# Electrostatics Notes

1 – Charges and Coulomb's Law

Ancient Greeks discovered that if amber (fossilized sap) is rubbed it will attract small objects. This is similar to when you run a comb through your hair…it will then attract bits of lint or dust. WHY?!?

Clearly this attraction is due to some FORCE at work. In this case it is electrostatic force which exists between electrically charged objects.

Conductors are materials that…allow electrons to flow.

Insulators are materials that…impede electron flow.

A negative charge is caused by…an excess of electrons.

A positive charge is caused by…a lack of electrons.

It is possible to build up a charge on insulators because electrons cannot…
easily flow off of (−) or onto (+) an insulator.

When a vinyl strip is rubbed with fur or wool the rod gains an excess of electrons and therefore is negative. If an acetate strip is rubbed with silk then it will lose electrons and become positive.

| Two charged vinyl strips | Two charged acetate strips | One charged vinyl and one charged acetate strip |
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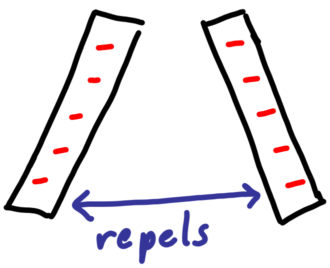
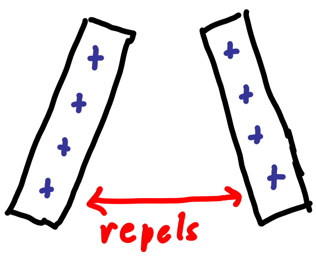
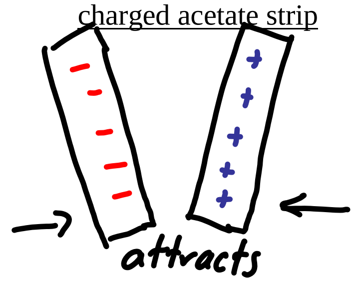
|  |  |  |

The Law of Charges states:

(1) Like charges repel.

(2) Opposite charges attract.

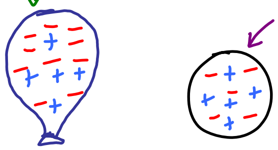
(3) Neutral charges are attracted to charged (+ or −) objects.

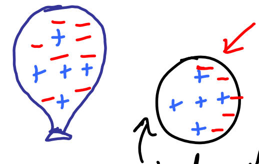
But what about that so-called amber effect? Why are seemingly uncharged objects attracted to charged amber (or combs for that matter)?

It has to do with something called… INDUCTION!

charged with extra electrons

Consider a rubber balloon that has been rubbed on someone's hair and a tin can.




Note that the electrons on the can… are able to move freely so that as the can rolls a + charge always faces the balloon.

Other examples of electrostatic charges in everyday life include:

(1) Rubbing your feet on the carpet then touching a door knob.

(2) Jumping on a trampoline then stepping on the metal rail.

(3) When you pull your clothes out of the dryer, the sock sticks to your sweater.

Ok enough playing around, where's the formulas?!?

Coulomb determined that the force between two charged objects is proportional to their charges and inversely proportional to the square of their distances or:

$$\boxed{F_E = \frac{K q_1 q_2}{r^2}}$$

Where:

$q_1$ = 1st charge, in Coulombs (C)
$q_2$ = 2nd charge, " " "
r = distance between charges
k = Coulomb's Constant
= $9.0 \times 10^9 \ N\cdot\frac{m^2}{C^2}$

There are two important things to notice from this equation.
First, this equation is quite similar to… universal gravitation

$$F_g = \frac{G m_1 m_2}{r^2}$$

Second, electrostatic forces are much stronger than gravitational forces.

$G = 6.67\times10^{-11}\ N\frac{m^2}{kg^2}$ whereas $K = 9.0\times10^9\ N\frac{m^2}{C^2}$

There is a very important difference between gravitational and electrostatic forces:

Gravity ALWAYS… attracts

Electrostatic force can.. attract or repels

When solving for electrostatic forces we will NOT… use +/− signs of charges even though force is a vector.

Instead we will determine the direction of the force based on…

whether it is an attraction or repulsion

Example:
Two 85 kg students are 1.0 m apart. What is the gravitational force between them?

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{(6.67\times10^{-11})(85)(85)}{(1.0)^2}$$
$$= 4.82\times10^{-7}\ N$$

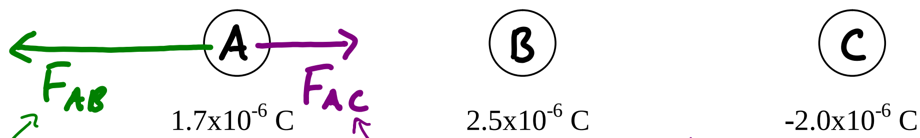
If these two students each have a charge of 2.0x10$^{-3}$ C, what is the electrostatic force between them?

$$F_E = \frac{k q_1 q_2}{r^2} = \frac{(9.0\times10^9)(2.0\times10^{-3})(2.0\times10^{-3})}{(1.0)^2}$$
$$= \underline{36000\ N}$$

Example:
Two point charges of 1.8x10$^{-6}$ C and 2.4x10$^{-6}$ C produce a force of 2.2x10$^{-3}$ N on each other. How far apart are these two charges?

$$F_E = \frac{k q_1 q_2}{r^2}$$
$$r = \sqrt{\frac{k q_1 q_2}{F_E}}$$
$$= \sqrt{\frac{(9.0\times10^9)(1.8\times10^{-6})(2.4\times10^{-6})}{(2.2\times10^{-3})}}$$
$$= \underline{4.2\ m}$$

Example:
A charge of 1.7x10$^{-6}$ C is placed 2.0x10$^{-2}$ m from a charge of 2.5x10$^{-6}$ C and 3.5x10$^{-2}$ m from a charge of -2.0x10$^{-6}$ as shown.




since A+B are positive they repel

since A+C are opposite they attract

What is the net electric force on the 1.7x10$^{-6}$ charge?

