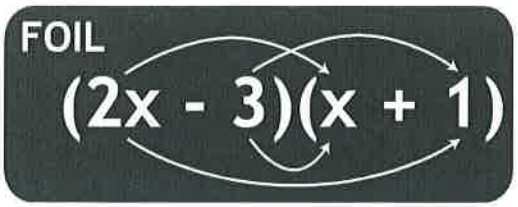


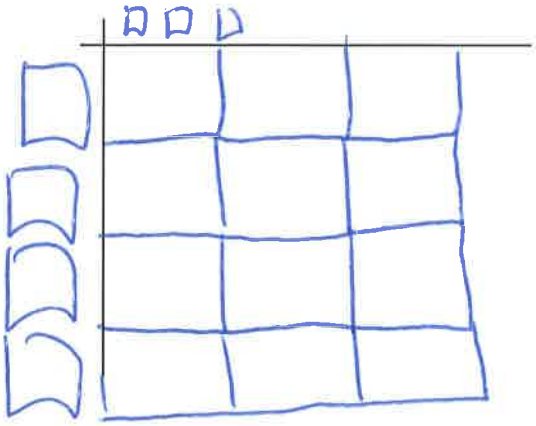
Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

Introduction

Find the product using algebra tiles:

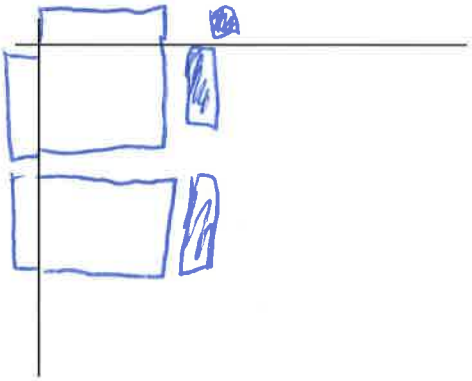
a) $3(4x^2)$

$= 12x^2$



b) $2x(x - 1)$

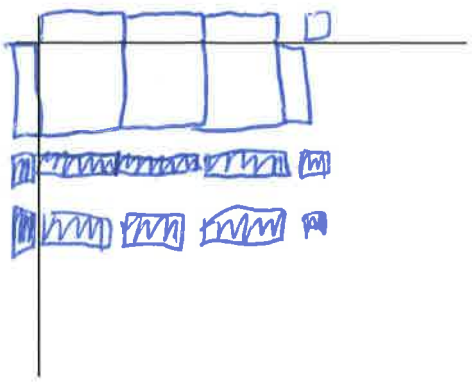
$2x^2 - 2x$



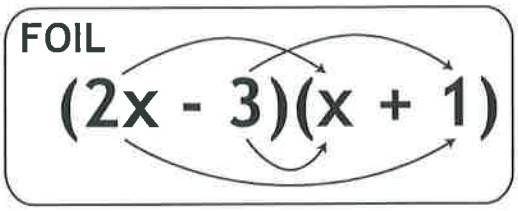
c) $(x - 2)(3x + 1)$

$3x^2 + x - 6x - 2$

$3x^2 - 5x - 2$



Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes



Example 1 ***Monomial* × *Monomial***. Determine the product.

a) $3(2x^2)$ $6x^2$

b) $(5x)(7x)$ $35x^2$

c) $(6a)(3ab)$ $18a^2b$

d) $(4x)^2$ $16x^2$

e) $2(3x)(5x)$ $30x^2$

Example 2 ***Monomial* × *Binomial***. Determine the product.

