work-Energy Ex 21 for answer)

Example #1. A 6.0 kg mass is raised from 1.5 m above the ground to 6.5 m high.

a) **What work is done?**

b) **What E_p does the mass now have?**

a) $W = \Delta E_p = mg\Delta h$

$= 6.0\,(9.8)(6.5 - 1.5)$

$$\boxed{W = 2.9 \times 10^2\ J}$$

b) $E_p = mgh = 6.0\,(9.8)(6.5)$

$$\boxed{E_p = 3.8 \times 10^2\ J}$$

Example #2. **A 150 kg object is pulled at constant velocity over a horizontal surface (μ = 0.12) for a distance of 7.0 m. How much heat energy was generated?**

$W = F_F d = \mu F_N d \Rightarrow F_N = F_g$ on horizontal surface

$\therefore \quad W = \mu F_g d$

$= .12(150)(9.8)(7)$

$$\boxed{W = 1.2 \times 10^3 \text{ J}}$$

Example #3: A 60.0 kg lab cart is moving at 5.00 m/s, and is accelerated to 12.0 m/s. How much work was done to cause this?

$$W = \Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}(60)\left[12^2 - 5^2\right]$$

$$\boxed{W = 3.6 \times 10^3 \text{ J}}$$

Example #4: A force of 100 N is applied on a 50 kg cart that is moving with a speed of 6.0 m/s and has a force of friction of 20. N acting on it. At the end of 10. seconds, the cart is going 22 m/s.

a) **How much work was done against inertia?**

b) **How much work was done in total?**

a) $W = \Delta E_k = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$= \frac{1}{2}(50)\left[22^2 - 6^2\right]$

$\boxed{W = 1.1 \times 10^4 \text{ J}}$ (11200 J)

b) → find distance travelled:

$d = v_{av} t = \left[\frac{22+6}{2}\right](10)$

$d = 140 \text{ m}$

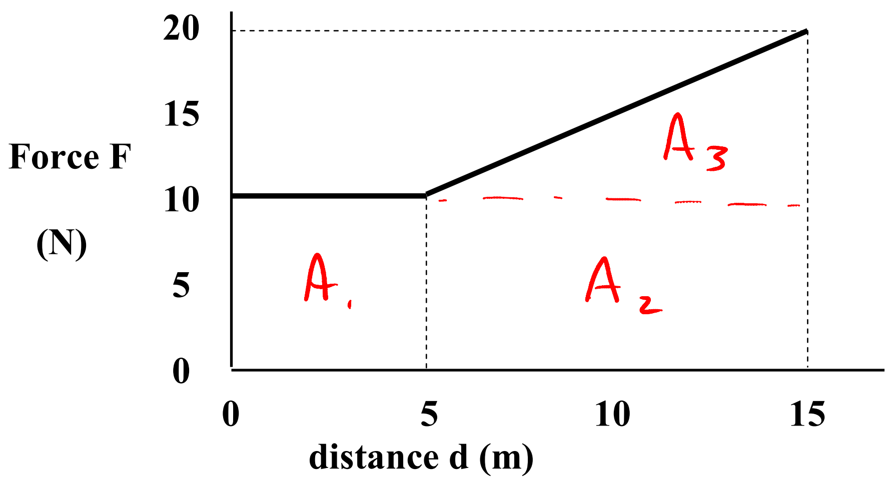
→ $W = F_F d = 20(140) = 2800 \text{ J}$

→ $W_{total} = 11200 + 2800$

$\boxed{W_{total} = 1.4 \times 10^4 \text{ J}}$

Example #5: A 5.0 kg cart is accelerated using a varying force. The force is a constant 10 N for 5 m, then increases at a constant rate up to 20 N for another 10 m.

a) **What is the total work done on the cart?**
b) **If the cart was going 24 m/s when this began, what is its speed now?**



a) $W = F \times d$ = area under the graph ($y \times x$)