Winner - Loser

$$F_{net} = F_{AB} - F_{AC}$$
$$= \frac{k q_A q_B}{r_{AB}^2} - \frac{k q_A q_C}{r_{AC}^2}$$

don't use negative sign!!!

$$= \frac{(9.0\times10^9)(1.7\times10^{-6})(2.5\times10^{-6})}{(2.0\times10^{-2})^2} - \frac{(9.0\times10^9)(1.7\times10^{-6})(2.0\times10^{-6})}{(3.5\times10^{-2})^2}$$
$$= \underline{71\ N}$$

# **Electrostatics Notes**

## 2 – Electric Field on a Single Charge

There are many similarities between **gravitational** and *electrostatic* forces. One such similarity is that both forces can be exerted on objects that are not in contact.

In the same way that any mass is surrounded by a **gravitational field**, we will imagine that any charge object is surrounded by an **electric field**.

Similar to gravitational fields, an electric field will depend on:
size of and distance to the charge.

In fact we define an electric field as the force per unit charge:

this is just like grav fields:

$$g = \frac{F_g}{m}$$

$$\boxed{\vec{E} = \frac{F_E}{q}}$$

Where: E = electric field (N/C)
$F_E$ = electrostatic force (N)
q = test charge (C)

We can substitute in Coloumb's Law to get:

$$\boxed{\vec{E} = \frac{Kq}{r^2}}$$

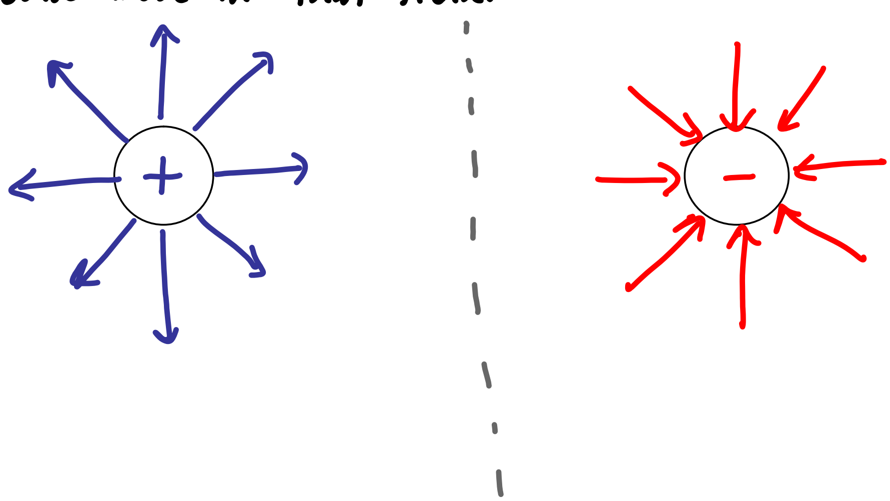
In the case of electric fields we are dealing with another example of a force field.

Therefore the field is a vector quantity.

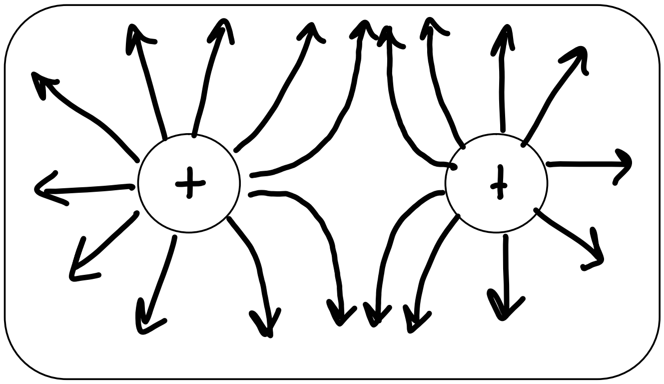
In order to show this we always draw the field lines as arrows.

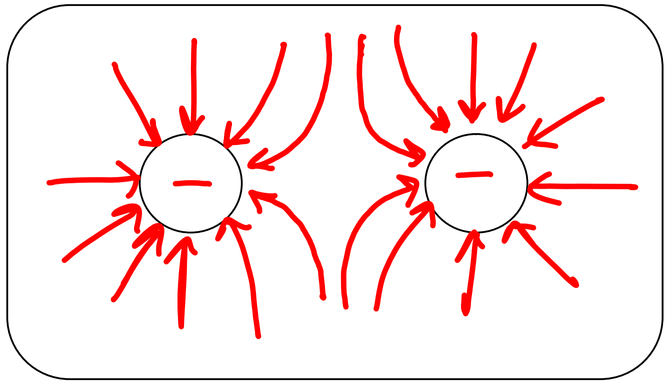
Again there is an important difference between gravitational fields and electric fields due to the fact that…

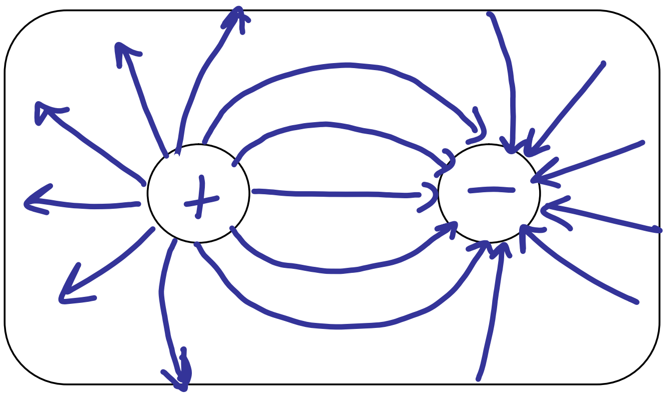
We therefore define the direction of an electric field as… the direction a positive charge would move in that field.



