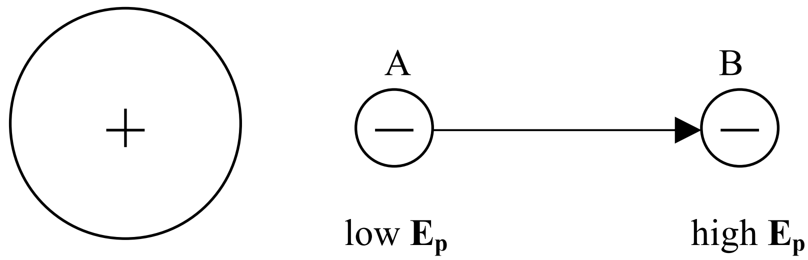
A Review of Circuitry

There is an attractive force between a positive and a negative charge. In order to separate these charges, a force *at least* equal to the attractive force must be applied to one of them:



Work = Fd **W = $\Delta \mathbf{E_p}$**

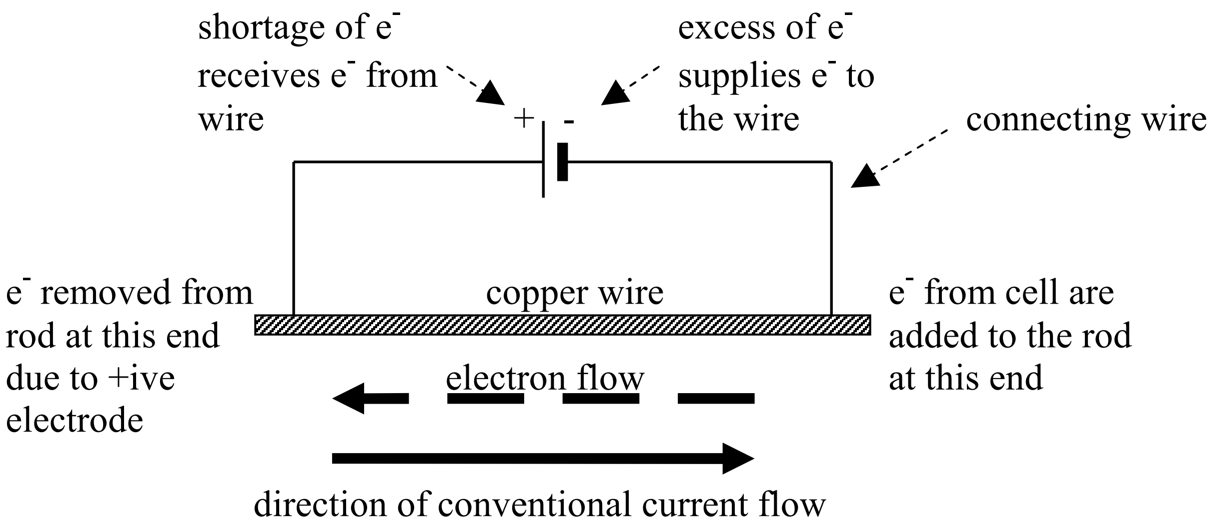
This force, acting over a distance, does work *on* the charge that is moved, so the charge receives potential energy. Such work can be done by chemical reactions taking place in a battery or dry cell, or by mechanical forces in a generator.

If the negative charge is subsequently released at **B**, it will move back to position **A**, *releasing* energy which is equal to the original work done to move the charge to **B**.

Now consider the inside of a dry cell:

- When charge is separated between the two electrodes through chemical reactions, an <u>electric field</u> is created between the different charges and the differently charged electrodes. In particular, the electrons that have accumulated at the negative electrode have gained potential energy.
- If a conducting wire then connects these two electrodes, another electric field is set up in the wire, causing a force to be exerted on every movable charge in the wire, creating electron flow. Electrons now move back through the wire from the negative electrode, or *terminal*, of the battery to its positive terminal, losing potential energy along the way.

A summary of this process is shown in the following diagram.



Note that as the charge moves back through a wire, energy is given off to heaters, light bulbs, motors, buzzers or any other circuit elements that can absorb energy.

Pay close attention to the direction arrows:

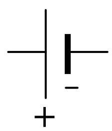
- *Electron flow* refers to the flow of negative charges, repelled from the negative terminal of a chemical cell and attracted to the positive terminal.
- *Conventional current* refers to the direction that an imaginary positive test charge would take if placed into the conductor (the same direction as the field **E** set up in the wire by the potential difference). This convention was first adopted by scientists in the 1800's, before there was a clear understanding of the sub-atomic structure of the atom.

For current flow to occur, an electric *circuit* must be set up. All circuits require the following:

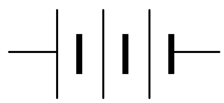
- a *power supply* to give the electrons potential energy.
- a *conductor*, which is a material that allows a free-flow of electrons with little resistance). The most common conductors include:
  - metal conductors (e.g., copper, gold, aluminum).
  - ionized gases or liquids (e.g., electrolytic solutions like salt water).
  - A gas discharge tube (such as a CRT that produces an electron beam).
- a *resistor* to use the energy. A resistor is a material that resists electron flow and converts electrical energy to some other form of energy. Examples of resistors include motors, heaters, light bulbs, etc.

