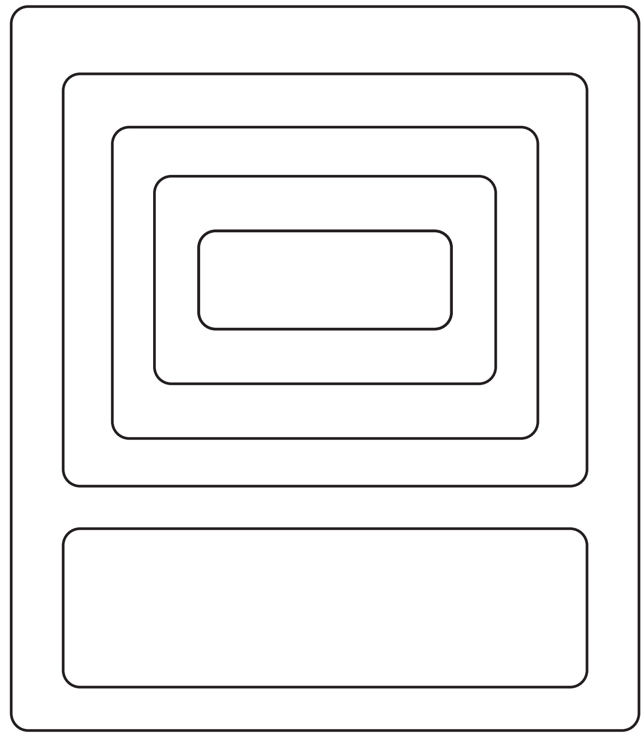
Numbers, Radicals, and Exponents
LESSON ONE - *Number Sets*
Lesson Notes

Introduction

Define each of the following sets of numbers and fill in the graphic organizer on the right.



a) Natural Numbers

b) Whole Numbers

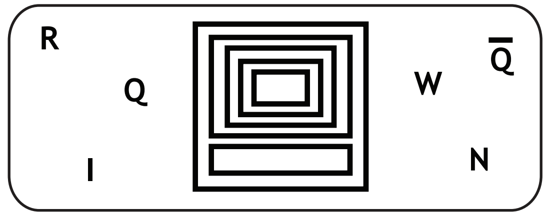
c) Integers

d) Rational Numbers

e) Irrational Numbers

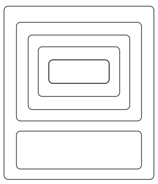
f) Real Numbers

Numbers, Radicals, and Exponents
LESSON ONE - *Number Sets*
Lesson Notes

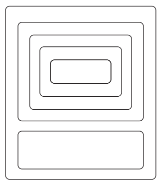


Example 1 Determine which sets each number belongs to.
In the graphic organizer, shade in the sets.

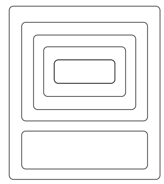
a) -4



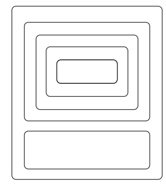
b) 0



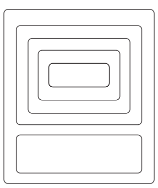
c) 1.273958...



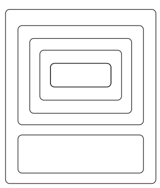
d) 7



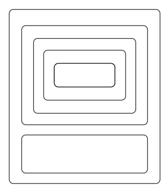
e) $7.\overline{4}$



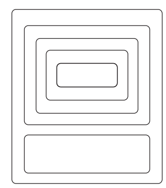
f) 4.93



g) $-\frac{2}{3}$



h) π



Example 2 For each statement, circle true or false.

a) All natural numbers are whole numbers. T F

b) All rational numbers are integers. T F

c) Some rational numbers are integers. T F

d) Some whole numbers are irrational numbers. T F

e) Rational numbers are real numbers, but irrational numbers are not. T F

Example 3 Sort the following numbers as rational, irrational, or neither.
You may use a calculator.