You will remember that the strength of a vector field is indicated by the density of the arrows, therefore the field is always strongest…




Example:

What is the electric field strength at a point where a -2.00 uC charge experiences an electric force of 5.30x10$^{-4}$ N?

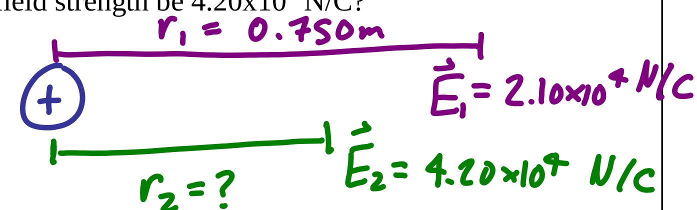
$$F_E = \vec{E}q$$

$$\vec{E} = \frac{F_E}{q} = \frac{5.30\times10^{-4}N}{2.00\times10^{-6}C}$$

$$= 265 \ N/C$$

Example:

At a distance of 7.50x10$^{-1}$ m from a small charged object the electric field strength is 2.10x10$^{4}$ N/C. At what distance from this same object would the electric field strength be 4.20x10$^{4}$ N/C?




$$\vec{E}_1 = \frac{kq}{r_1^2} \qquad q = \frac{\vec{E}_1 r_1^2}{k} = 1.3125\times10^{-6}C$$

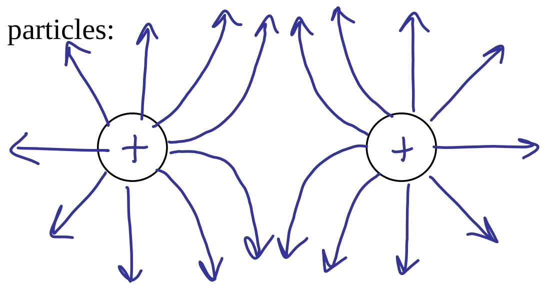
$$\vec{E}_2 = \frac{kq}{r_2^2} \qquad r_2 = \sqrt{\frac{kq}{E_2}} = \underline{0.53m}$$

# Electrostatics Notes

3 – Electric Field from Multiple Charges

We have already seen how charged particles emit electric fields, but how do these fields interact when two or more charges act on each other?

Consider two positively charged particles:

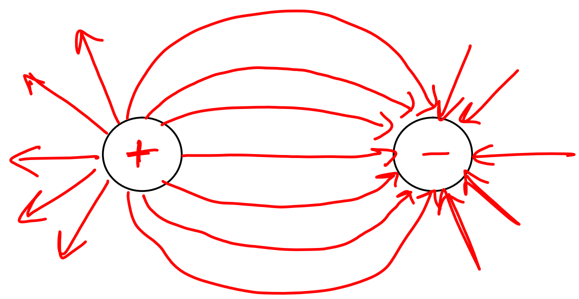



Note that the electric lines of force work… in opposition to each other.

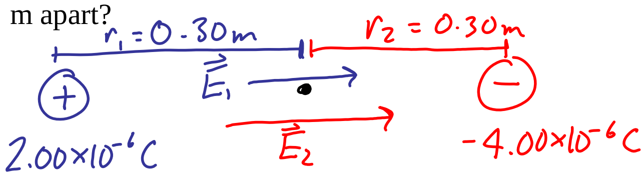
Because this electric field is a force field, it is a vector. So when multiple fields overlap we simply…
add them up as vectors.

OK, now try two opposite charges:

Again the two fields interact, only this time they…rei force each other




Example:
What is the strength of an electric field midway between a 2.00 uC charge and a -4.00 uC that are 0.60 m apart?

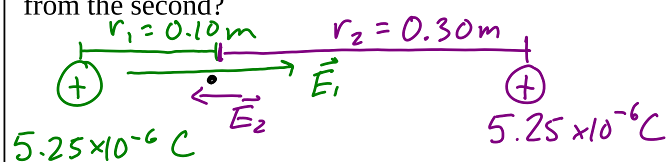



$\vec{E}_1 = \frac{kq_1}{r_1^2} = 200\,000\ N/C$

$\vec{E}_2 = \frac{kq_2}{r_2^2} = 400\,000\ N/C$ * Note that we don't use the "-" sign for the charge

$\vec{E}_T = \vec{E}_1 + \vec{E}_2 = 600\,000\ N/C$

Example:
Two 5.25 uC charges are 0.40 m apart. What is the strength of the electric field between them at a point 0.10 m away from the first charge and 0.30 m away from the second?

