Example 1. Find the electrostatic force between a +3.0 µC charge and a +8.0 µC charge, 0.25 m apart.

$$F_E = \frac{kQq}{r^2}$$

$$= \frac{(9\times10^9)(3\times10^{-6})(8\times10^{-6})}{(.25)^2}$$

$$\boxed{F_E = 3.5\text{ N}, \text{ repulsive (like charges)}}$$

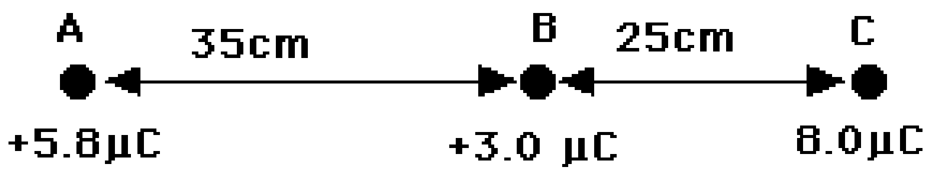
Example 2. A -6.0 μC charge and a -4.0 μC charge repel each other with a force of 7.0 N. How far apart are these point charges?

$$F_E = \frac{kQq}{r^2}$$

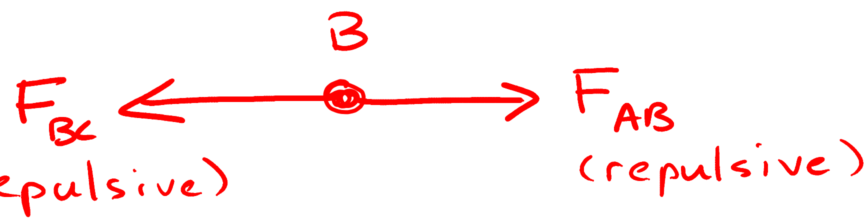
$$7.0 = \frac{(9\times10^{9})(6\times10^{-6})(4\times10^{-6})}{r^2}$$

$$\boxed{r = 0.18\ m}$$

Example 3. What is the force on the 3.0 μC charge if the charges are positioned along one line as follows.



start with f. b. d. on "B":



$$F_{AB} = \frac{(9\times10^{9})(5.8\times10^{-6})(3.0\times10^{-6})}{.35^{2}}$$

$$= 1.28\ N \longrightarrow$$

$$F_{BC} = \frac{(9\times10^{9})(3.0\times10^{-6})(8.0\times10^{-6})}{.25^{2}}$$

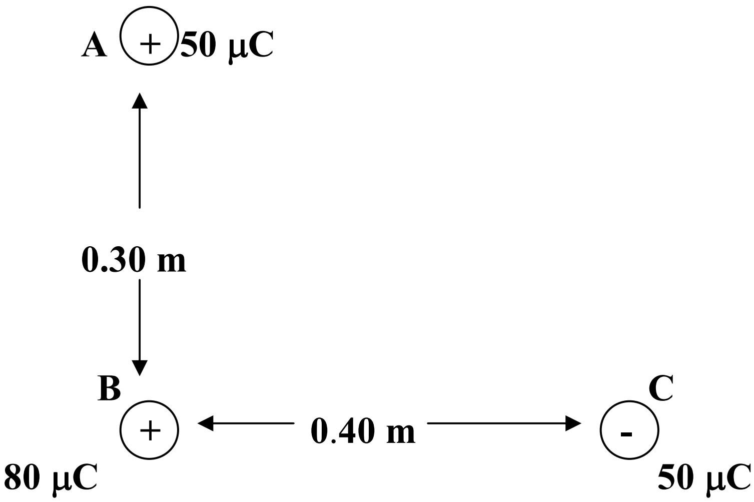
$$= 3.46\ N \longleftarrow$$

From analysis of f. b. d. :

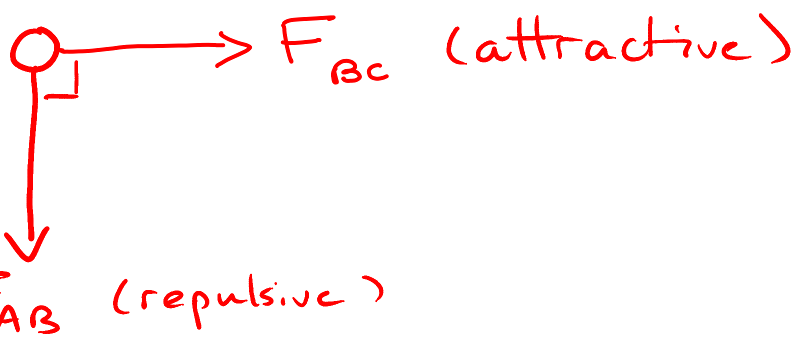
$$F_{Net} = F_{BC} - F_{AB} = 3.46 - 1.28$$

$$\boxed{F_{Net} = 2.2N \text{ to the left}}$$

Example 4. Three charges are laid out as in the following diagram. Find the resultant force due to the other two charges on charge B.