$\frac{1}{4}$ $\sqrt[3]{8}$ 0 $-\sqrt{2}$ $\frac{3}{0}$ $\sqrt[5]{-0.03125}$ $27^{\frac{1}{3}}$

$-\sqrt{\frac{3}{5}}$ $\sqrt{49}$ 5 $\sqrt{-2}$ $\frac{0}{3}$ $\sqrt{0.13}$ $(-2)^{\frac{1}{2}}$

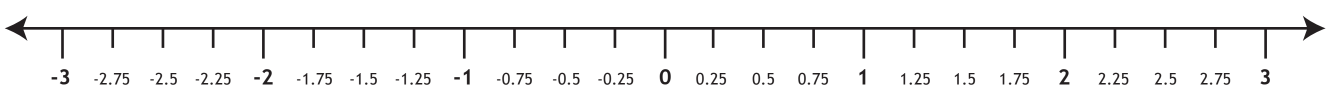
Rational	Irrational	Neither

Numbers, Radicals, and Exponents
LESSON ONE - *Number Sets*
Lesson Notes

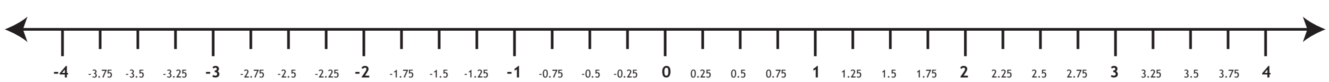
Example 4 Order the numbers from least to greatest on a number line.
You may use a calculator.

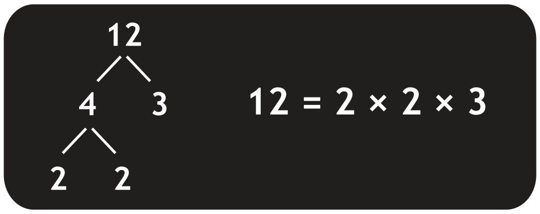
a) $-0.\overline{75}$ $\quad -\frac{1}{4}$ $\quad 1.3572...$ $\quad -\frac{1}{3}$

b) $-\sqrt{3}$ $\quad \sqrt[4]{5}$ $\quad \sqrt{\frac{4}{5}}$ $\quad \sqrt[3]{-6}$ $\quad 2\sqrt[3]{2}$ $\quad -2\sqrt{2}$ $\quad \sqrt{8}$



c) -2^2 $\quad (-2)^2$ $\quad (-3)^{\frac{1}{3}}$ $\quad 2^{\frac{1}{2}}$ $\quad 4^{-1}$ $\quad (-32)^{\frac{1}{5}}$





Numbers, Radicals, and Exponents
LESSON TWO - *Primes*, *LCM*, *and GCF*
Lesson Notes

Introduction

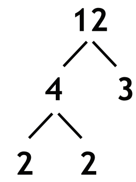
Prime Numbers, Least Common Multiple, and Greatest Common Factor.

a) What is a prime number?

b) What is a composite number?

c) Why are 0 and 1 not considered prime numbers?

Numbers, Radicals, and Exponents
LESSON TWO - *Primes, LCM, and GCF*
Lesson Notes

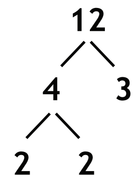


12 = 2 × 2 × 3

d) What is prime factorization? Find the prime factorization of 12.

e) What is the LCM? Find the LCM for 9 and 12 using two different methods.

f) What is the GCF? Find the GCF for 16 and 24 using two different methods.



$12 = 2 \times 2 \times 3$

Numbers, Radicals, and Exponents
LESSON TWO - *Primes*, *LCM*, *and GCF*
Lesson Notes

Example 1

Determine if each number is prime, composite, or neither.

a) 1

b) 14

c) 13

d) 0

Example 2

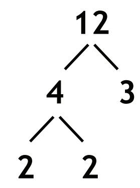
Find the least common multiple for each set of numbers.

a) 6, 8

b) 7, 14

c) 48, 180

d) 8, 9, 21



12 = 2 × 2 × 3

Example 3 Find the greatest common factor for each set of numbers.

a) 30, 42

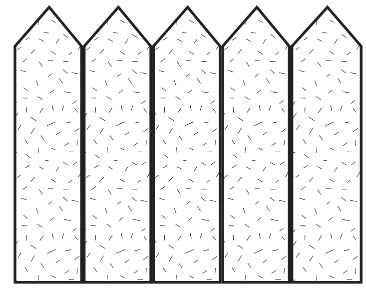
b) 13, 39

c) 52, 78

d) 54, 81, 135

Example 4

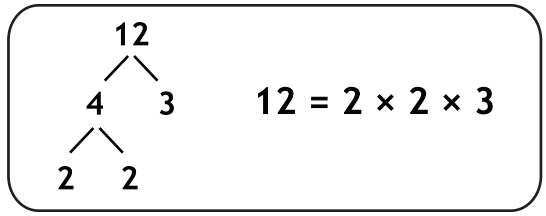
Problem solving with LCM



a) A fence is being constructed with posts that are 12 cm wide.
A second fence is being constructed with posts that are 15 cm wide.
If each fence is to be the same length, what is the shortest fence that can be constructed?