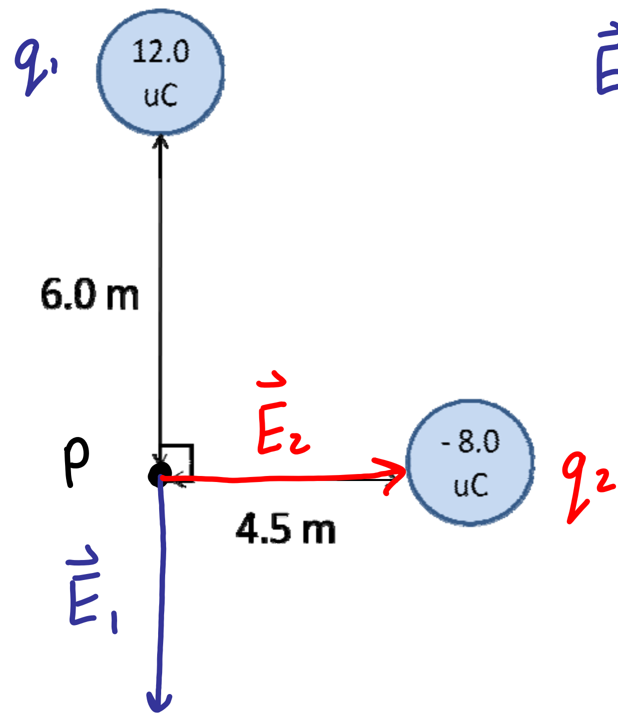


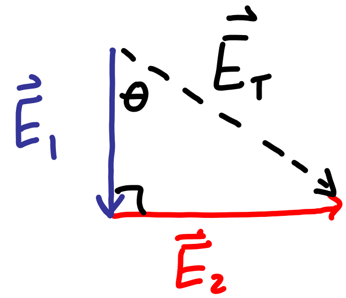

$\vec{E}_1 = \frac{kq_1}{r_1^2} = 4\,725\,000\ N/C$

$\vec{E}_2 = \frac{kq_2}{r_2^2} = 525\,000\ N/C$

$\vec{E}_T = \vec{E}_1 - \vec{E}_2 = 4.20\times10^6\ N/C$

<u>Example:</u>
Find the magnitude and direction of the electric field at the point P due to the charges as shown.




$$\vec{E}_1 = \frac{kq_1}{r_1^2} = 3000 \text{ N/C}$$

$$\vec{E}_2 = \frac{kq_2}{r_2^2} = 3556 \text{ N/C}$$

$$\vec{E}_T = \sqrt{\vec{E}_1^{\,2} + \vec{E}_2^{\,2}}$$

$$= \boxed{4650 \text{ N/C}}$$

$$\tan\theta = \frac{\vec{E}_2}{\vec{E}_1}$$

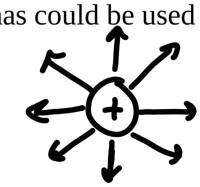
$$\theta = \tan^{-1}\left(\frac{\vec{E}_2}{\vec{E}_1}\right) = \tan^{-1}\left(\frac{3556}{3000}\right)$$

$$= \boxed{50^\circ}$$

# Electrostatics Notes

4 – Electric Potential, Electric Potential Difference and
Electric Potential Energy

First let's examine **electric potential energy**. If a charged object is in an electric field it has electric potential energy - that is it has the potential to move in that field. Note that the potential energy it has could be used to… attract or repel depending on the charges.



non - uniform

A non-uniform field, such as that provided by a point, is one which has a different… strengths and directions (depending on position)

In this case we can derive a formula for the electric potential energy in a NON-UNIFORM FIELD:

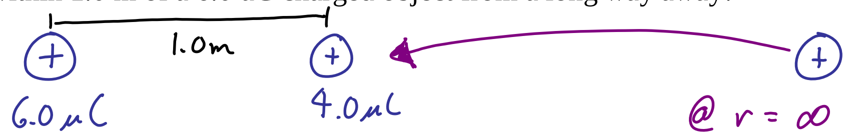
$$W = E_p = F_E \cdot d$$

$$= \left(\frac{Kq_1q_2}{r^{\cancel{2}}}\right)\cancel{d} = \frac{Kq_1q_2}{r}$$

$$\boxed{E_p = \frac{Kq_1q_2}{r}}$$

Again note the similarities between…

Example:
How much work must be done to bring a 4.0 uC charged object to within 1.0 m of a 6.0 uC charged object from a long way away?



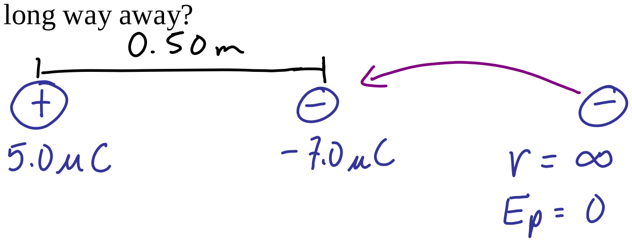

$$W = E_{pf} = \frac{Kq_1q_2}{r}$$

$$= \frac{(9.0\times10^9)(6.0\times10^{-6})(4.0\times10^{-6})}{1.0} = \underline{0.216\ J}$$

NOTE:
1. Potential energy is a … Scalar
2. We **WILL** … use + and - signs of charges

In this case, bringing a positive charge near another positive charge requires input therefore the work is positive.

Example:
How much work must be done to bring a -7.0 uC charged object to within 0.5 m of a 5.0 uC charged object from a long way away?




$$E_p = \frac{kq_1q_2}{r}$$

$$= \frac{(9.0\times10^9)(5.0\times10^{-6})(-7.0\times10^{-6})}{0.5}$$

$$= \underline{-0.63\ J}$$

In this case, bringing a negative charge near a positive charge releases energy therefore work is negative.