Start with F.b.d. on "B":



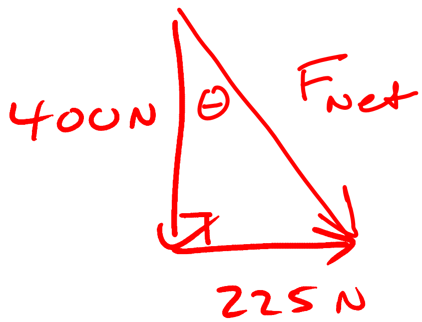
$$F_{AB} = \frac{(9\times10^{9})(50\times10^{-6})(80\times10^{-6})}{.3^2}$$

$$= 400\ \text{N} \downarrow$$

$$F_{BC} = \frac{(9\times10^{9})(80\times10^{-6})(50\times10^{-6})}{.4^2}$$

$$= 225\ \text{N} \longrightarrow$$

Now draw a forces triangle to find F_{Net}:



$$F_{Net} = \sqrt{400^2 + 225^2}$$

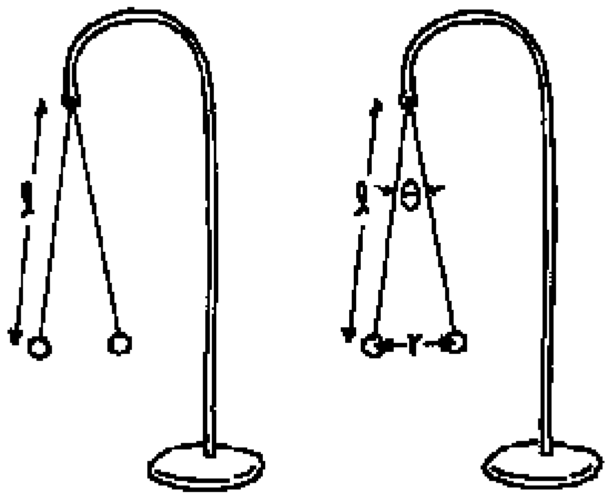
$$= 459\ N$$

$$\theta = \tan^{-1}\left[\frac{225}{400}\right]$$

$$= 29°$$

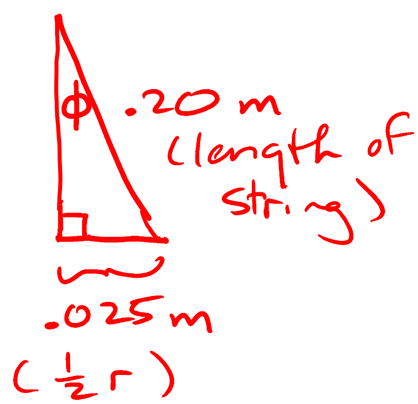
$$\therefore \boxed{F_{Net} = 4.6 \times 10^2\ N\ @\ 29° \text{ from line A-B}}$$

Example 5. Examine the electroscope arrangement below, where two pith balls have identical charges and are repelling each other.



If each string has a length l = 0.20 m, the distance of separation between the charged pith balls is r = 0.05 m, and the mass of the balls is 0.010 kg each, find the magnitude of the charge on each pith ball.

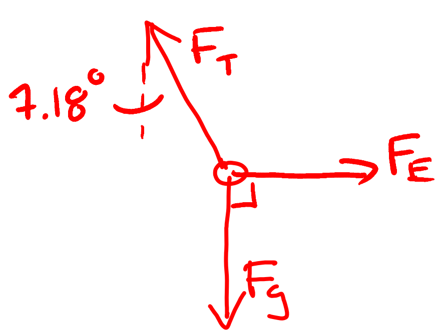
Start: find $\frac{\theta}{2}$ (called "ϕ")