b) Stephanie can run one lap around a track in 4 minutes. Lisa can run one lap in 6 minutes.
If they start running at the same time, how long will it be until they complete a lap together?

c) There is a stack of rectangular tiles, with each tile having a length of 84 cm and a width of 63 cm.
If some of these tiles are arranged into a square, what is the smallest side length the square can have?



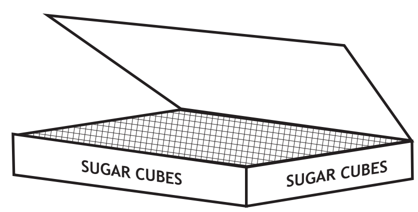
Example 5

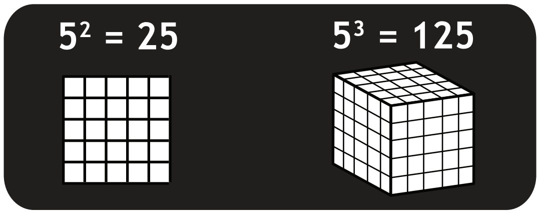
Problem solving with GCF

a) A fruit basket contains apples and oranges. Each basket will have the same quantity of apples, and the same quantity of oranges. If there are 10 apples and 15 oranges available, how many fruit baskets can be made?
How many apples and oranges are in each basket?

b) There are 8 toonies and 20 loonies scattered on a table. If these coins are organized into groups such that each group has the same quantity of toonies and the same quantity of loonies, what is the maximum number of groups that can be made? How many loonies and toonies are in each group?

c) A box of sugar cubes has a length of 156 mm, a width of 104 mm, and a height of 39 mm. What is the edge length of one sugar cube? Assume the box is completely full and the manufacturer uses sugar cubes with the largest possible volume.





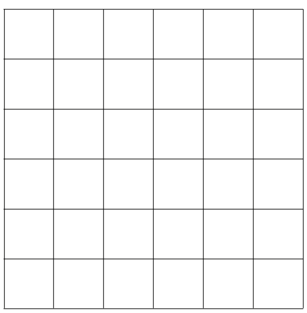
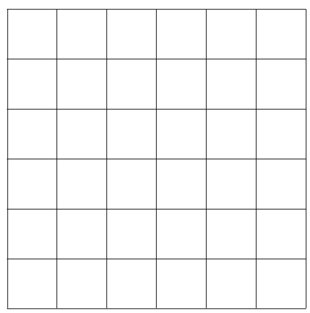
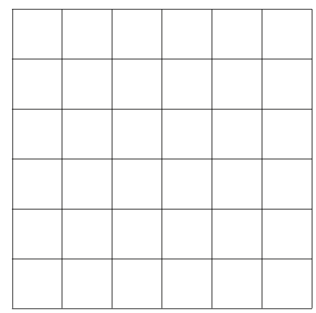
Numbers, Radicals, and Exponents

LESSON THREE - *Squares, Cubes, and Roots*

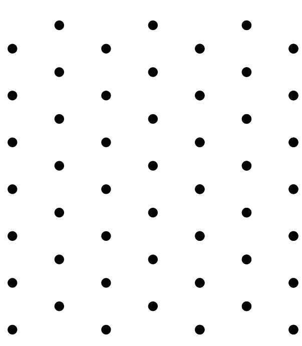
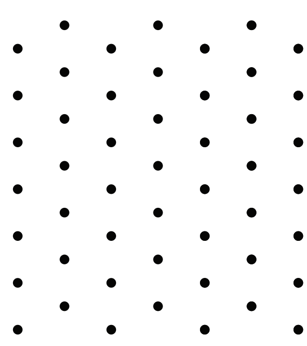
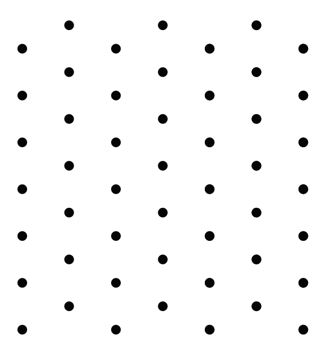
Lesson Notes

Introduction Perfect Squares, Perfect Cubes, and Roots.

a) What is a perfect square? Draw the first three perfect squares.

b) What is a perfect cube? Draw the first three perfect cubes.