# Electric Potential

Now we need to consider a new quantity, electric potential (V). Electric potential is defined as the electric potential energy per unit charge.

$$\boxed{V = \frac{E_p}{q}}$$

$$V = \frac{\frac{kq_1\cancel{q_2}}{r}}{\cancel{q_2}}$$

Which becomes,

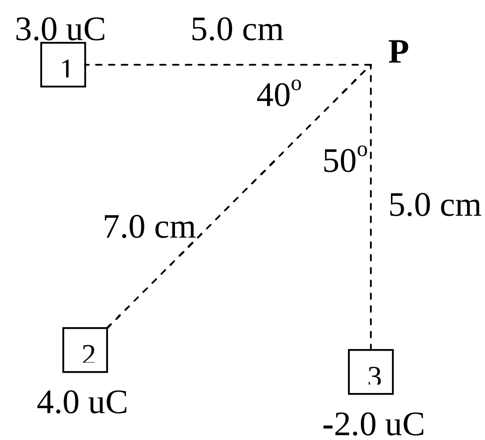
$$\boxed{V = \frac{kq}{r}}$$

**NOTE**:

(1) The electric potential is defined in terms of the moving of a positive charge. Therefore…
  + charges…move towards low potential
  - charges...move towards high potential

(2) The unit for potential is… volts (V)

Example:
Calculate the potential at point P as shown in the diagram.




$$V_T = V_1 + V_2 + V_3$$

$$V_1 = \frac{kq_1}{r_1} = \frac{(9.0\times10^9)(3.0\times10^{-6})}{0.05} = 540\,000\,V$$

$$V_2 = \frac{kq_2}{r_2} = \frac{(9.0\times10^9)(4.0\times10^{-6})}{0.070} = 514\,000\,V$$

$$V_3 = \frac{kq_3}{r_3} = \frac{(9.0\times10^9)(-2.0\times10^{-6})}{0.050} = -360\,000\,V$$

$$V_T = 540\,000 + 514\,000 - 360\,000 = \underline{\underline{694\,000\,V}}$$

NOTE:
(1) Potentials are… scalar quantities
(2) We WILL use… + and - signs of charges

<u>Potential Difference</u>

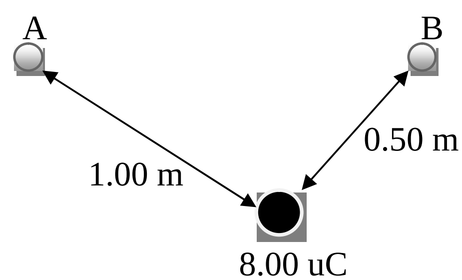
We sometimes want to determine the electric potential between two points. This is known as the **potential difference**.

For example, given two points A and B, the potential difference between A and B is:

$$V_{AB} = V_B - V_A$$

NOTE: When we talk about potential at a point we are talking about the potential difference between that point and infinity, where the potential at infinity is ZERO.

<u>Example:</u> What is the potential difference between points A and B due to the charge shown?




$$V_{AB} = V_B - V_A$$

$$= \frac{kq}{r_B} - \frac{kq}{r_A}$$

$$= \frac{(9.0\times10^9)(8.00\times10^{-6})}{0.50} - \frac{(9.0\times10^9)(8.00\times10^{-6})}{1.00}$$

$$= \underline{72\,000\text{ V}}$$

# Electrostatics Notes
## 5 – Equipotential Lines and Changes in Energy

Potential Difference

We sometimes want to determine the electric potential between two points. This is known as the **potential difference**.

For example, given two points A and B, the potential difference between A and B is:

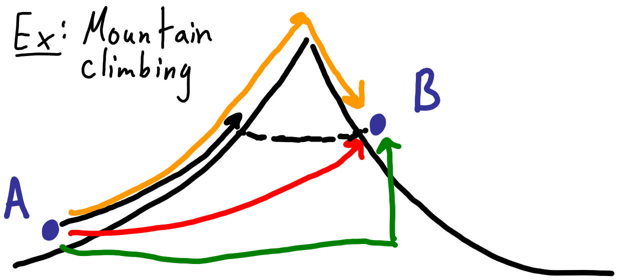
$$\boxed{\Delta V = V_2 - V_1}$$

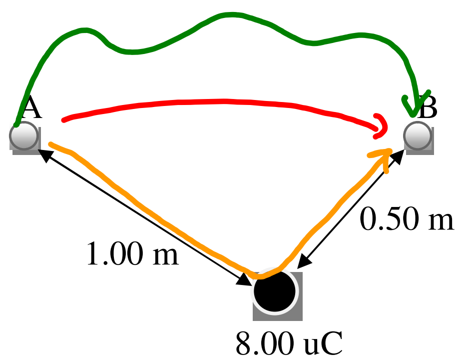
NOTE: When we talk about potential at a point we are talking about the potential difference between that point and infinity, where the potential at infinity is ZERO.

Example: What is the potential difference between points A and B due to the charge shown?

Since V is scalar, path does not matter!

Ex: Mountain climbing




$$\Delta V_{AB} = V_B - V_A = \frac{kq}{r_B} - \frac{kq}{r_A} = \frac{(9\times10^9)(8\times10^{-6})}{0.50} - \frac{(9\times10^9)(8\times10^{-6})}{1.00}$$

$$= 144\,000 - 72000 =$$

$$= \boxed{72000\ V}$$

Equipotential Lines

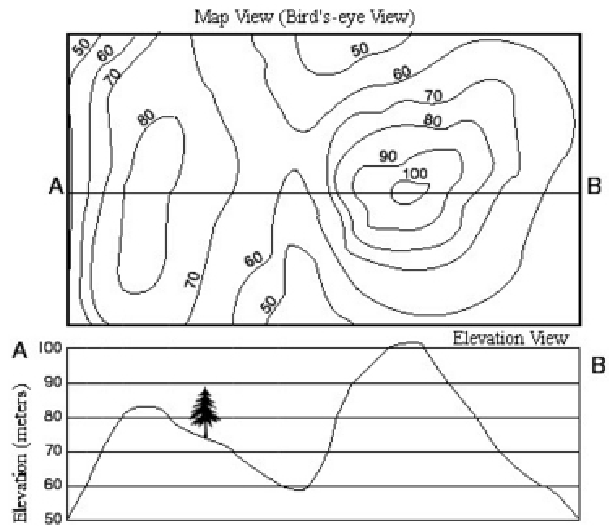
Theory:

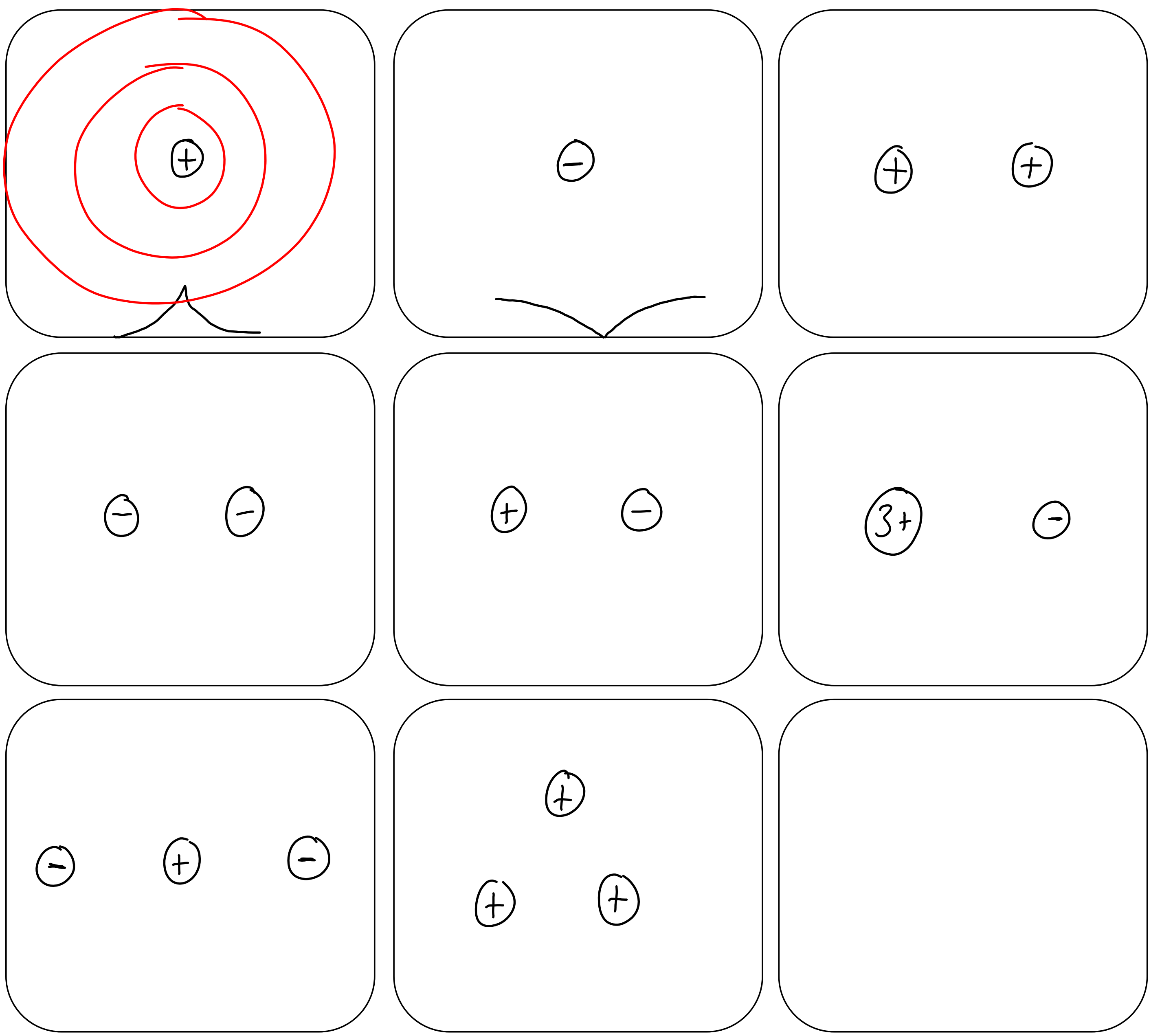
- As a charge moves along an electric field line, work is done by the electrical force. The energy gained or lost by this charge moving in the field is a form of *potential energy*, and so associated with the electric field is an *electric potential*, V, which has units of Energy per charge or Joules per Coulomb (also call Volts).
- Since voltage is potential energy per unit charge, voltage increases when going from a negative charge towards a positive charge. (The kinetic energy of a positive charge would increase when going from a higher potential to a lower potential.)
- A surface along which the potential is constant is called an *Equipotential*. On a piece of paper, the equipotential is represented by a line on which the voltage is constant.

Topographical Maps:

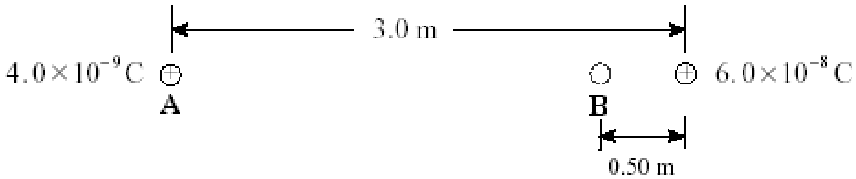
- Since gravitational potential energy depends on height, lines of constant height would be gravitational equipotentials. A map of such lines is called a topographical map. Typically, a topographical map shows equally spaced lines of constant elevation.