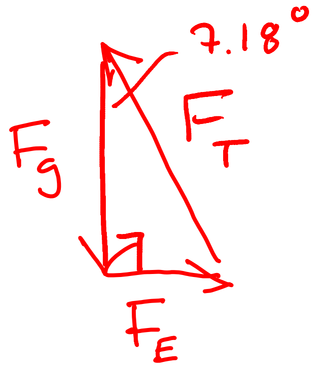
$$\sin\phi = \frac{.025}{.2}$$

$$\phi = 7.18°$$

Next: draw a f.b.d. of one pith ball:



→ since pith ball is stationary, draw a forces triangle to show how the 3 forces cancel out:



$$\Rightarrow \frac{F_E}{F_g} = \tan\theta$$

$$\Rightarrow F_E = (.010)(9.8)\tan 7.18$$

$$= .0123\ N$$

Finally,

$$F_E = \frac{kQq}{r^2} = \frac{kQ^2}{r^2}$$ → charges are equal

$$.0123 = \frac{(9\times10^9)\,Q^2}{.05^2}$$

$$\boxed{Q = 5.9\times10^{-8}\ C}$$

Note: the repulsive nature tells us each force has the same charge, but the type of charge (+ or −) is unknown.

Example 6. Two unknown charges have a force between them of 5.6 N. How will that force change if:
a) one of the charges is tripled?
b) one charge is halved and the other quadrupled?
c) the distance between them halved?
d) both charges are doubled and the distance tripled?

a) $F \propto Q$

$\therefore$ new $F = 5.6 \times 3 = \boxed{16.8\ N}$

b) $F \propto Q$ and $F \propto q$

$\therefore$ new $F = 5.6 \times \frac{1}{2} \times 4 = \boxed{11.2N}$

c) $F \propto \frac{1}{r^2}$

$\therefore$ new $F = 5.6 \times \frac{1}{.5^2} = \boxed{22.4N}$

d) new $F = 5.6 \times 2 \times 2 \times \frac{1}{3^2}$

$= \boxed{2.49\ N}$

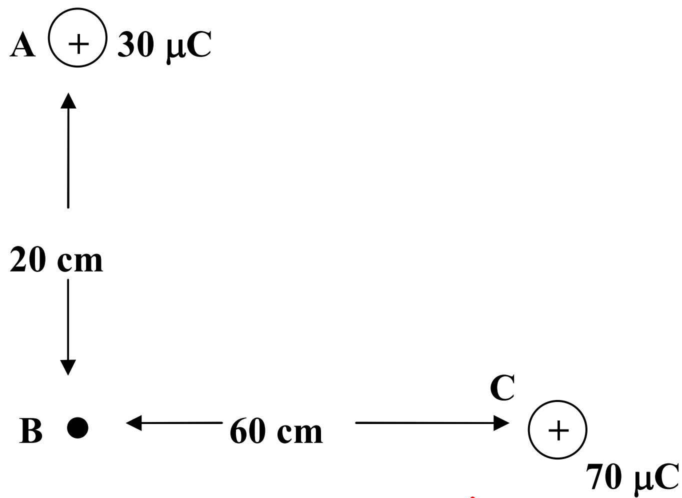
Example 7. A 6.0 μC charge and a 4.5 μC charge are positioned 1.6 cm apart. If the smaller charge is removed, what is the electric field strength at the location of the 4.5 μC charge, due to the larger charge?

$$E = \frac{kQ}{r^2}$$ ← use 6.0μC charge

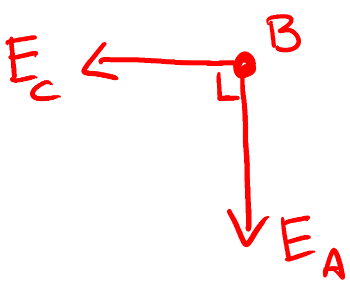
$$= \frac{(9\times10^{9})(6\times10^{-6})}{.016^{2}}$$

$$\boxed{E = 2.1\times10^{8}\ N/C}$$

Example 8. Find the resultant field at point B due to the two charges.