a) $-2x(3x - 1)$ $-6x^2 + 2x$

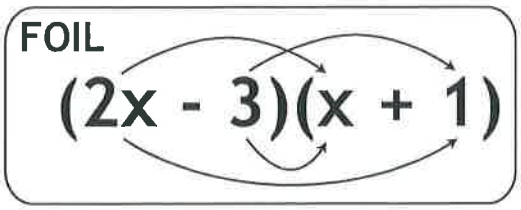
b) $-8a(a - ab)$ $-3a^2 + 8a^2b$

c) $x^2(x^2 - 4)$ $x^4 - 4x^2$

d) $(3x)^2(2x - 1)$

$(3x)(3x)$

$= 9x^2(2x-1) = 18x^3 - 9x^2$



Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

Example 3

***Binomial* × *Binomial*.** Determine the product.

a) $(x + 1)(x + 2)$

x^2+3x+2

c) $(3x - 2)^2$

$9x^2-12x+4$

b) $(2x - 3)(x + 4)$

$2x^2+5x-12$

d) $2(2x + 1)(4x - 5)$

$2(8x^2-6x-5) = 16x^2-12x-10$

Example 4

***Binomial* × *Binomial continued*.** Determine the product.

a) $(5x - 8)(5x + 8)$

$25x^2-64$

c) $(2x + y)(x - 3y)$

$2x^2-6xy-3y^2$

b) $(3x - 2)(1 - 2x)$

$6x^2+7x-2$

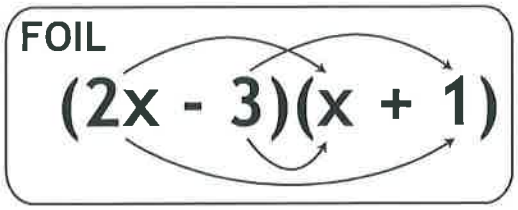
d) $3x(-5 - 2x)^2$

$(-5-2x)(-5-2x)$

$3x(25+4x^2+20x)$

$= 12x^3+60x^2+75x$

Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes



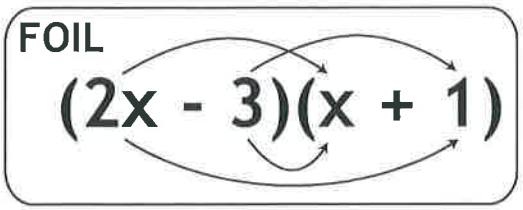
Example 5 ***Multiplying with Trinomials.*** Determine the product.

a) $(4x - 3y)(2 + 3x - y)$

c) $(3x - 1)^2(2x + 1)$

b) $(2x - 3)^3$

d) $(-2x^2 - x + 1)(-3x^2 + 3x - 2)$



Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

Example 6 *Multi-term Expansions*

a) $2x - 1 - (3x - 2)$

$-1x + 1$

b) $(x + 1)(4x - 3) + 4(x - 2)^2$

c) $3(x - 1)^2 - 2(2x - 3)^2$

$3(x^2 - 2x + 1) - 2(4x^2 - 12x + 9)$

$3x^2 - 6x + 3 - (8x^2 - 24x + 18)$

$3x^2 - 6x + 13 - 8x^2 + 24x + 18$

$-5x^2 + 18x + 31$

d) $2x(x - y) - (3x - 2y)(5x + y)$

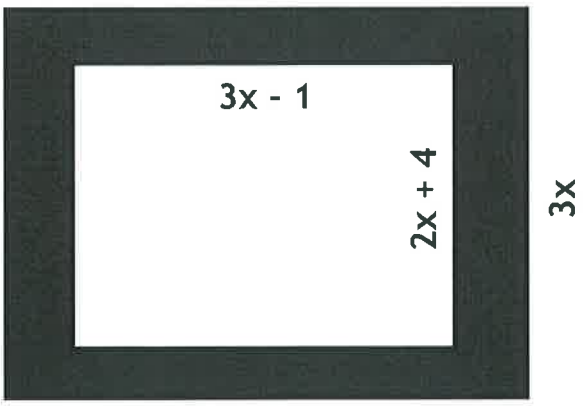
Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

FOIL

$(2x - 3)(x + 1)$

Example 7 Determine an expression for the shaded area.

a)



Dark - Light

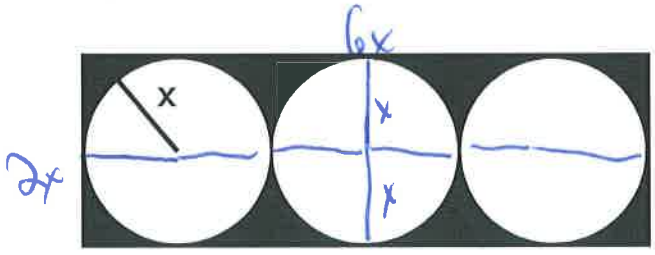
Dark: $4x \cdot 3x = 12x^2$

Light $(3x-1)(2x+4) = 6x^2 + 10x - 4$

$12x^2 - 6x^2 + 10x - 4$

$= 6x^2 + 10x - 4$

b)



Dark - Light:

Dark: $6x \cdot 2x = 12x^2$

Light = Circles $3(\pi r^2)$ $r = x$

$3(\pi x^2)$

$12x^2 - 3\pi x^2$ $9x^2 - \pi$

FOIL

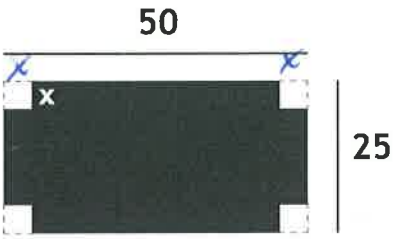
$(2x - 3)(x + 1)$

Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

Example 8

A piece of cardboard is made into an open box by cutting out squares from each corner.