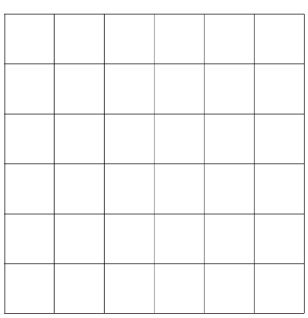




c) Complete the table showing all perfect squares and perfect cubes up to 10. The first three are completed for you.

Number	Perfect Square	Perfect Cube
1	$1^2 = 1$	$1^3 = 1$
2	$2^2 = 4$	$2^3 = 8$
3	$3^2 = 9$	$3^3 = 27$

d) What is a square root? Find the square root of 36.

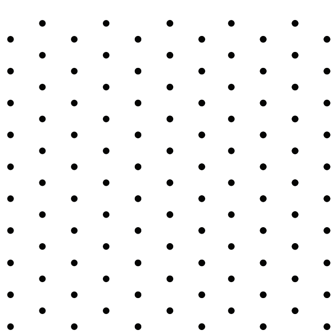
i) Using a geometric square.



ii) Using the formula $A = s^2$

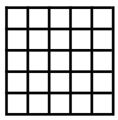
e) What is a cube root? Find the cube root of 125.

i) Using a geometric cube.

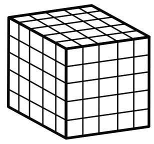


ii) Using the formula $V = s^3$

$5^2 = 25$



$5^3 = 125$



Numbers, Radicals, and Exponents

LESSON THREE - *Squares, Cubes, and Roots*

Lesson Notes

Example 1 Evaluate each power, without using a calculator.

a) 3^2

b) $(-3)^2$

c) -3^2

d) 3^3

e) $(-3)^3$

f) -3^3

Example 2 Evaluate each expression, without using a calculator.

a) $2(2)^3$

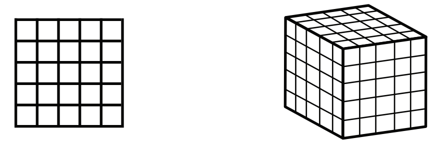
b) $-2(-4)^2$

c) $1 - 5^2$

d) $\frac{1}{4^3}$

e) $\frac{1}{2^2 + 2^3}$

f) $\frac{5(-2)^3}{-2^2}$



Example 3 Evaluate each root using a calculator.

a) $\sqrt{8}$ b) $\sqrt{-8}$ c) $\sqrt[3]{8}$ d) $\sqrt[3]{-8}$

e) What happens when you evaluate $\sqrt[4]{-8}$ and $\sqrt[5]{-8}$?
Is there a pattern as to when you can evaluate the root of a negative number?

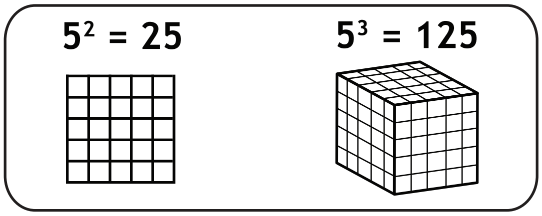
Example 4 Evaluate each expression, without using a calculator.

a) $2\sqrt{49} + \sqrt{36}$

b) $\dfrac{\sqrt{25} - \sqrt[3]{8}}{3^2}$

c) $\dfrac{1 - \sqrt{36}}{5(-2)^2}$

d) $\dfrac{3\sqrt[3]{27} - (-4)^2}{-3^2 - (-1)^2}$

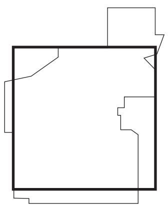


Numbers, Radicals, and Exponents
LESSON THREE - *Squares*, *Cubes*, *and Roots*
Lesson Notes

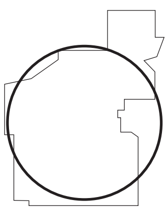
Example 5

The area of Edmonton is 684 km^2

a) If the shape of Edmonton is approximated to be a square, how wide is the city?