$= A_1 + A_2 + A_3$

$= 5(10) + 10(10) + \frac{1}{2}(10)(10)$

$$\boxed{W = 200\ \text{J}}$$

b) $W = \Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$200 = \frac{1}{2}(5)\left[v_f^2 - 24^2\right]$

$$\boxed{v_f = 26\ \text{m/s}}$$

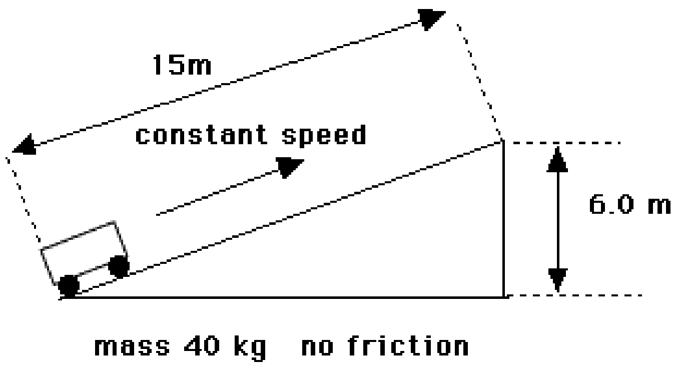
Example #6: A cart accelerates from 0 to 15 m/s in 60 sec. What power is developed if the mass of the car is 20. kg?

$$P = \frac{W}{t} = \frac{\Delta E_k}{t} = \frac{\frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{\,0}}{t}$$

$$= \frac{\frac{1}{2}(20)(15)^2}{60}$$

$$\boxed{P = 38\ W}$$

Example #7: For the diagram below, if the cart goes from the bottom to the top in 16 seconds, how much power was developed?



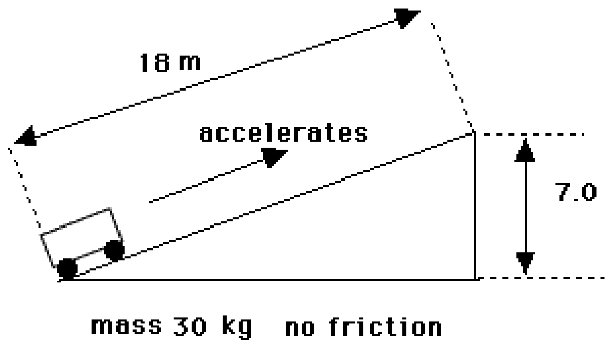
→ work is only done against gravity, so

$$P = \frac{W}{t} = \frac{\Delta E_p}{t} = \frac{mg\Delta h}{t}$$

$$= \frac{40(9.8)(6.0)}{16}$$

$$\boxed{P = 1.5\times10^{2}\ W}$$

Example #8: The cart below has an initial speed of 2.0 m/s and accelerates to 5.0 m/s by the time it is at the top of the ramp. How much power is developed? Hint: to find time, use kinematics.



→ find time: $d = v_{av} t$

$$18 = \left[\frac{5+2}{2}\right] t$$

$$t = 5.14\ s.$$

→ work is done against gravity & inertia:

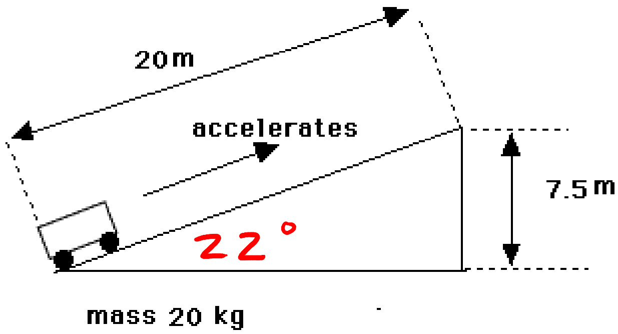
$$P = \frac{W}{t} = \frac{\Delta E_p + \Delta E_k}{t}$$

$$= \frac{mg\Delta h + \frac{1}{2}m\left[v_f^2 - v_i^2\right]}{t}$$

$$= \frac{30(9.8)(7) + \frac{1}{2}(30)\left[5^2 - 2^2\right]}{5.14}$$

$$\boxed{P = 4.6 \times 10^2\ W}$$

Example #9: Here there is a 22° slope. The cart starts from rest at the bottom of the ramp and accelerates to 4.0 m/s by the time it reaches the top of the ramp. With a coefficient of friction μ = 0.21, how much power was developed?