The various ways in which electricity can travel from a source throughout a circuit can be represented by a schematic diagram. Common circuit symbols include the following:

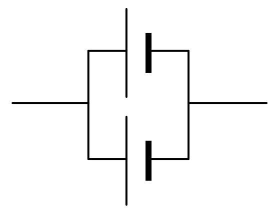
cell



cells in series

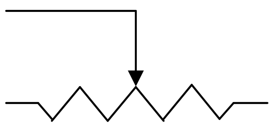


cells in parallel

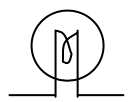


resistor

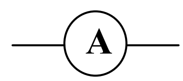
variable resistor



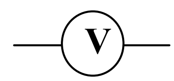
lamp



ammeter

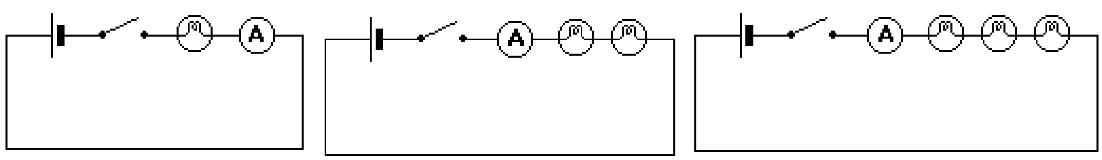


voltmeter

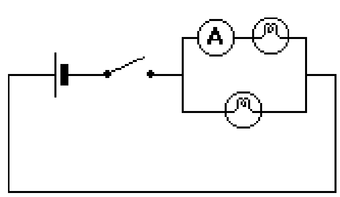


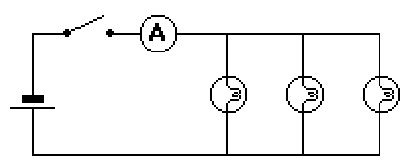
Two types of circuits you should be familiar with are as follows:

- *Series Circuit* - A single current path for electron flow; e.g.:



- *Parallel Circuit* - more than one current path for electron flow to occur; e.g.:





Electric Current

Electric current is the *rate* of movement of electric charge. As an equation,

$$\mathbf{I} = \frac{\mathbf{q}}{\mathbf{t}}$$

→ **q** is the charge, measured in coulombs (**C**)

→ **I** is the current measured in amperes, or amps (**A**), where one ampere = one coulomb/sec

Example #1: A 10 ampere current flows through a wire in 60 seconds. Determine:

a) The amount of charge that moves in 60 seconds.
b) The number of electrons that pass in 60 seconds.

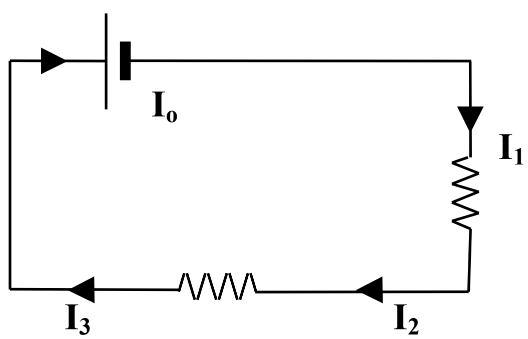
(see Circuitry Ex 1 for answer)

Example #2: If a current of 5.0 A flows for 20 minutes, what charge was transferred?

(see Circuitry Ex 2 for answer)

Kirchhoff's Current Law

It was German physicist Gustav Robert Kirchhoff who first determined that electrons are conserved. Since electrons have only one path to travel in a series circuit, electric current at all points remains the same.

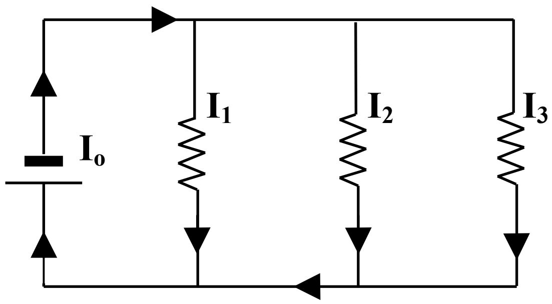


$\mathbf{I_o = I_1 = I_2 = I_3}$

where $\mathbf{I_o}$ = the total current in the circuit, as produced by the battery

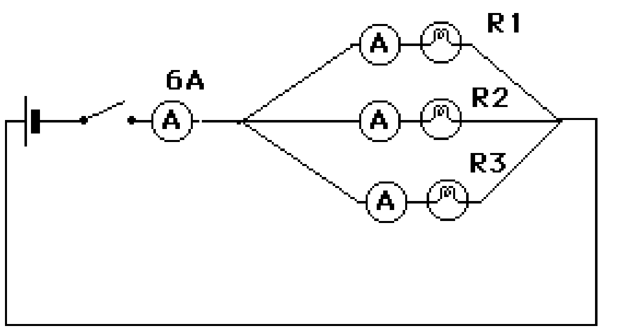
To measure current properly, an *ammeter* must be placed in series into the circuit. For the above circuit, the device can be placed anywhere along the path, and will display the same current reading, regardless of its location.