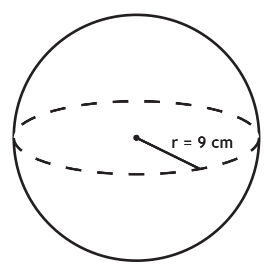
b) If the shape of Edmonton is approximated to be a circle, how wide is the city?



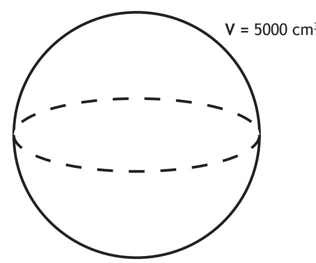
Example 6

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$

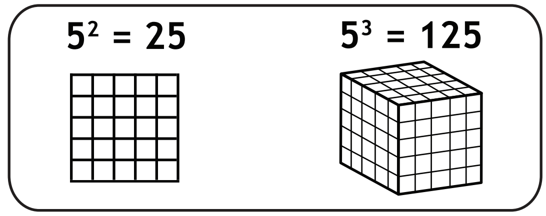
a) If a sphere has a radius of 9 cm, what is the volume?



b) If a sphere has a volume of approximately 5000 cm^3, what is the radius?



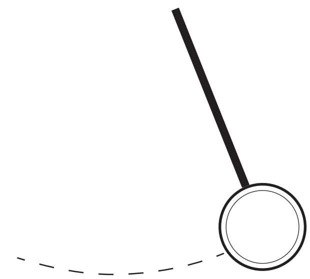
Numbers, Radicals, and Exponents
LESSON THREE - *Squares, Cubes, and Roots*
Lesson Notes



Example 7

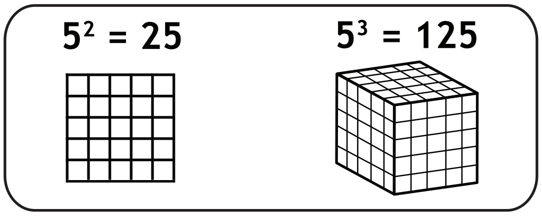
The amount of time, T, it takes for a pendulum to swing back and forth is called the period.

The period of a pendulum can be calculated with the formula: $T = 2\pi\sqrt{\frac{l}{9.8}}$



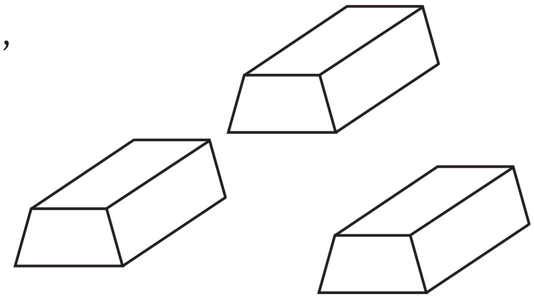
a) What is the period of the pendulum if the length, l, is 1.8 m?

b) What is the length of the pendulum if the period is 2.4 s?



Example 8 The total volume of gold mined throughout history is approximately 8340 m^3.

a) If all the gold was collected, melted down, and recast as a cube, what would be the edge length?



b) If the density of gold is 19300 kg/m^3, what is the mass of the cube?

The density formula is density = $\frac{\text{mass}}{\text{volume}}$

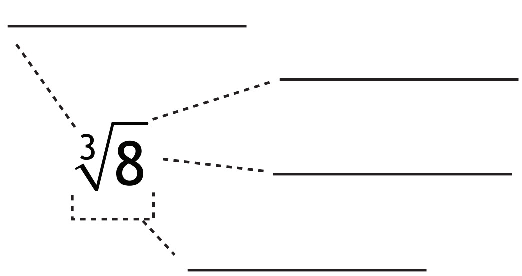
c) In 2011, 1 kg of gold costs about \$54 000. What is the value of all the gold ever extracted?

$\sqrt[3]{16} = 2\sqrt[3]{2}$

Numbers, Radicals, and Exponents
LESSON FOUR - *Radicals*
Lesson Notes

Introduction
Understanding Radicals

a) Label each of the following parts of a radical.



b) What is the index of $\sqrt{5}$?

c) What is the difference between an entire radical and a mixed radical?

d) Is it possible to write a radical without using the radical symbol $\sqrt{}$?