The length of the piece of cardboard is 50 cm and the width is 25 cm. Each square has a side length of x cm.



a) Write expressions for the length and width of the box.

$L = 50 - 2x$

$W = 25 - 2x$

b) Write an expression for the area of the base.

$A = L \times W = (50 - 2x)(25 - 2x) = 4x^2 - 150x + 1250$

c) Write an expression for the volume of the box.

d) What is the volume of the box if each removed corner square had a side length of 3 cm?

Polynomials
LESSON ONE - *Expanding Polynomials*
Lesson Notes

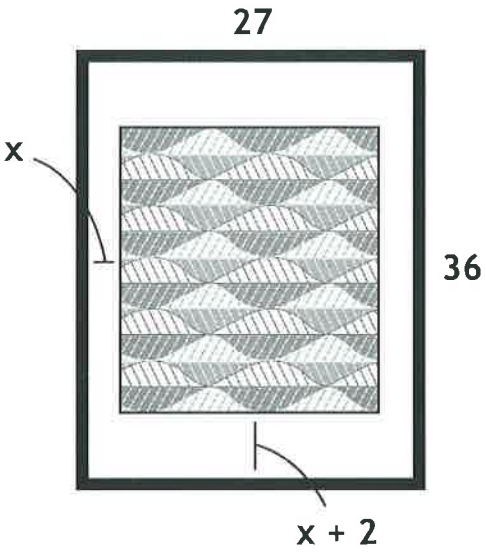
FOIL

$(2x - 3)(x + 1)$

Example 9

A picture frame has a white mat surrounding the picture.

The frame has a width of 27 cm and a length of 36 cm. The mat is 2 cm wider at the top and bottom than it is on the sides.



a) Write expressions for the width and length of the picture.

$27 + 2x = W$

$36 + 2(x+2) = L$

b) Write an expression for the area of the picture.

$(27 + 2x)(2x + 40$

$= 4x^2 + 134x + 1080$

c) Write an expression for the area of the mat.

$27 \times 36 = 972$ (mat)

$- 4x + 134x + 1080$

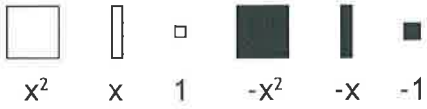
$= -4x^2 + 134x - 108$

$3x^3 - 6x^2$ (Expand) / (Factor) $3x^2(x - 2)$

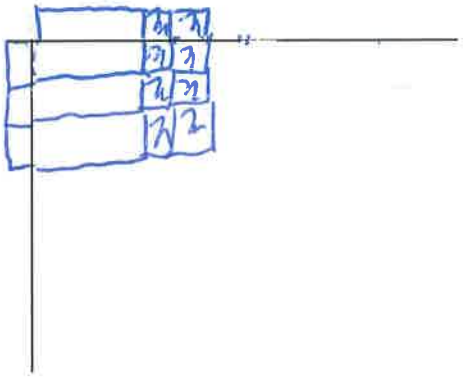
Polynomials
LESSON TWO - *Greatest Common Factor*
Lesson Notes

Introduction

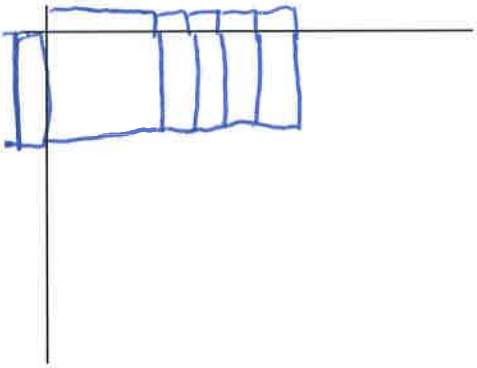
Factor each expression using algebra tiles.