→ find t:

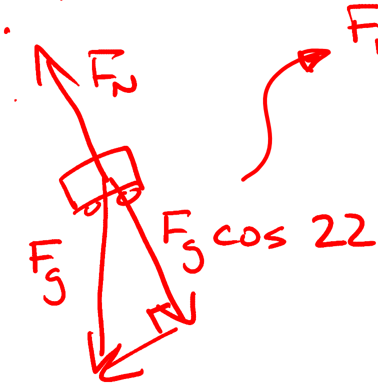
$$d = \left[\frac{v + \cancel{v_0}^{\,0}}{2}\right] t$$

$$20 = \frac{4}{2} t$$

$$t = 10 \text{ s.}$$

Work is done against gravity, inertia, & friction.

→ find friction: $F_F = \mu F_N$



$F_N = F_g \cos 22$, so:

$F_F = .21(20)(9.8)\cos 22$

$F_f = 38$ N

$$\Rightarrow P = \frac{\Delta E_k + \Delta E_p + F_F d}{t}$$

$$= \frac{\frac{1}{2}(20)\left[4^2 - 0\right] + 20(9.8)(7.5) + 38(20)}{10}$$

$$\boxed{P = 2.4 \times 10^2 \text{ W}}$$

Example #10: A motor driven sled of mass 10.0 kg <u>moves at a constant speed</u> of 15 m/s over a horizontal surface of coefficient of friction μ = 0.12. What power would the motor have to develop to cause this to happen?

$$P = \frac{W}{t} = \frac{F_f d}{t} = F_f v_{av}$$

$$\rightarrow F_F = \mu F_N = \mu F_g \quad \text{(horizontal surface)}$$

$$\text{so } P = \mu F_g v_{av} = .12(10)(9.8)(15)$$

$$\boxed{P = 1.8 \times 10^2 \, W}$$

Example #11: A 35 kg mass falls 4.0 m to the ground.

a) **How much kinetic energy does it have when it strikes the ground?**

b) **With what speed does it strike the ground?**

a) At start, $E_{total} = E_p = mgh$

$= 35(9.8)(4)$

$= 1372 \text{ J}$

→ at bottom, $\boxed{E_{Total} = E_k = 1.4 \times 10^3 \text{ J}}$

b) $E_k = \frac{1}{2}mv^2$

$1372 = \frac{1}{2}(35)v^2$

$\boxed{v = 8.9 \text{ m/s}}$

Example #12: In the diagram above, if the right ball has a mass of 5.2 kg and an initial speed of 1.4 m/s at the top of the 2.8-m high ramp, what will its speed be at the bottom of the ramp?

At top, $E_T = E_p + E_k$

$= mgh + \frac{1}{2}mv^2$

$= 5.2(9.8)(2.8) + \frac{1}{2}(5.2)(1.4)^2$

$= 148 \text{ J}$

At bottom, $E_T = E_k = \frac{1}{2}mv^2$

$148 = \frac{1}{2}(5.2)v^2$

$\boxed{v = 7.5 \text{ m/s}}$

Example #13: If a cart of mass 10 kg and with an initial speed of 3.5 m/s rolls down a 50 m high frictionless incline and then proceeds to roll up another similar incline to a height of 20 m, what is the speed of the cart at this point?