On the other hand, at a parallel connection, the total current flowing into the connection must equal the sum of the currents flowing out of the connection.



$$I_o = I_1 + I_2 + I_3$$

Note that when all resistors are equal, current is divided equally through the different paths. Examine the diagram below.



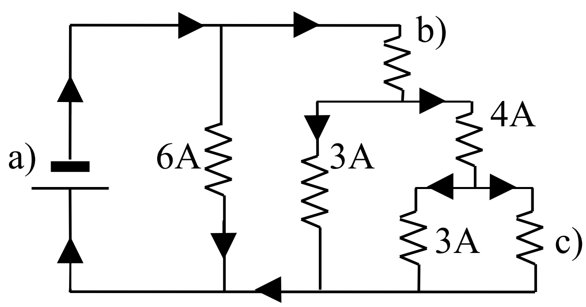
An ammeter placed by the battery will display the total current, which in this case is 6 A. An ammeter placed along any parallel path (in series with the device in that path) will only display the current through that path.

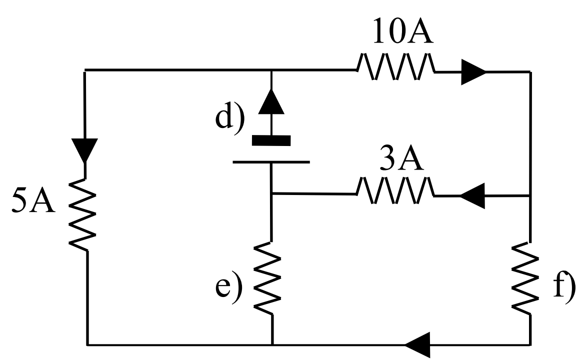
If all the lamps have the same resistance in the above circuit, then each of the three ammeters placed in the parallel branches will display a current of 2 A (2 + 2 + 2 = 6 A).

If the resistors in the diagram are <u>not</u> equal, then other methods can be used (later). Always keep in mind, though that *less* resistance means a *greater* flow of electrons.

From this, *Kirchhoff's Current Law* was developed: At any connection in an electric circuit, the total current into a connection must equal the total current out of that connection.

Example #3: Determine the unknown currents for each of the following circuits.





(see Circuitry Ex 3 for answer)

Potential Difference, or Voltage

Because electrons are very small and carry very little energy, it is more useful to describe the work done on a *coulomb* of charge (recall from electrostatics that one coulomb (C) = 6.24×10^{18} electrons). This is the definition of potential difference, or voltage **ΔV**:

$$\Delta V = \frac{\Delta E_p}{q} \qquad \text{or} \qquad \Delta E_p = q\Delta V$$

→ where **ΔV** is measured in volts (**V**) or Joules/coulomb

Since the work done to separate charges is positive, the potential difference between the terminals in a cell is also positive. On the other hand, as charges flow through devices contained within a circuit, work is done *by* the system and the potential difference across each device is negative.

Example #4: If a chemical cell gives 600 J of energy to a charge of 50 C, what is the potential difference of this cell?

(see Circuitry Ex 4 for answer)

Note that in circuitry, the symbol for potential difference (also called *voltage*) is simply "**V**"; the delta sign **Δ** is dropped for simplicity.

Kirchhoff's Voltage Law

Kirchhoff also examined the potential difference in circuits. He found that, from the law of conservation of energy: the sum of the gains and drops in potential energy around any closed circuit path must equal zero, **or** in any complete circuit loop, the sum of the gains in potential difference (batteries) is equal to the sum of the potential drops (resistors). We can use this general principle to describe voltage rules for both series and parallel circuits:

- In a simple, one loop series circuit, the voltage drops across the various resistors = the potential difference of the voltage supply. Put another way, the potential difference of the voltage supply V_o is equal to the sum of the potential "drops" across each of the components V_1, V_2, V_3...etc. That is,

$$V_o = V_1 + V_2 + V_3 + \ldots.$$

➢ In a circuit with parallel circuit paths, the total potential difference across each parallel branch must be equal; that is,

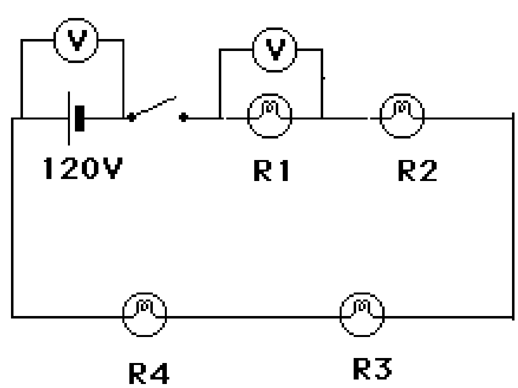
$$\mathbf{V_o = V_1 = V_2 = V_3 = ...}$$

There is a simple skiing analogy that can be used to determine voltage drops at various locations in a circuit:

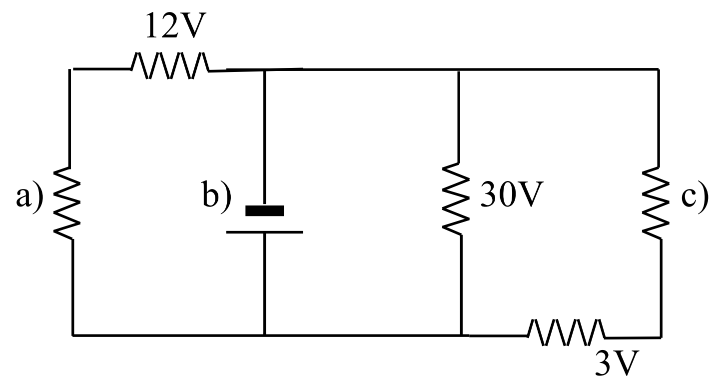
- A chairlift (*battery*) gives skiers (*electrons*) the gravitational potential to "drop" down a ski-slope once they have reached the top of the chair.
- How those skiers get to the bottom depends on the number of runs (*circuit paths*) available to them and which hills (*resistors*, *light bulbs*, etc) they decide to "drop" down, losing potential along the way.
- Connecting wires are analogous to horizontal trails that lead skiers to their runs, since electrons lose almost no potential here. Note that all skiers will have "dropped" their potential once they have reached the bottom of the chair again.

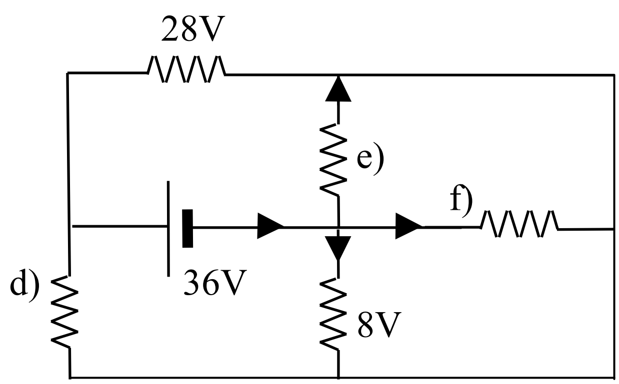
From all this, *Kirchhoff's Voltage Law* was developed: At any connection in an electric circuit, the sum of the potential difference around any closed pathway or loop = 0.

To measure the voltage drop or gain across any device in a circuit properly, a *voltmeter* must be placed in parallel with the device that is being analyzed. The diagram below shows how a voltmeter is attached to correctly measure the potential difference across a 120 V power supply as well as resistor $\mathbf{R_1}$.



Example #5: Determine the unknown voltages for each of the following circuits.





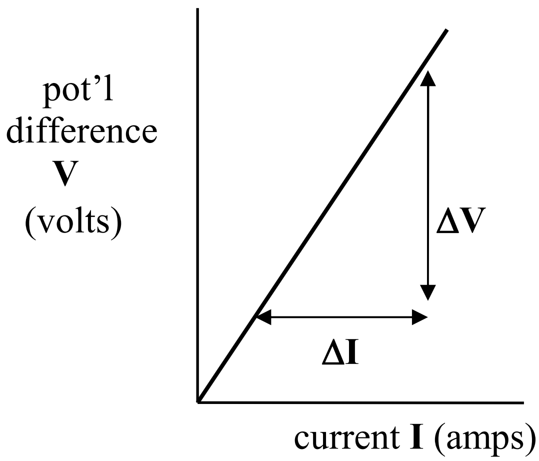
(see Circuitry Ex 5 for answer)

Ohm’s Law

This famous formula results from attempts to find a math relationship relating potential difference to current flow.

Recall the Ohm’s Law lab from physics 11. In it, you are to apply different voltages across a given resistor and measure the changing current flow.

Graphing **V vs. I** we get the graph that follows:



The straight line through the origin expresses a direct relationship, so long as the resistor does not significantly overheat. From $\mathbf{y = kx + b}$ we get

$$\mathbf{V = kI}$$

By definition, this constant slope represents the resistance **R** of the circuit element and is measured in **ohms** (**Ω**), so that

$$\mathbf{V = IR}$$

Note that resistance stays constant only for one temperature; changing temperature changes resistance.

Example #6: A small light bulb is connected to 3.0 V and will draw 150 mA.

(a) What is the net resistance of the bulb?

(b) If the voltage dropped to 2.0 V, how would the current change?

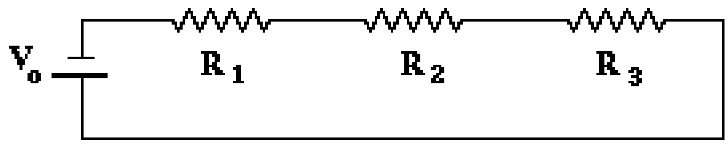
(see Circuitry Ex 6 for answer)

Calculating Resistance in a Circuit

We can use Kirchhoff's Laws in combination with Ohm's Law to find the total resistance in circuits.

Resistors in Series

Consider the diagram below.