- Where the lines are most closely spaced the elevation is changing most sharply, in other words the terrain is steep.



Changes in Energy




A 4.0 x10$^{-9}$ C charge of mass 2.4 x10$^{-21}$ kg, is initially located at point **A**, 3.0 m from a stationary 6.0x10$^{-8}$ C charge.

a) How much work is required, by an external agent, to move the 4.0x10$^{-9}$ C charge to a point **B**, 0.50 m from the stationary charge?

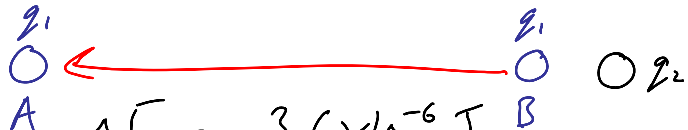
$$W = \Delta E_p = E_f - E_i = \frac{kq_1q_2}{r_B} - \frac{kq_1q_2}{r_A} = 4.32\times10^{-6} - 7.2\times10^{-7}$$

$$= \boxed{3.6\times10^{-6}\,J}$$

$$E_f = \frac{kq_1q_2}{r} = \frac{(9\times10^{9})(4\times10^{-9})(6\times10^{-8})}{0.5} = 4.32\times10^{-6}\,J$$

$$E_{p_i} = \frac{(9\times10^{9})(4\times10^{-9})(6\times10^{-8})}{3.0} = 7.2\times10^{-7}\,J$$

b) If the 4.0x10$^{-9}$ C charge is now released from point **B**, what will be its velocity when it passes back through point **A**?




$$\Delta E_p = -3.6\times10^{-6}\,J$$

$$\Delta E_K = -\Delta E_p = 3.6\times10^{-6}\,J \qquad E_{K_i} = 0$$

$$\therefore E_{K_f} = \frac{1}{2}mv_f^2 \qquad V = \sqrt{\frac{2E_K}{m}}$$

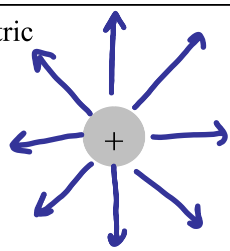
$$= \sqrt{\frac{2(3.6\times10^{-6})}{2.4\times10^{-21}}}$$

$$= \boxed{5.48\times10^{7}\,m/s}$$

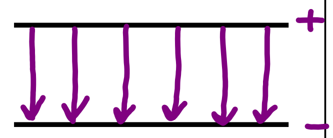
# Electrostatics Notes

## 6 – Electric Potential in Uniform Electric Fields

We have seen that the electric field surrounding a point charge is not uniform – that it…varying strength and direction



If we examine the electric field between charged plates we will find that it is..uniform in strength and direction




Notice that the density of the lines is also… uniform

In a uniform electric field we cannot use our previous formula:

$$\vec{E} = \frac{kq}{r^2}$$

← individual charge
← distance from charge

This formula is only valid for describing the strength of non-uniform fields (point charges only!!!)

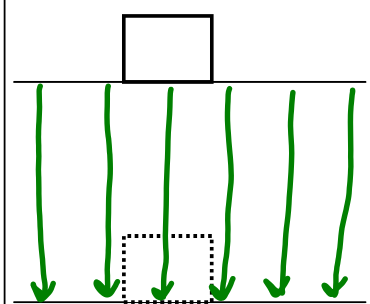
To find an equation for uniform fields, we will once again draw a parallel with gravitational potential energy.

Consider a mass sitting in a uniform *gravitational* field at some height.

The mass will tend to move from…high potential (height) to low potential

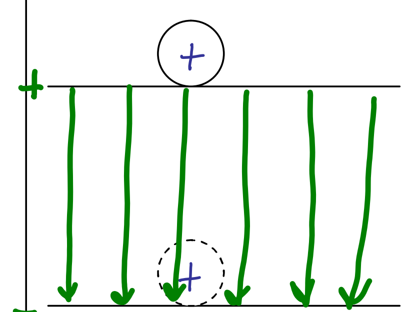
As it does it… potential energy is converted into kinetic

If we allow the mass to fall the work done on it (W = $\Delta E_p$ ) is negative. If we want to lift the mass to a certain height we need to do positive work on it.



A charged object in an electric field will behave in the same way, accelerating from an area of…high potential to low potential

As it does it..potential energy is converted into kinetic




In the same way that we would do positive work on an object to lift it against gravity, we need to do work to bring a positive charge near a plate with positive potential.

To calculate the work done in this case we can use the formula:

$$W = \Delta E_p = Fd$$

It is often easier, however, to describe the work done in a uniform field using the potential difference between the two plates.

Recall that potential difference:

$$\Delta V = \frac{\Delta E_p}{q}$$

A potential difference is generated any time we have areas of high and low potential energy, just like those generated by gravitational fields.

In order to determine the electric field between two charged plates we must use the formula:

$$\boxed{\vec{E} = \frac{\Delta V}{d}}$$

Where:
E = electric field (N/c)
ΔV = potential difference (V)
d = distance between plates (m)

Example:
Calculate the electric field strength between two parallel plates that are 6.00x10$^{-2}$ m apart. The potential of the top plate is 6.0 V and the bottom plate is -6.0 V.

6.0V
$d = 6.00\times10^{-2}$ m
-6.0V

$$\vec{E} = \frac{\Delta V}{d} = \frac{12.0\,V}{6.00\times10^{-2}\,m} = 200\ N/c$$

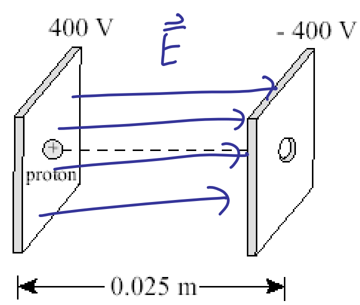
Example:
An electron is accelerated from rest through a potential difference of 3.00x10$^{4}$ V. What is the kinetic energy gained by the electron?