At start: $E_T = E_p + E_k$

$= mgh + \frac{1}{2}mv^2$

$= 10(9.8)(50) + \frac{1}{2}(10)(3.5)^2$

$= 4961 \text{ J}$

At end: $E_T = E_p + E_k$

$4961 = 10(9.8)(20) + \frac{1}{2}(10)v^2$

$\boxed{v = 25 \text{ m/s}}$

Example #14: A pendulum bob of mass 5.0 kg falls through a height of 25 cm as it swings from maximum height to lowest position.

a) **How fast is it going at the bottom?**
b) **What is the energy of the bob at the bottom of the swing?**
c) **What is the speed of the bob as it swings up past the bottom of its arc and rises 10 cm from the bottom position?**
d) **What is the total energy at this position?**
e) **What is the potential energy at this position?**

→ first find E_T at start:

$$E_T = E_p = mgh = 5(9.8)(.25)$$

$$E_T = 12.25 \text{ J.}$$

a) at bottom, $E_T = E_K = \frac{1}{2}mv^2$

$$12.25 = \frac{1}{2}(5)v^2$$

$$\boxed{v = 2.2 \text{ m/s}}$$

b) $\boxed{E_K = 12 \text{ J}}$

c) $E_T = E_p + E_K$

$$12.25 = 5(9.8)(.10) + \frac{1}{2}(5)v^2$$

$$\boxed{v = 1.7 \text{ m/s}}$$

d) $\boxed{E_T = 12 \text{ J}}$

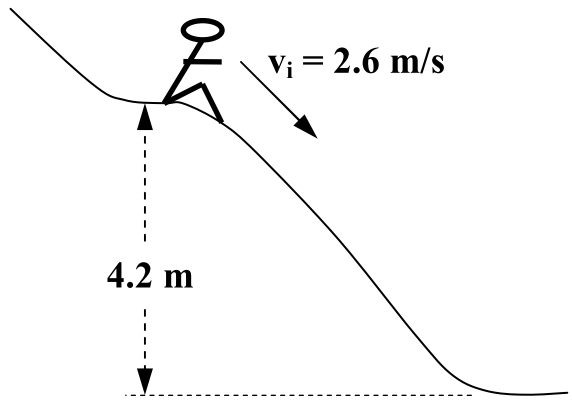
e) $E_p = 5(9.8)(.10)$

$$\boxed{E_p = 4.9 \text{ J}}$$

Example #15: Consider the diagram to the right showing a 60 kg student on a large slide.

a) In the absence of friction, what would her speed be at the bottom?

b) If her actual speed at the bottom is 6.0 m/s, how much heat was generated on the section shown?



a) at start, $E_T = E_p + E_k$

$$= 60(9.8)(4.2) + \frac{1}{2}(60)(2.6)^2$$

$$= 2672.4 \text{ J}$$

at bottom, $E_T = E_k$

$$2672.4 = \frac{1}{2}(60)v^2$$

$$\boxed{v = 9.4 \text{ m/s}}$$

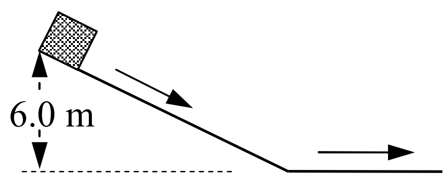
b) at bottom: $E_T = E_k + \text{heat}$

$$2672.4 = \frac{1}{2}(60)(6)^2 + \text{heat}$$

$$\boxed{\text{heat} = 1.6 \times 10^3 \text{ J}}$$

Example #16: An object of mass 12 kg starts from rest and slides down a ramp that has a vertical drop of 6.0 m. Heat generated as the object moves down the ramp is 310 J.

a) **How fast will the object be going at the bottom of the ramp?**
b) **If the object *then* slides along a horizontal surface of $\mu = 0.25$, how far will it travel before coming to a rest?**



a) at top, $E_T = E_p = 12(9.8)(6)$

$= 705.6$ J

at bottom, $E_T = E_k + \text{heat}$

$705.6 = \frac{1}{2}(12)v^2 + 310$

$\boxed{v = 8.1 \text{ m/s}}$

b) $F_f = \mu F_N = \mu F_g = .25(12)(9.8)$

$F_f = 29.4$ J

→ along horizontal surface, remaining $E_k = 705.6 - 310 = 395.6$ J → will go up in heat

Heat generated : $W = F_f d$

$395.6 = 29.4\, d$

$\boxed{d = 13 \text{m}}$

Example #17: Consider the slide from Example #15. Using the information from part (b) only, what is the efficiency of this section of the slide?