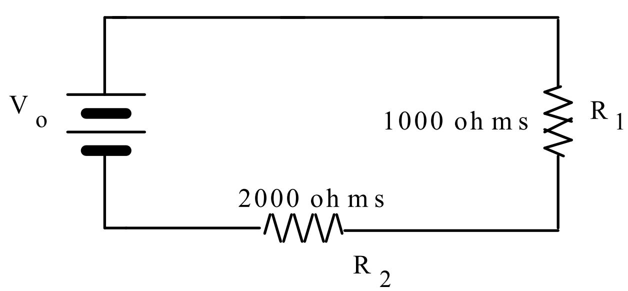
The voltage law states that $\mathbf{V_o = V_1 + V_2 + V_3}$, and the current law states that the current is the same through each resistor. Substituting Ohm's Law into this equation gives:

$$\mathbf{IR_o = IR_1 + IR_2 + IR_3}$$

Since the current cancels out on both sides we have the equation for finding the total resistance in series:

$$\mathbf{R_o = R_1 + R_2 + R_3 + \ldots etc.}$$

Example #7: Consider the following circuit diagram showing two resistors attached in series to a battery of two 1.5 V cells. Determine all unknown voltages, currents and resistances for each apparatus in the circuit.



V_o = p.d. of 2-1.5 cells in series

Since cells connected in series have their voltages added together, the total voltage,

$$V_o = 2 \times 1.5 \text{ V}$$
$$= 3.0 \text{ V}$$

Some helpful hints:

- First find the total effect of resistance, $\mathbf{R_o}$. This should be listed with the power supply.
- Second, use Ohm's Law to determine the current $\mathbf{I_o}$ supplied by the battery.
- Next, use Kirchhoff's Law to list the current through each resistor.
- Finally, use Ohm's Law again to find the voltage drop across each resistor; note that the sum of these voltages = $\mathbf{V_o}$.

(see Circuitry Ex 7 for answer)

In summary, for a *series* circuit,

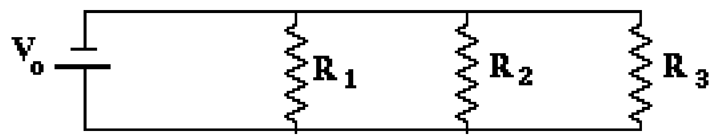
$\mathbf{V_o = V_1 + V_2 + ...etc.}$ (voltage is added)

$\mathbf{I_o = I_1 = I_2 = ...etc.}$ (current is the same throughout the circuit)

$\mathbf{R_o = R_1 + R_2 + ...etc.}$ (resistor values are added)

Resistors in Parallel

Consider this new circuit diagram, showing three resistors in parallel with a power supply of voltage $\mathbf{V_o}$.



The current law states that current generated at $\mathbf{V_o}$ will split so that some of the total current $\mathbf{I_o}$ goes through each resistor. In effect,

$$\mathbf{I_o = I_1 + I_2 + I_3}.$$

The voltage law states that the voltage drop is the same through each resistor. Substituting Ohm's Law into this equation gives:

$$\frac{\mathbf{V}}{\mathbf{R_o}} = \frac{\mathbf{V}}{\mathbf{R_1}} + \frac{\mathbf{V}}{\mathbf{R_2}} + \frac{\mathbf{V}}{\mathbf{R_3}}$$

Since the voltage cancels out on both sides we have the equation for finding the total resistance in series:

$$\frac{\mathbf{1}}{\mathbf{R_o}} = \frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}} + \frac{\mathbf{1}}{\mathbf{R_3}}$$

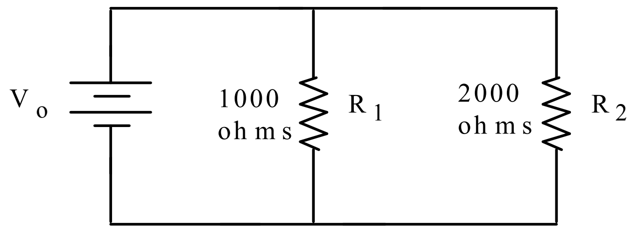
Example #8: Use your calculator to add these resistors in parallel:

(a) 25 Ω, 30 Ω, 50 Ω

(b) 50 Ω, 68 Ω, 270 Ω, 569 Ω

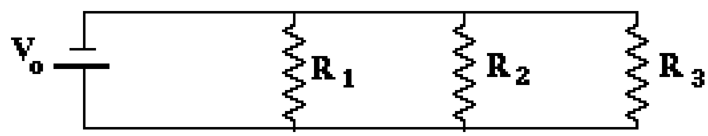
(see Circuitry Ex 8 for answer)

Example #9: Consider the following circuit diagram showing two resistors attached in parallel to a battery of two 1.5 V cells. Determine all unknown currents, voltages and resistances.



(see Circuitry Ex 9 for answer)

Example #10: In this example R_1= 5.0 Ω, R_2 =10 Ω and R_3 = 15 Ω and the total current is 10 A. Find the current in each branch.



(see Circuitry Ex 10 for answer)

In summary, for a *parallel* circuit,

$$\mathbf{V_o = V_1 = V_2 = ...etc.}$$

$$\mathbf{I_o = I_1 + I_2 + ...etc.}$$

$$\mathbf{\frac{1}{R_o} = \frac{1}{R_1} + \frac{1}{R_2} + ...etc.}$$

Also note the following:

- When resistors are added in series the total resistance is always *larger* than the largest resistor.
- When resistors are added in parallel the total resistance is always *smaller* than the smallest resistor.