First: draw a vectors diagram of the field lines at B:



Next: calculate E due to each charge:

$$E_A = \frac{(9\times10^9)(30\times10^{-6})}{.20^2}$$

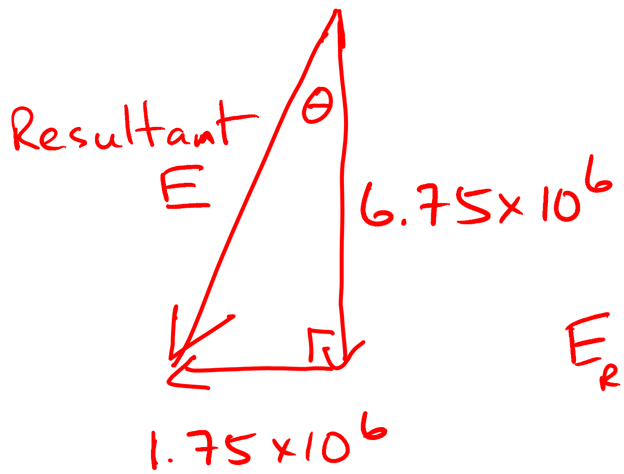
$$= 6.75\times10^6 \text{ N/C} \downarrow$$

$$E_c = \frac{(9\times10^9)(70\times10^{-6})}{.60^2}$$

$$= 1.75\times10^6 \text{ N/C} \leftarrow$$

continued on next page . . .

Now draw the vector-sum of the two field lines:



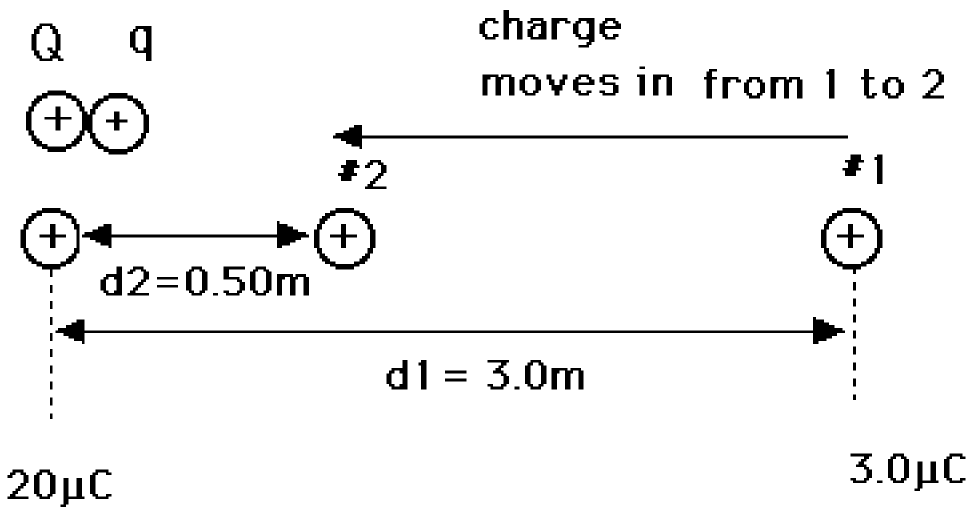
$$E_R = \sqrt{(1.75\times10^6)^2 + (6.75\times10^6)^2}$$

$$= 6.97\times10^6 \text{ N/C}$$

$$\theta = \tan^{-1}\left[\frac{1.75}{6.75}\right] = 14.5°$$

$$\therefore \boxed{E = 7.0\times10^6 \text{ N/C @ } 15° \text{ from line A-B}}$$

Example 9. Find the work done to move a charge (q) from position #1 to #2 under the influence of the field of charge Q. (0.90 J)

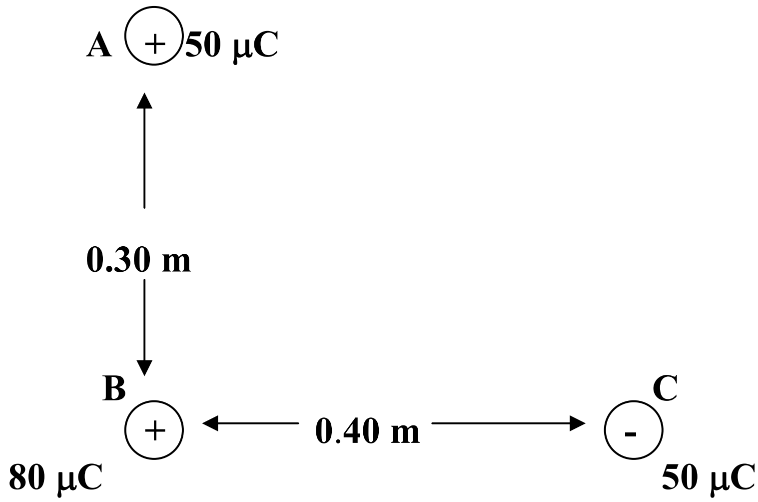


$$W = \Delta E_p = \frac{kQq}{r_2} - \frac{kQq}{r_1} = kQq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= (9\times10^{9})(20\times10^{-6})(3\times10^{-6})\left[\frac{1}{0.5} - \frac{1}{3}\right]$$

$$\boxed{W = 0.90\ \text{J}}$$

Example 10. Re-examine the diagram from Example 4 (see below). Find the potential energy of particle B due to the other charges.