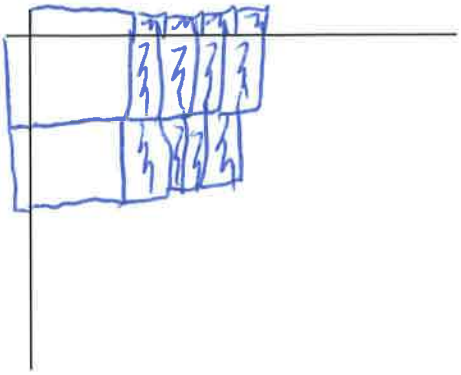
a) $3x - 6$



b) $x^2 + 4x$



c) $2x^2 - 8x$



Polynomials
LESSON TWO - *Greatest Common Factor*
Lesson Notes

$3x^3 - 6x^2$ ↖ Expand

Factor ↘ $3x^2(x - 2)$

Example 1 Find the greatest common factor of each pair.

a) 36 and 48

6

b) 15 and 45

15

c) $16x^2$ and $24x$

$8x$

d) $3a^2b^3$ and $6a^4b^3$

$3a^2b^3$

e) πr^2 and πrs

πr

Example 2 Factor each binomial.

a) $3x - 12$

3

$3(x-4)$

b) $-4x^2 + 24x$

$4x$

$4x(-x + 6)$

c) $15x^4 + 60x^2$

$15x^2$

$15x^2(x^2 + 4)$

d) $-12x^3 - 27x$

$-3x$

$-3x(x^2 + 9)$

$3x^3 - 6x^2$ ← Expand

Factor → $3x^2(x - 2)$

Polynomials
LESSON TWO - *Greatest Common Factor*
Lesson Notes

Example 3 Factor each polynomial.

a) $a^2b - a^2c + a^2d$

$a^2(b - c + d)$

c) $-13ab^2c^3 + 39bc^2 - 26ab^4$

$13b(-1abc^3 + 3c^2 - 2ab^3)$

b) $6x^2y^2 + 18xy$

$6xy(xy + 3)$

d) $-xy^3 - x^2y^2$

$-xy^2(y + x)$

Example 4 Factor each polynomial.

a) $3x(x - 1) + 4(x - 1)$

$(x-1)(3x+4)$

c) $5ax - 15a - 3x + 9$

b) $4x(2x + 3) - (2x + 3)$

$(2x+3)(4x-1)$

d) $4x^4 + 4x^2 - 3x^2 - 3$

none

Polynomials
LESSON TWO - *Greatest Common Factor*
Lesson Notes

$3x^3 - 6x^2$ ↖ Expand

Factor ↘ $3x^2(x - 2)$

Example 5

The height of a football is given by the equation $h = -5t^2 + 15t$, where h is the height above the ground in metres, and t is the elapsed time in seconds.

a) Write the factored form of this equation.

$h = -5t(t - 3)$

b) Calculate the height of the football after 2 s.

$h = -5(2)^2 + 15(2)$

$h = -20 + 30$ $h = 10m$

$3x^3 - 6x^2$ ← Expand

Factor → $3x^2(x - 2)$

Polynomials

LESSON TWO - *Greatest Common Factor*

Lesson Notes

Example 6

A pencil can be thought of as a cylinder topped by a cone.

a) Write a factored expression for the total visible surface area.

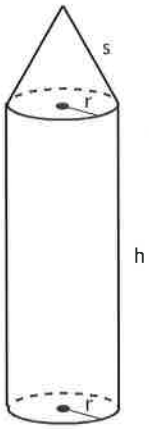
later

From Formula Sheet:

$SA_{Cylinder} = 2\pi r^2 + 2\pi rh$

$SA_{Cone} = \pi r^2 + \pi rs$

Hint: The top of the cylinder *(and the bottom of the cone)* are internal to the pencil and do not contribute to the surface area.



b) Calculate the visible surface area if the radius of the pencil is 0.5 cm, the cylinder height is 9 cm and the slant height of the cone is 2 cm.

later

Polynomials
LESSON TWO - *Greatest Common Factor*
Lesson Notes

$3x^3 - 6x^2$ ↖ Expand

Factor ↘ $3x^2(x - 2)$

Example 7

Laurel is making food baskets for a food drive. Each basket will contain boxes of spaghetti, cans of beans, and bags of rice.

Each basket must contain exactly the same quantity of items. *(**example**: all baskets have 2 spaghetti boxes, 3 cans of beans, and 2 bags of rice).*

If there are 45 boxes of spaghetti, 27 cans of beans, and 36 bags of rice, what is the maximum number of baskets that can be prepared? What quantity of each item goes in a basket?

GCF of 45, 27, 36

27: 1, 3, 9, 27 ✓ ×

~~Each~~ ~~each~~

9 baskets.