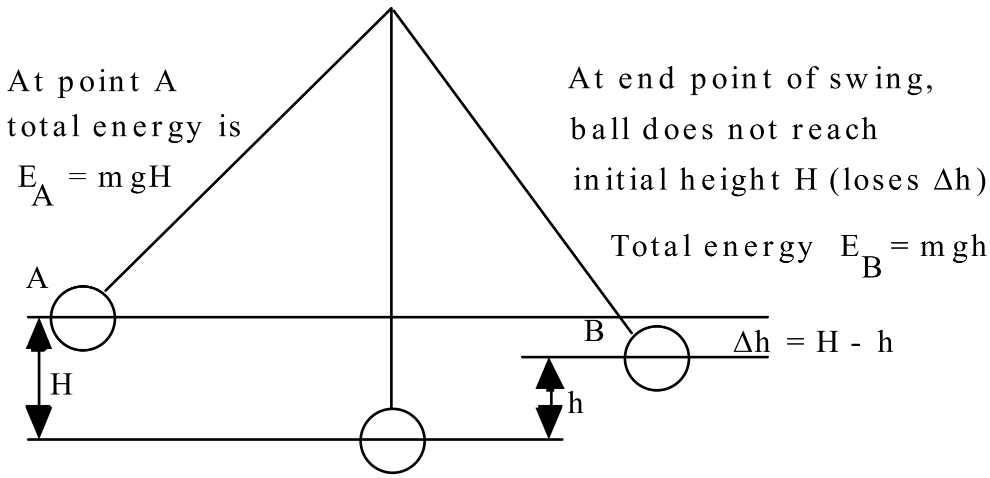
$$\% \text{ eff.} = \frac{\text{useful energy out}}{\text{total energy in}} \times 100$$

$$= \frac{E_k \text{ at bottom}}{E_T \text{ at start}} \times 100$$

$$= \frac{1080}{2672.4} \times 100$$

$$\boxed{\text{Eff.} = 40\%}$$

Example #18: In the following system, if H = 25 cm and h = 23 cm, what is the efficiency?



At start "A": $E_T = E_p = m(9.8)(.25)$

$= 2.45m$

At "B": $E_p = m(9.8)(.23)$

$= 2.254\ m$

$$\text{Eff.} = \frac{E_p}{E_T} \times 100 = \frac{2.254\cancel{m}}{2.45\cancel{m}} \times 100$$

$$\boxed{\text{Eff} = 92\%}$$

Example #19: Find the % efficiency of a long hit baseball of mass 200 g; the ball leaves the bat at 18 m/s and is caught in the field (same height as when it was hit) at a speed of 14 m/s.