E_p is a scalar quantity, so no vector analysis is needed.

→ find the potential energy "B" contains due to each particle:

$$E_{p\,(AB)} = \frac{(9\times10^{9})(50\times10^{-6})(80\times10^{-6})}{.30}$$

$$= 120\ J$$

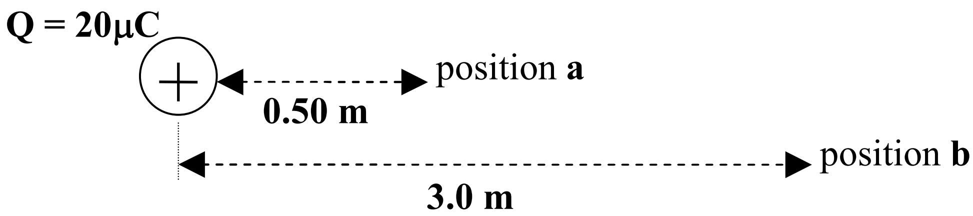
note!

$$E_{p\,(BC)} = \frac{(9\times10^{9})(80\times10^{-6})(-50\times10^{-6})}{.40}$$

$$= -90\ J$$

$$\therefore \boxed{E_{p\,(total)} = 120 - 90 = 30\ J}$$

Example 11. What is the potential difference between the two positions in the following example?



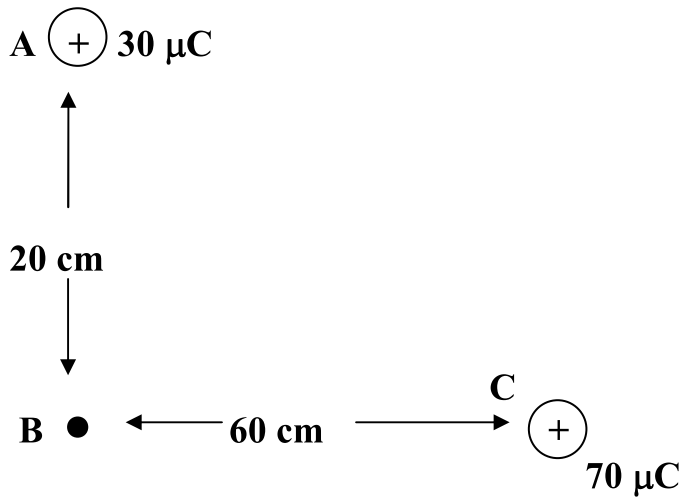
$$\Delta V = V_b - V_a$$

$$= \frac{kQ}{r_b} - \frac{kQ}{r_a} = kQ\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= (9\times10^{9})(20\times10^{-6})\left[\frac{1}{3} - \frac{1}{.5}\right]$$

$$\boxed{\Delta V = -3.0\times10^{5}\ \text{V}}$$

Example 12. Re-examine the diagram from Example 8 (see below). Find the electric potential at point B due to the other charges. Hint: remember, electric potential is a scalar quantity. No vector analysis is needed here.

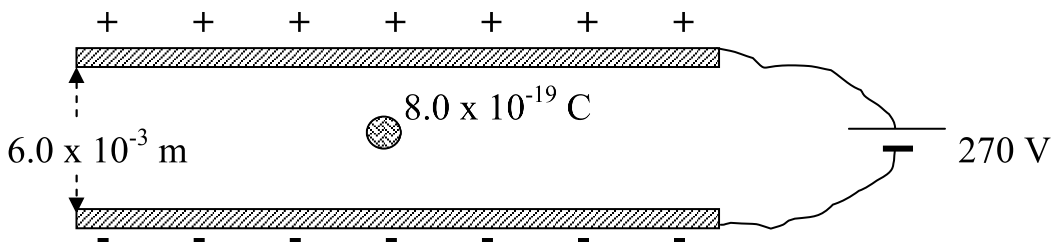


$$V_A = \frac{(9\times10^{9})(30\times10^{-6})}{.20} = 1350000\ V$$

$$V_B = \frac{9\times10^{9}(70\times10^{-6})}{.60} = 1050000\ V$$

$$\boxed{V_{total} = 2.4\times10^{6}\ V}$$

Example 13. A charged particle of 8.0×10^{-19} C is held stationary inside an electric field produced by two electric plates. The voltage between the plates is 270 V and they are separated by a distance of 6.0×10^{-3} m.