$\frac{45}{9} = 6$ Spag

$\frac{27}{9} = 3$ beans

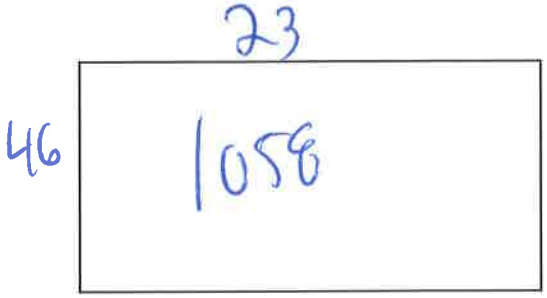
$\frac{36}{9} = 4$ rice

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	√

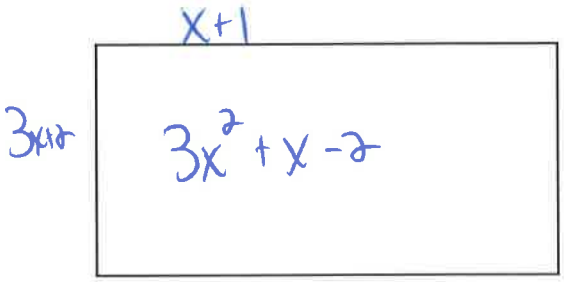
Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

Introduction

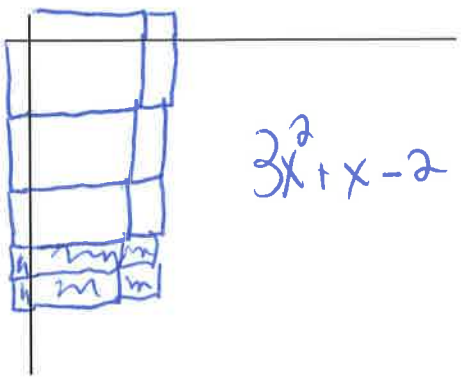
a) Multiply 23 and 46 using an area model.



b) Expand (x + 1)(3x - 2) using an area model.



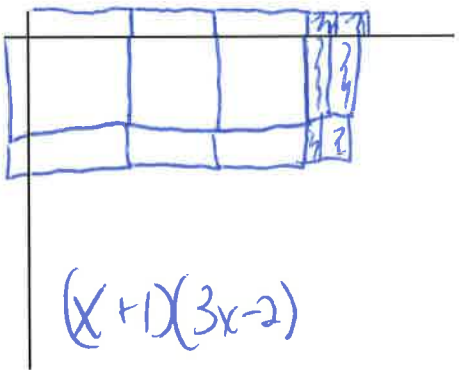
c) Expand (x + 1)(3x - 2) using algebra tiles.



d) What generalizations can be made by comparing the area model from part b with the tile grid in part c?

Same

e) Factor $3x^2 + x - 2$ using algebra tiles.



$(x+1)(3x-2)$

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Example 1

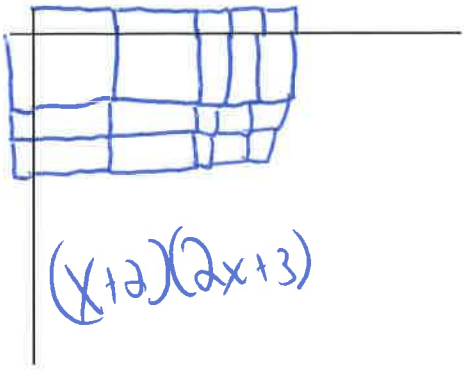
If possible, factor each trinomial using algebra tiles.

How to Factor: See

a) $2x^2 + 7x + 6$

$2 \cdot 6 = 12$

$4 + 3 = 7$ ✓



$(x+2)(2x+3)$

$4x^2 - 3x - 1$

A B C

$A \times C = -4$

need factors of $A \cdot C$ that add to 'b'

$A \cdot C = -4$

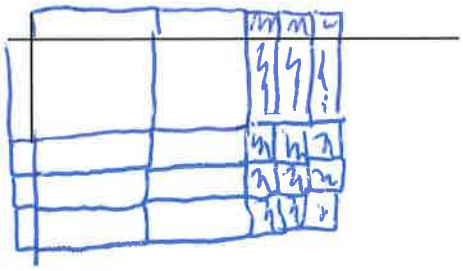
$-4 + 1 = -3$ ✓

Factorable.