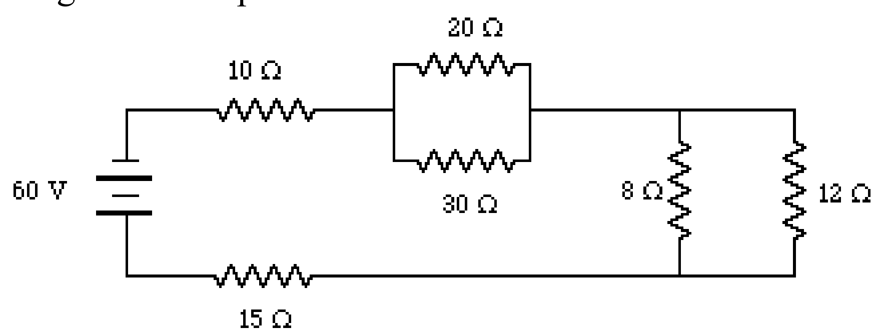
Finally, you have probably noticed that most circuits contain a mixture of resistors in series and in parallel. Such arrangements are called *combination* circuits.

In combination circuits, we want to find:

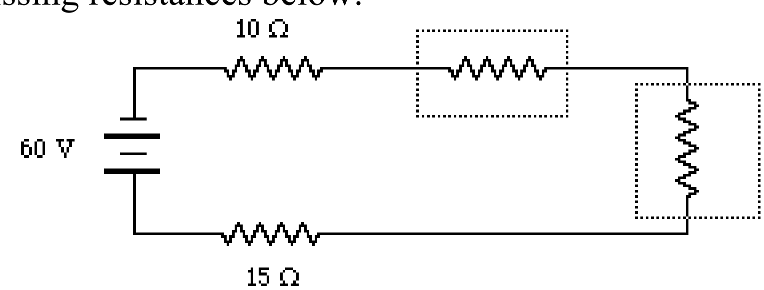
- the equivalent resistance for various parts of the circuit;
- the total resistance **Ro** in the circuit;
- the current flow through each device in the circuit;
- the voltage gain or drop through each device in the circuit.

In performing these tasks, you will use a combination of Ohm's Law and Kirchhoff's Laws. Ohm's Law will apply for an entire circuit if the total values are used, or apply to an individual resistor if the individual values are used.

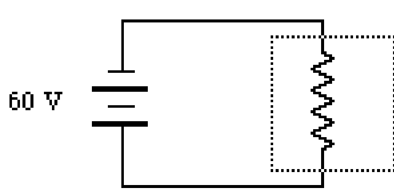
The following are the steps to use to solve a combination circuit:



- ➢ Reduce the individual resistor networks to the equivalent of one resistor by applying the appropriate equations. You should now have only a group of series resistors. Redraw the circuit in this simplified way. Determine the missing resistances below:



- ➢ Add these series resistors to get the equivalent of one resistor. Determine this value:



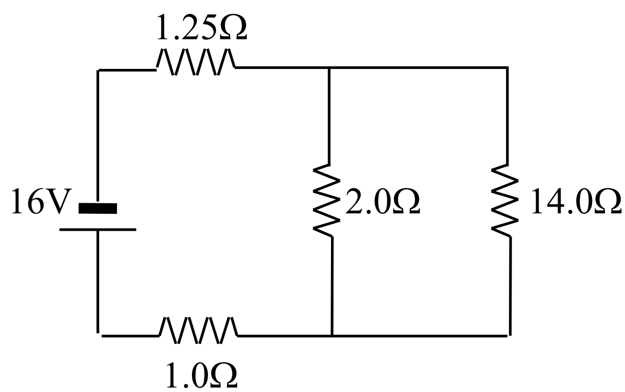
- Find the current in the circuit using Ohm's Law.
- Finally, work backwards through the circuit using Ohm's Law and/or Kirchhoff's Laws to find the current and voltage across each resistor.

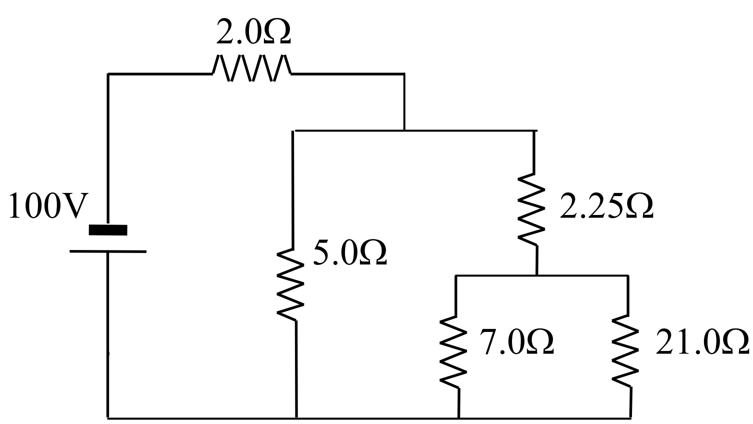
Example #11: Complete the table for the circuit on the previous page:

R (Ω)	V (V)	I (A)
10	____	____
20	____	____
30	____	____
8	____	____
12	____	____
15	____	____

(see Circuitry Ex 11 for answer)

Example #12: Calculate all unknown voltages, currents and resistances for the following circuits:





(see Circuitry Ex 12 for answer)

Calculating Power

When electric charges move through a circuit, potential energy is converted to other forms of energy. The *rate* at which this energy is converted is defined as the **power** used. As an equation,

$$\mathbf{P = \frac{\Delta E_p}{t}} \qquad \rightarrow \text{recall from energy unit}$$

→ but also recall that $\mathbf{\Delta E_p = qV}$ so $\mathbf{P = \frac{qV}{t}}$

→ and, since $\mathbf{\frac{q}{t} = I}$, by substitution we get $\boxed{\mathbf{P = IV}}$

This is the general formula for power transformed by any electrical device, measured in *watts* (W).