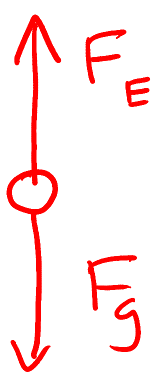


a) **What constant electric field strength exists between the plates?**
b) **What is the mass of the particle? Hint: first draw a f.b.d. of the particle to determine its weight.**

a) $E = \dfrac{\Delta V}{d} = \dfrac{270}{6.0 \times 10^{-3}}$

$$\boxed{E = 45000 \text{ N/C}}$$

b)



$\Rightarrow$ particle is stationary,

so

$F_g = F_E$

continued on next page . . .

$\rightarrow$ since $F_E = qE$

then $F_g = F_E = (8 \times 10^{-19})(45000)$

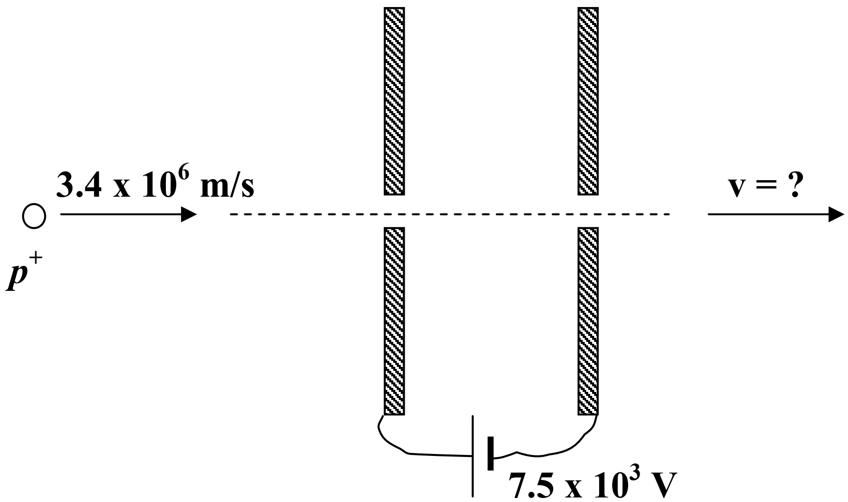
$= 3.6 \times 10^{-14}\ N$

Since $F_g = mg$,

$$m = \frac{3.6 \times 10^{-14}}{9.8}$$

$$\boxed{m = 3.7 \times 10^{-15}\ kg}$$

Example 14. A proton travelling at 3.4 x 10^6 m/s passes through an electric field as shown below. How fast will the proton be going after it emerges from the field?



This is a conservation of energy problem:

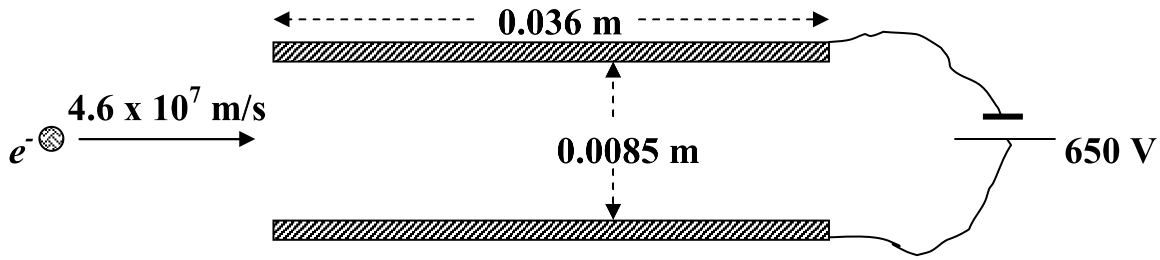
$$\Delta E_p = \Delta E_k$$

$$q\Delta V = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$(1.6\times10^{-19})(7500) = \frac{1}{2}(1.67\times10^{-27})\left[v_f^2 - (3.4\times10^6)^2\right]$$

$$\boxed{v_f = 3.6\times10^6 \text{ m/s}}$$

Example 15. An electron travelling at 4.6×10^7 m/s enters a constant electric field between two charged plates spread 0.0085 m apart, as shown below. The voltage between the plates is 650 V and the plates are 0.036 m long.



a) What is the electric force acting on the electron?
b) How much time is taken for the electron to pass through the plates?
c) How far will the electron "fall" from its path while in-between the two plates?