b) $2x^2 + 3x - 9$

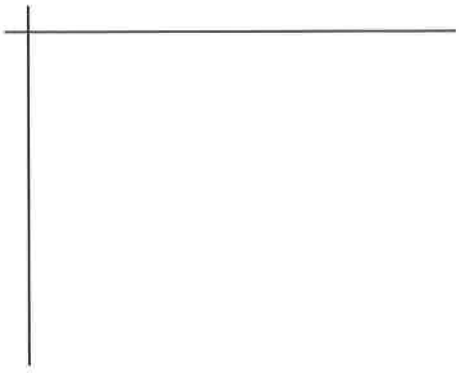
$A \cdot C = -18$

$6 + -3 = 3$ ✓



$(x+3)(2x-3)$

c) $x^2 - 8x + 4$



$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

Example 2 If possible, factor each trinomial using decomposition.
Note: In this example, we are factoring the trinomials from Example 1 algebraically.

a) $2x^2 + 7x + 6$

A×C = 12	B = 7	works? ✓

Decomp: $A \cdot C = 12$

$4 + 3 = 7$

$= (2x^2 + \underbrace{4x}_{\text{still } = 7x})(\underbrace{3x}_{\text{still } = 7x} + 6)$ → Bracket term 1,2 term 3,4 then factor those

$\frac{(2x^2+4x)}{2x} + \frac{(3x+6)}{3} = \underbrace{2x(x+2)}_{x+2} + \underbrace{3(x+2)}_{x+2}$

Factor out common $(x+2)$

$(x+2)(2x+3)$

leftovers from taking out $(x+2)$

b) $2x^2 + 3x - 9$

A×C =	B =	works?

c) $x^2 - 8x + 4$

A×C = 4	B = -8	works?
		X no factors of 4 that add to -8

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Example 3 Factor each trinomial using the indicated method.

a) $x^2 - 8x + 12$

A×C = 12	B = -8	works?
1		✓
-6 + -2	= -8	

i) shortcut

If $a = 1$ then

$(x \pm \_)(x \pm \_)$

factors of C that add to B

$(x-6)(x-2)$

ii) decomposition

$AC = +12$

$-6 + -2 = -8$

$(x^2 - 6x)(-2x + 12)$

$x(x-6) - 2(x-6)$

$(x-2)(x-6)$

b) $x^2 - x - 20$

A×C =	B =	works?

i) shortcut

ii) decomposition

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

Example 4 Factor each trinomial using the indicated method.

a) $6a - 4a^2 - 2a^3$

* Factor out $a^1 - 2a$ to start

$-2(a^2 + 2a - 3)$

A×C = ☐	B = 2	works?
-3		
+3 -1	2	✓

i) shortcut

a -3 3 -1

$-2(a+3)(a-1)$

ii) decomposition

b) $x^2y^2 - 5xy + 6$

A×C = ☐	B = ☐	works?
Don't do		

i) shortcut

ii) decomposition

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Example 5 Factor each trinomial using decomposition.

a) $10a^2 - 17a + 3$

$AC = 30$

$-15 + -2 = -17$

$(10a^2 - 15a)(-2a + 3)$

$5a \quad -1$

$5a(2a - 3) - 1(2a - 3)$

$(5a - 1)(2a - 3)$

b) $\frac{24x^2 - 72x + 54}{6}$

$6(4x^2 - 12x + 9)$ Doable ✓

$4 \cdot 9 = 36$

$-6 + -6 = -12$

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

Example 6 Factor each trinomial using decomposition.

a) $12 + 21x - 6x^2$ → $\frac{-6x^2 + 21x + 12}{3}$

$3(-2x^2 + 7x + 4)$ $\quad -2 \cdot 4 = -8$, $8 + -1 = 7$

$3(-2x^2 + 8x)(-x + 4)$

$-2x \quad -1$

$3(2x(x-4) - 1(x-4)$

$= 3(2x-1)(x-4)$

b) $\frac{8a^2 - 10ab - 12b^2}{2}$ $\quad 2(4a^2 - 5ab - 6b^2)$ $\quad 4 \cdot 6 = 24$, $-8 + 3 = -5$

$2(\underbrace{4a^2 - 8ab}_{4a} + \underbrace{3ab - 6b^2}_{3b})$

$2(4a(a-2b)\ 3b(a-2b)$

$2(4a+3b)(a-2b)$

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Example 7 Find up to three integers that can be used to replace k so each trinomial can be factored.

a) $3x^2 + kx - 10$

$-10 \times 3 = -30$

$+ \quad = k$

So factors of -30

$-1, 30 = +29$ or $-30, 1 = -29$

$-2, 15 = +13$ or $-15, +2 = -13$

$-3, 10 = 7$ or -7

$-6, 5 = -1$ or $+1$

b) $x^2 + 4x + k$

$1 \cdot k = 1k$

$- + - = 4$

need # that have factors add to 4

So: $4 = 1, 2, 2, 4$ $\quad 2 + 2 = 4$

$-12 = +6 \times 2$ $\quad +6 + -2 = 4$

...

c) $3x^2 - 8x + k$

Same as above.