at start: $E_T = E_k = \frac{1}{2}(.2)(18)^2$

$= 32.4\ J$

when caught: $E_k = \frac{1}{2}(.2)(14)^2$

$= 19.6\ J$

$$\%\ \text{efficiency} = \frac{E_k\ \text{out}}{E_T\ \text{in}} \times 100$$

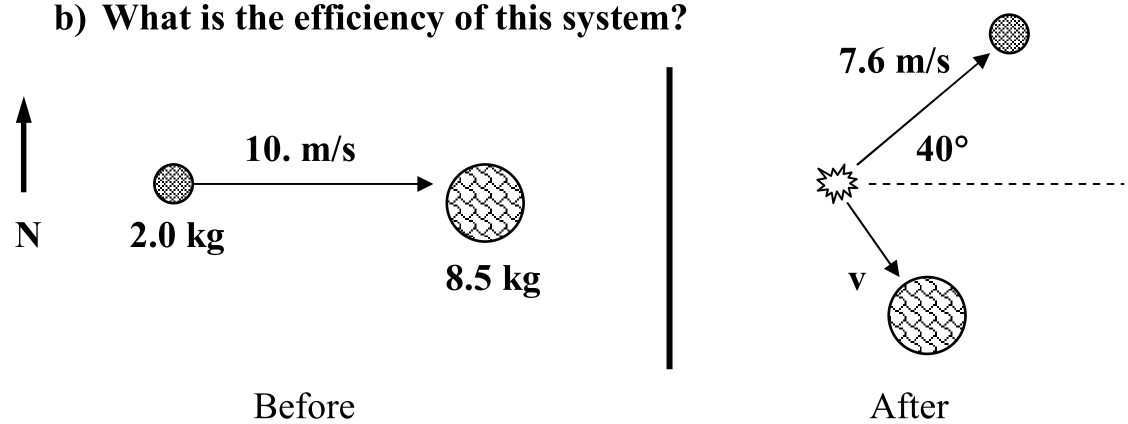
$$= \frac{19.6}{32.4} \times 100$$

$$\boxed{\text{Eff.} = 60\ \%}$$

Example #20: A 2.0 kg ball collides at 10. m/s with a much larger stationary 8.5 kg ball as shown to the right. After the collision, the 2.0 kg ball changes its speed to 7.6 m/s @ 40° N of E.

a) At what speed 'v' does the 8.5 kg ball move after the collision?

b) What is the efficiency of this system?

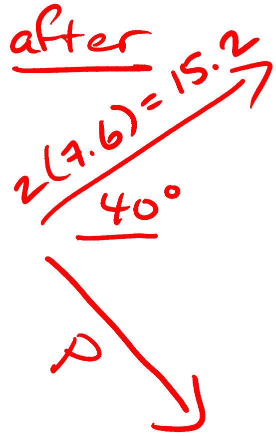


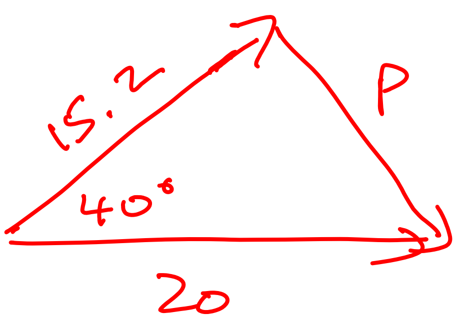
a) a collision: use cons. of momentum

before

$P_T = 10(2) = 20$

after





$p^2 = 15.2^2 + 20^2 - 2(15.2)(20)\cos 40$

$p = 12.9 \frac{kg \cdot m}{s}$

$p = mv$

$12.9 = 8.5v$

$\boxed{v = 1.5\ m/s}$

b) before: $E_T = E_k$ of 2 kg ball

$$= \frac{1}{2}(2)(10)^2 = 100 \text{ J}$$

after: $E_{out} = E_k$ for each ball after collision

$$= \frac{1}{2}(2)(7.6)^2 + \frac{1}{2}(8.5)(1.5)^2$$

$$= 67 \text{ J}$$

$$Eff = \frac{E_{out}}{E_T} = \frac{67}{100} \times 100$$

$$\boxed{Eff = 67\%}$$

Example #21: A 0.015 kg bullet is fired horizontally into a 3.0 kg block of wood suspended by a long cord. The bullet sticks in the block. Compute the original velocity of the bullet if the impact causes the block to swing 10 cm above its initial level.

→ assume negligible energy lost in swing, so

$$E_T = E_k \text{(bottom)} = E_p \text{(top)}$$

block and bullet

$$\frac{1}{2}(3+.015)v^2 = (3+.015)(9.8)(.10)$$

$v = 1.4$ m/s at bottom of swing, after bullet collides with block.

→ use cons. of momentum to find bullet velocity before the collision

before

$$p_T = .015\,v$$

after

$$p_T = (3+.015)(1.4)$$

$$= 4.221$$

$$.015v = 4.221$$

$$\boxed{v = 2.8 \times 10^2 \text{ m/s}}$$