0V ⊖ 30 000 V

$\Delta E_p = \Delta V q$
$= (30\,000\,V)(-1.6\times10^{-19}\,C)$
$= -4.8\times10^{-15}$

$\Delta E_k = -\Delta E_p$
$= -(-4.8\times10^{-15})$
$= \boxed{4.8\times10^{-15}\ J}$

Example:
A proton, initially at rest, is released between two parallel plates as shown.
a) What is the magnitude and direction of the electric field?




• Field is + to − ∴ right

$$\vec{E} = \frac{\Delta V}{d} = \frac{800\,V}{0.025\,m} = 32\,000\ N/c$$

b) What is the magnitude of the electrostatic force acting on the proton?

$F_E = \vec{E}q = (32\,000\ N/c)(1.6\times10^{-19}\ c)$
$= 5.12\times10^{-15}\ N$

c) What is the velocity of the proton when it exits the - 400 V plate?

$\Delta E_p = \Delta V q$
$= (-800\,V)(1.6\times10^{-19}\,C)$
$= -1.28\times10^{-16}\ J$

$\Delta E_k = -\Delta E_p$
$\Delta E_k = 1.28\times10^{-16}\ J$

$\Delta E_k = E_{k_f} - \cancel{E_{k_i}}^{\,0}$

$E_{k_f} = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(1.28\times10^{-16})}{1.67\times10^{-27}}}$

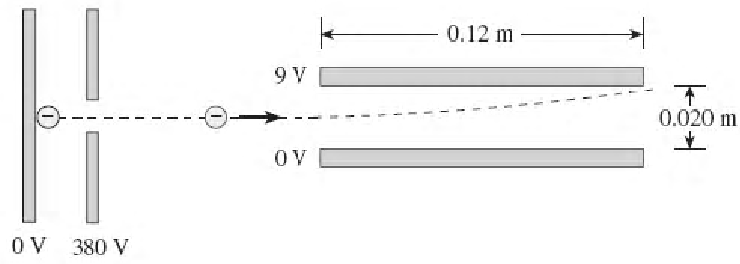
$v = 3.92\times10^{5}\ m/s$

# Electrostatics Notes

## 8 – Cathode Ray Tubes

Non-flat screen TVs and monitors work by directing a beam of high speed particles at a film of fluorescing chemicals. These charged particles are accelerated by electrically charged plates. After they are sped up, the beam can be directed by very precise control of another set of charged plates. Consider the following problem:

Example:
A beam of electrons is directed to a region between oppositely charged parallel plates as shown in the diagram below.




1) The electron beam is produced by accelerating electrons through an electric potential difference of 380 V. What is the speed of the electrons as they leave the 380 V plate?

$\Delta E_p = \Delta V q$
$= (380V)(-1.6\times10^{-19}C)$
$= -6.08\times10^{-17}$ J

$\Delta E_k = -\Delta E_p$
$= 6.08\times10^{-17}$ J

$\Delta E_k = E_{k_f}$ $\quad V_f = \sqrt{\frac{2E_k}{m}}$

$E_{k_f} = \frac{1}{2}mv_f^2$ $\quad = \sqrt{\frac{2(6.08\times10^{-17}J)}{9.11\times10^{-31}kg}} = 1.16\times10^{7}$ m/s

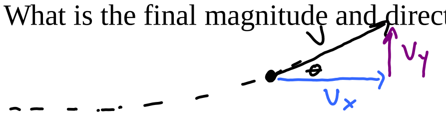
2) What is the electrostatic force on electrons in the region between the horizontal plates when they are connected to a 9.0 V potential difference? 4(deflecting)

$\vec{E} = \frac{\Delta V}{d} = \frac{9V}{0.020m} = 450$ N/C $\qquad F_E = \vec{E}q = 7.2\times10^{-17}$ N

3) What is the acceleration of the electrons between the deflecting plates?

$F_{net} = F_E = ma \qquad a = \frac{F_E}{m} = \frac{7.2\times10^{-17}N}{9.11\times10^{-31}kg} = 7.90\times10^{13}$ m/s²

4) What is the final magnitude and direction of the velocity of the electrons as it leaves the second set of plates?



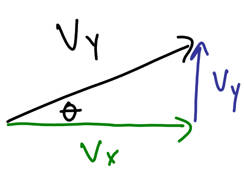

| x | y |
|---|---|
| $V_x = 1.16\times10^7$ | $V_y = ?$ |
| $d_x = 0.12$ | $V_{y0} = 0$ |
| $t =$ | $a_y = 7.90\times10^{13}$ m/s² |
| | $d_y =$ |
| | $t =$ |

$V_x = \frac{d_x}{t}$

$t = \frac{d_x}{V_x} = \frac{0.12\,m}{1.16\times10^7 m/s}$

$= 1.034\times10^{-8}$ s

$V_y = V_{y0} + at$
$= 0 + (7.90\times10^{13})(1.034\times10^{-8})$
$V_y = 8.169\times10^{5}$ m/s




$V = \sqrt{V_y^2 + V_x^2}$
$= 1.2\times10^{7}$ m/s

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 4.0°$

5) How could you cause the beam to bend
a. more?
i) increase deflecting voltage
ii) decrease accelerating voltage

b. less?
i) decrease deflecting voltage
ii) increase accelerating voltage