$k =$

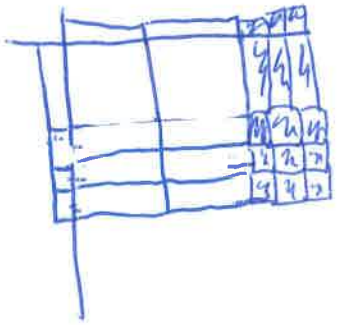
$4x^2 - 3x - 1$		
A×C = -4	B = -3	works?
-4 and 1	-3	✓

Polynomials
LESSON THREE - *Factoring Trinomials*
Lesson Notes

Example 8 Factor each expression to find the dimensions.

a) rectangle

$A = 2x^2 + 3x - 9$



$2x^2 + 3x - 9$

$2 \cdot -9 = -18$

$+6 + -3 = 3$

$(2x^2 + 6x)(-3x - 9)$

$2x \quad -3$

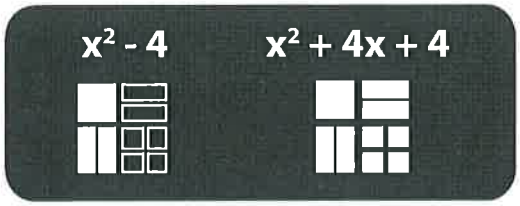
$2x(x+3) - 3(x+3)$

$(2x-3)(x+3)$

$(x+3)(2x-3)$

b) rectangular prism later

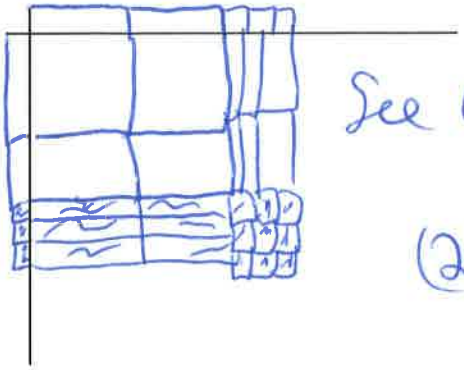
$V = 4x^3 - 40x^2 + 36x$



Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes

Introduction Factor each expression using algebra tiles first, then use the shortcut.

a) $4x^2 - 9$



See $6x$ and $-6x$
Cancel
$(2x+3)(2x-3)$

Difference of Squares Shortcut

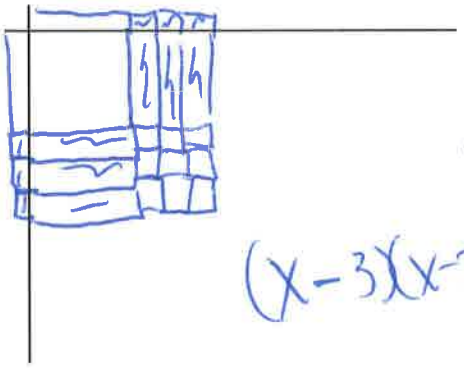
When a diff of Square
$(\sqrt{A}+\sqrt{B})(\sqrt{A}-\sqrt{B})$

a $4x^2 - 9$
A B

$\sqrt{4} = 2$ $\sqrt{9} = 3$
$\sqrt{x^2} = x$

$(2x+3)(2x-3)$

b) $x^2 - 6x + 9$



See when $A \cdot C = 9$

Sum → $3 + 3$

So $(\sqrt{A}+\sqrt{B})(\sqrt{A}+\sqrt{B})$

$(x-3)(x-3)$

Perfect Square Trinomial Shortcut

When $a = 1$ then
(ax^2+bx+c) factors of 'c' add +B

$(x-3)(x-3)$
A B
Big easy

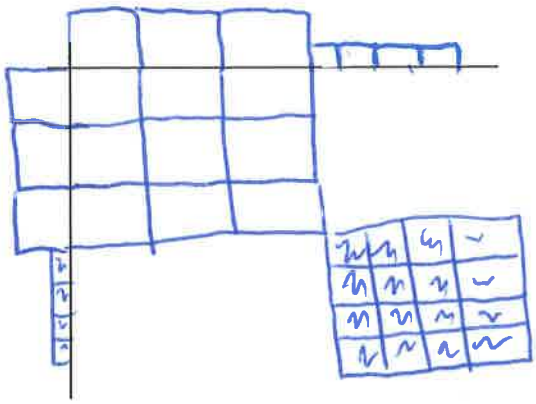
Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes

$x^2 - 4$ 

$x^2 + 4x + 4$ 

Example 1 Factor each expression using algebra tiles.