Hint: for b) and c), you'll have to examine horizontal and vertical components, just like for objects fired horizontally off a cliff.

a) $F_E = qE$ and $E = \frac{\Delta V}{d}$ between the plates

so $F_E = q\frac{\Delta V}{d}$

$= \frac{(1.6 \times 10^{-19})(650)}{.0085}$

$= 1.22 \times 10^{-14}$ N

b) speed is constant "horizontally" because F_E acts $\perp$ to motion

$\therefore d = v_{av}t \qquad t = \frac{.036}{4.6 \times 10^7}$

$\boxed{t = 7.83 \times 10^{-10} \text{ s}}$

c) → find "vertical" acceleration:

$$F_{net} = F_E = ma$$

$$a = \frac{1.22 \times 10^{-14}}{9.11 \times 10^{-31}} = 1.34 \times 10^{16}\ m/s^2$$

→ also in the vertical direction:

$$V_o = 0$$

$$t = 7.83 \times 10^{-10}\ s$$ → time in plates where electron "falls"

$$d = \cancel{v_o t} + \frac{1}{2}at^2$$

$$= \frac{1}{2}(1.34 \times 10^{16})(7.83 \times 10^{-10})^2$$

$$\boxed{d = 4.1 \times 10^{-3}\ m}$$

Example 16. Given this information:

V_a = 100 V **distance between Y-plates = 0.040 m**

V_d = 10.0 V **length of Y-plates = 0.100 m**

a) use accelerating voltage V_a to find electron velocity in the x-direction v_x after leaving the anode.

b) since v_x is <u>constant</u> after leaving the anode, calculate the time taken for an electron to pass through the deflecting Y-plates.

c) use deflecting voltage V_d to find the force F_y on the electron between the Y-plates.

d) find the acceleration a_y of the electron between the Y-plates.

e) At this point, you have enough kinematics information to find the y-deflection d_y between the Y-plates.

f) If the accelerating voltage is now doubled, while the deflecting voltage is reduced to 3/4 of its original value, what is the new magnitude for d_y?

a) use conservation of energy : $\Delta E_p = \Delta E_k$

$$q\Delta V_a = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} \quad (v_i = 0)$$

$$(1.6\times10^{-19})(100) = \frac{1}{2}(9.11\times10^{-31})\,v_f^2$$

$$\boxed{v = 5.9\times10^{6}\ m/s} \quad (5.93\times10^{6})$$

b) speed from a) is constant in horizontal direction through deflecting plates, so

$$d = v_{av}t \qquad \text{where } d = \text{length of plates}$$

$$.100 = (5.93\times10^{6})\,t$$

$$\boxed{t = 1.7\times10^{-8}\ s.} \quad (1.69\times10^{-8}\ s)$$

c) $F_E = qE$ (deflecting force) and $E = \frac{\Delta V_d}{d}$

so $F_E = q\frac{\Delta V_d}{d}$ ← distance between deflecting plates

$= \frac{(1.6\times10^{-19})(10)}{.040}$

$$\boxed{F_E = 4.0\times10^{-17}\,N}$$

d) $F_{net} = F_E = ma$

$4.0\times10^{-17} = 9.11\times10^{-31}\,a$

$$\boxed{a = 4.4\times10^{13}\,m/s^2}$$ $(4.39\times10^{13}\,m/s^2)$

e) "vertically": $v_0 = 0$

$t = 1.69\times10^{-8}\,s$ (to "fall" through plates)

$a = 4.39\times10^{13}\,m/s^2$

$d = \cancel{v_0 t} + \frac{1}{2}at^2$

$= \frac{1}{2}(4.39\times10^{13})(1.69\times10^{-8})^2$

$$\boxed{d = 6.3\times10^{-3}\,m}$$ (6.27×10^{-3})

f) $d \propto \Delta V_d$ and $V \propto \frac{1}{\Delta V_a}$

so $d = (6.27 \times 10^{-3}) \times .75 \times \frac{1}{2}$

$$\boxed{d = 2.35 \times 10^{-3} \text{ m}}$$