The rate of energy transformation in a resistance **R** can also be written in two other ways. By substituting Ohm's Law,

$$\mathbf{P = I^2R} \qquad \text{and} \qquad \mathbf{P = \frac{V^2}{R}}$$

Example #13: From the circuit diagram for example 11, calculate the power used by each resistor connected in the circuit.

(see Circuitry Ex 13 for answer)

Since electrical devices use electrical energy, we can find the cost of using electrical devices. However, the basic unit of energy (the joule) is very small for this purpose, so we use a larger unit called the *kilowatt hour* **(kWh)**. The cost of electricity then is found by:

$$\begin{aligned}\mathbf{Cost} &= \mathbf{Energy \cdot rate}\\ &= \mathbf{Pt \cdot rate}\\ &= \mathbf{IVt \cdot rate}\end{aligned}$$

Example #14: Find the cost of operating a kettle for 15 min if it draws 10 A from a standard 120 V outlet, and the cost is 5.5¢/kWh.

(see Circuitry Ex 14 for answer)

Example #15: An electric fan draws 2.0 A of current from a 120 V source. Determine the following:

(a) the power rating of the fan.
(b) its electrical resistance.
(c) the cost of operation of the fan during the month of August, assuming it is run continuously and electric energy costs 10 cents per kilowatt hour.

(see Circuitry Ex 15 for answer)

Finally, note that electrical energy and power can be converted to other forms of energy and power. Electrical energy, which is given by $\mathbf{\Delta E = Pt}$ (where $\mathbf{P = IV}$), can be equated with:

$$\Delta E = Fd \qquad \Delta E = mg\Delta h \qquad \Delta E = \frac{1}{2}m(v_f^2 - v_i^2)$$

Example #16: A 6.0 volt motor is used to winch a 0.056 kg mass a vertical distance of 0.65 m in 5.62 sec. What current will the motor draw?

(see Circuitry Ex 16 for answer)

Terminal Voltage & Internal Resistance

In a cell, chemical reactions cause a charge separation to occur between the two terminals. This potential difference is called the *Electromotive Force* (the term is misleading; it is not a force), often referred to as *EMF*, with symbol '**ε**' and measured in Volts.

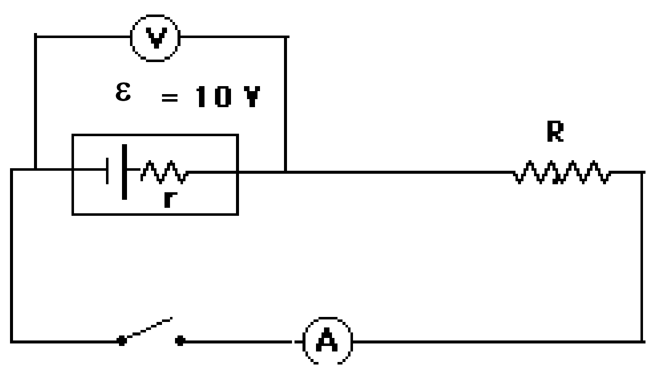
EMF = the voltage across a supply when there is no current flowing.

$\mathbf{V_T}$ (terminal voltage) = the *actual* potential difference across the terminals of the supply when a current is being supplied.

When a closed circuit is set up so that electrons flow from negative to positive terminals, the terminal voltage drops below **EMF** value. Here's why:
- Chemical reactions within the cell cannot separate charges fast enough to keep maximum charge separation.
- The charges must flow between electrolyte and terminals, and there is always some resistance to this, called *internal resistance* (**r**).
- As a result, when current flows, there is an internal voltage drop equal to **ir**, and from this,

$$V_T = \varepsilon - Ir \qquad \rightarrow \text{note that when } I = 0,\ V_T = \varepsilon$$

Examine the following schematic. Note that the small box represents the battery. The terminal voltage $\mathbf{V_T}$ is measured from one end of the box to the other, and is read by the voltmeter. To find the internal resistance **r** of the power supply, perform the following steps:



- find the potential difference with the switch open; this is the **EMF**. Note that the voltmeter can't do this (it draws some current), so other techniques must be used.
- close switch and measure the potential difference with current flowing **(V_T)** and measure current with the ammeter.

EMF - V_T = lost voltage (Ir) → **$\varepsilon - V_T = Ir$**

→ so **$r = (\varepsilon - V_T)/I$**

We can also use use internal and external resistance to find the current supplied by a power source. Start with

$TV = \varepsilon - Ir$ → **$\varepsilon = V_T + Ir = IR_o + Ir$**

→ which becomes **$I = \varepsilon/(R_o + r)$** where **$R_o$** is the total external resistance of the circuit.