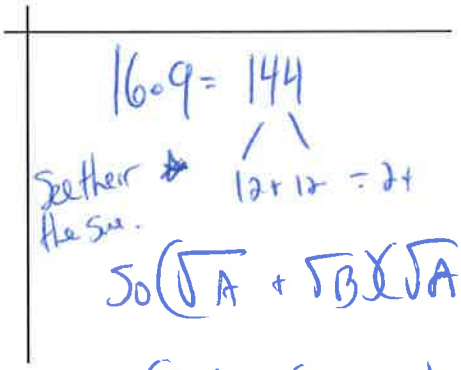
a) $9x^2 - 16$



$(3x-4)(3x+4)$

b) $16 - 9x^2$

c) $16x^2 + 24x + 9$ Tooby to do w/tiles



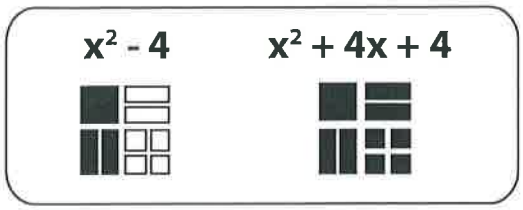
$16 \cdot 9 = 144$

$12 + 12 = 24$

Sol $(\sqrt{A} + \sqrt{B})(\sqrt{A} + \sqrt{B})$

$(4x+3)(4x+3)$

d) $1 - 16x + 64x^2$



Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes

Example 2 Factor each expression using decomposition.

Note: In this example, we are factoring the trinomials from Example 1 algebraically.

a) $9x^2 - 16$

Diff of Squares

$(\sqrt{A}+\sqrt{B})(\sqrt{A}-\sqrt{B})$ ex $(3x-4)(3x+4)$.

A×C =	B =	works?
-144	0	✓

b) $16 - 9x^2$

$(4-3x)(4+3x)$

A×C =	B =	works?
-144	0	✓

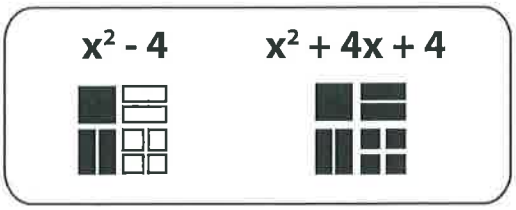
c) $16x^2 + 24x + 9$

A×C =	B =	works?

d) $1 - 16x + 64x^2$

A×C =	B =	works?

Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes



Example 3

Factor each expression using a shortcut.
Note: In this example, we are factoring the trinomials from Examples 1 & 2 with a shortcut.

a) $9x^2 - 16$

$(3x-4)(3x+4)$

c) $16x^2 + 24x + 9$

b) $16 - 9x^2$

d) $1 - 16x + 64x^2$

Example 4

If possible, factor each of the following

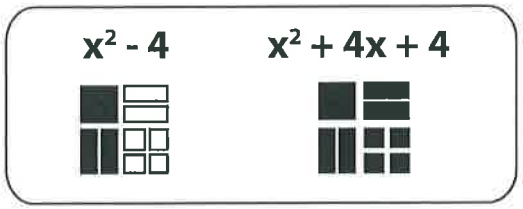
+ + not a difference.

a) $x^2 + 9$

$AC = 9$
$+ \cdot + = 9$
not possible.

b) $x^2 - 8x + 4$

$1 \cdot 4 = 4$
$+ \cdot + = -8$
not possible.



Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes

Example 5 If possible, factor each of the following

a) $9x - 4x^3$

x

$x(9 - 4x^2)$

$x(3 - 2x)(3 + 2x)$

b) $4x^2 + 16$

4

$4(x^2 + 4)$

$4(x + 2)(x - 2)$

c) $2x^4 - 32$

d) $16x^2 + 8xy + y^2$

e) $9x^4 - 24x^2 + 16$

Polynomials
LESSON FOUR - *Special Polynomials*
Lesson Notes

$x^2 - 4$ 

$x^2 + 4x + 4$ 

Example 6 Find a value for k that will make each expression a perfect square trinomial.

a) $9x^2 + kx + 49$

b) $25x^2 + 10x + k$

c) $kx^2y^2 - 48xy + 9$