Example #17: When a 6.0 V EMF battery was connected to a 15 Ω resistance, a current of 375 mA occurred and the voltmeter reading was 5.625 V.

(a) Find the internal resistance r of this supply.

(b) If this battery is now connected to a 5.0 Ω resistor, what current will flow?

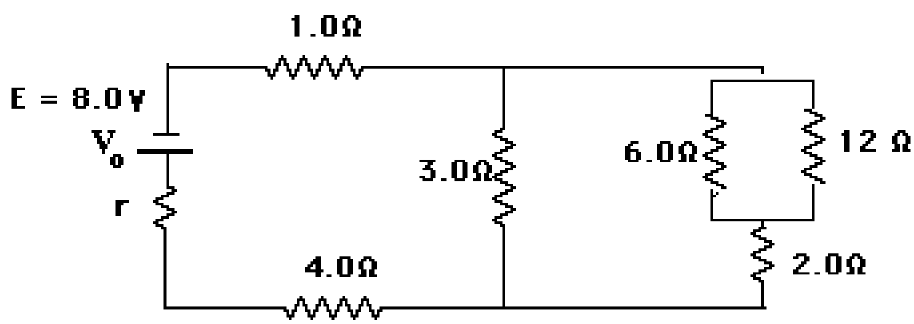
(see Circuitry Ex 17 for answer)

Example #18: A battery of EMF 8.0 V and internal resistance r = 1.0 Ω is connected to an external circuit as shown. Find:

(a) the equivalent resistance of the circuit.

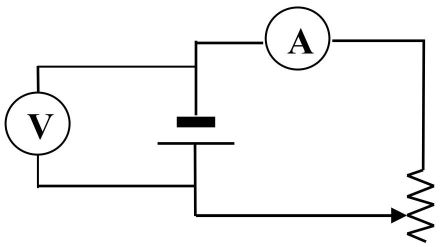
(b) the total current leaving the battery.

(c) the potential difference between the terminals of the battery.



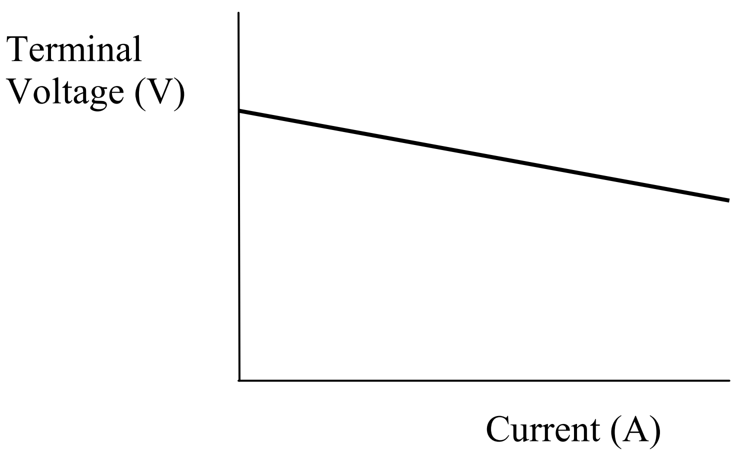
(see Circuitry Ex 18 for answer)

Note that the equation $\mathbf{V_T = \varepsilon - Ir}$ can be derived from graphing. Using a variable resistor in the following set-up:



→ammeter reads current
→voltmeter reads terminal voltage
→variable resistor changes resistance $\mathbf{R_o}$

As you would expect, changing the external resistance $\mathbf{R_o}$ in the circuit changes the current drawn; i.e., more resistance means less current, and vice versa. However, what you might *not* expect is that the voltage reading $\mathbf{V_T}$ *also* changes, due to the changing current. Graphing terminal voltage vs. current:



As current increases, terminal voltage slowly decreases in a linear manner. Using the standard graphing equation $y = kx + b$, we obtain

$\mathbf{V_T = -\boldsymbol{k}I + \boldsymbol{b}}$ →where $\boldsymbol{k}$ is a negative slope

Compare this equation with $\mathbf{V_T = \varepsilon - Ir}$, we see:

- slope $\boldsymbol{k}$ = internal resistance **r**
- y-intercept $\boldsymbol{b}$ = **EMF ε**

This technique is commonly used to find the **EMF** of any battery or cell.

Two last points:
Consider a small 1.5 V cell connected to a closed circuit with another, much larger power supply that moves electrons from positive to negative terminals in the small cell. In this case, $\mathbf{V_T}$ is <u>greater</u> than the **EMF** value, due to the opposite-to-normal flow of current; as a result,

$$\mathbf{V_T = \varepsilon + Ir}$$

In some batteries, this reversed current will cause the chemical reactions to be reversed, effectively *recharging* the batteries; in other cases, the reactions cannot be reversed due to the chemicals involved.

Finally, a complete circuit <u>without</u> an external resistance $\mathbf{R_0}$ is called a *short circuit*. Only the internal resistance (**r**) of the supply limits the flow of current in a short circuit.