Numbers, Radicals, and Exponents
LESSON FOUR - *Radicals*
Lesson Notes

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Example 1 Convert each entire radical to a mixed radical.

a) $\sqrt{20}$

Prime Factorization Method	Perfect Square Method

b) $\sqrt{32}$

Prime Factorization Method	Perfect Square Method

c) $\sqrt[3]{16}$

Prime Factorization Method	Perfect Cube Method

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Example 2 Convert each entire radical to a mixed radical using the method of your choice.

a) $\sqrt{24}$

b) $\sqrt{72}$

c) $\sqrt{49}$

d) $\sqrt[3]{81}$

e) $\sqrt[3]{64}$

f) $\sqrt[4]{48}$

Numbers, Radicals, and Exponents
LESSON FOUR - *Radicals*
Lesson Notes

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Example 3 Convert each mixed radical to an entire radical.

a) $3\sqrt{3}$

Reverse Factorization Method	Perfect Square Method

b) $6\sqrt{2}$

Reverse Factorization Method	Perfect Square Method

c) $2\sqrt[3]{5}$

Reverse Factorization Method	Perfect Cube Method

$$\sqrt[3]{16} = 2\sqrt[3]{2}$$

Numbers, Radicals, and Exponents
LESSON FOUR - *Radicals*
Lesson Notes

Example 4

Convert each mixed radical to an entire radical using the method of your choice.

a) $4\sqrt{2}$

b) $5\sqrt{3}$

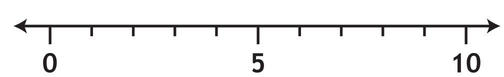
c) $3\sqrt[3]{3}$

d) $2\sqrt[4]{3}$

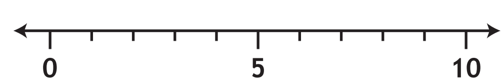
Example 5

Estimate each radical and order them on a number line.

a) $\sqrt{42}$ $\sqrt{20}$ $\sqrt{8}$ $\sqrt{14}$



b) $\sqrt[3]{92}$ $\sqrt[3]{169}$ $\sqrt[3]{54}$ $\sqrt[3]{35}$



Example 6 Simplify each expression without using a calculator.

a) $\frac{2\sqrt{12}}{4}$

b) $\frac{3\sqrt[3]{27}}{36}$

c) $\frac{3}{4}\sqrt{32}$

d) $\sqrt{\frac{49}{81}}$

e) $\frac{3\sqrt[3]{72}}{\sqrt{64}}$

Example 7 Write each power as a radical.

a) $3^{\frac{1}{2}}$

b) $(-4)^{\frac{1}{3}}$

c) $2^{\frac{4}{3}}$

d) $(-7)^{\frac{2}{5}}$

e) $\left(\frac{2}{3}\right)^{\frac{3}{2}}$

f) $16^{0.25}$

Example 8 Write each radical as a power.

a) $\sqrt{5}$

b) $\sqrt[4]{9}$

c) $\sqrt[3]{2^2}$

d) $\left(\sqrt[5]{-3}\right)^4$

e) $\left(\sqrt[3]{\frac{5}{7}}\right)^2$

f) $\sqrt{\left(\frac{3}{4}\right)^2}$

$a^m \times a^n = a^{m+n}$ $\quad (a^m b^n)^p = a^{mp}b^{np}$

$\frac{a^m}{a^n} = a^{m-n}$ $\quad \left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

$(a^m)^n = a^{mn}$ $\quad a^0 = 1$

Numbers, Radicals, and Exponents
LESSON FIVE - *Exponents I*
Lesson Notes

Introduction

Exponent Laws I

a) Product of Powers

$2^3 \times 2^4 =$

$\left(\frac{3}{4}\right)^5\left(\frac{3}{4}\right)^2 =$

General Rule:

b) Quotient of Powers

$(-6)^8 \div (-6)^5 =$

$\frac{7^9}{7^7} =$

General Rule:

c) Power of a Power

$(2^5)^3 =$

$\left((-3)^2\right)^4 =$

General Rule:

d) Power of a Product

$(a^2b^5)^3 =$

$(4a^3b^2)^4 =$

General Rule:

e) Power of a Quotient

$\left(\frac{a^3}{b^5}\right)^3 =$

$\left(\frac{2a^6}{3b^4}\right)^3 =$

General Rule:

f) Exponent of Zero

$2^0 =$

$\left(\frac{3mn^2}{7p^6q^4}\right)^0 =$

General Rule:

Numbers, Radicals, and Exponents
LESSON FIVE - *Exponents I*
Lesson Notes

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$\left(a^m\right)^n = a^{mn}$

$\left(a^m b^n\right)^p = a^{mp}b^{np}$

$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

$a^0 = 1$

Example 1

Simplify each of the following expressions.

a) $2^3 \times 2^4$

b) $\frac{3^9}{3^6}$

c) $\left(\frac{2a^2}{b}\right)^3$

d) $3\left(3^5\right)$

e) $\frac{7^4}{7}$

f) $\left(3a^2\right)^3$

$a^m \times a^n = a^{m+n}$	$\left(a^m b^n\right)^p = a^{mp} b^{np}$
$\dfrac{a^m}{a^n} = a^{m-n}$	$\left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$
$\left(a^m\right)^n = a^{mn}$	$a^0 = 1$

Example 2 Simplify each of the following expressions.

a) $5\left(3a^2b\right)$

b) $\left(4a\right)\left(4b^2\right)$

c) $\left(7a^2b^5\right)\left(-3ab^6\right)$

d) $\dfrac{36ab^2}{6b}$

e) $\dfrac{10a^8b}{15a^6c}$

f) $\dfrac{\left(3ab\right)\left(2ab\right)^2}{2\left(ab\right)^3}$

Numbers, Radicals, and Exponents
LESSON FIVE - *Exponents I*
Lesson Notes

$a^m \times a^n = a^{m+n}$ $\qquad (a^m b^n)^p = a^{mp} b^{np}$

$\dfrac{a^m}{a^n} = a^{m-n}$ $\qquad \left(\dfrac{a^m}{b^n}\right)^p = \dfrac{a^{mp}}{b^{np}}$

$(a^m)^n = a^{mn}$ $\qquad a^0 = 1$

Example 3 Simplify each of the following expressions.

a) $(3a^2b^3)^2$

b) $\left(\dfrac{4a}{5b}\right)^2$

c) $\left(\dfrac{16a^2b^5}{20ab^3}\right)^3$

d) $\left(-\dfrac{3a}{2b}\right)^0$

e) $\left(\dfrac{2a}{b}\right)^2 (ab)^0 \left(-\dfrac{1}{2}\right)^3$

f) $\dfrac{1}{25a^6}(5a^5)^2$

$a^m \times a^n = a^{m+n}$	$(a^m b^n)^p = a^{mp} b^{np}$
$\frac{a^m}{a^n} = a^{m-n}$	$\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$
$(a^m)^n = a^{mn}$	$a^0 = 1$

Example 4

For each of the following, find a value for ***m*** that satisfies the equation.

a) $(a^2)^m = a^{10}$

b) $a^{2m} \times a^8 = a^{14}$

c) $\left(\frac{a^7}{a^{3m}}\right) = a$

d) $\left(\frac{a^m \times a^{2m}}{a}\right) = a^{20}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ OR } \left(\sqrt[n]{a}\right)^m$$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

Introduction

Exponent Laws II

a) Negative Exponents

$3^{-5} =$

$(-12)^{-4} =$

General Rule:

$\frac{1}{7^{-2}} =$

$\left(\frac{2}{3}\right)^{-5} =$

b) Rational Exponents

$6^{\frac{1}{2}} =$

$(-5)^{\frac{1}{3}} =$

General Rule:

$3^{\frac{4}{5}} =$

$\sqrt{7^5} =$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 1

Simplify each of the following expressions. Any variables in your final answer should be written with positive exponents.

a) $(-4)^{-2}$

b) $\left(\frac{3}{2}\right)^{-3}$

c) $\left(\frac{a^2b}{c^3}\right)^{-1}$

d) $\left(3a^3\right)^{-2}$

e) $\left(\frac{3^{-1}}{5}\right)^{-2}$

f) $\frac{5(-4)^0}{2^{-1}}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 2

Simplify. Any variables in your final answer should be written with positive exponents.

a) $2^3(5)^{-2}$

b) $\dfrac{2^{-3}}{a^4}$

c) $\dfrac{(2a)^3}{(2a)^{-2}}$

d) $\left(a^5\right)^{-\frac{3}{5}}$

e) $\left(\dfrac{a^{-4}}{(ab)^2}\right)^{\frac{3}{2}}$

f) $\left(5a^2\right)^{-\frac{3}{2}}\left(a^{\frac{1}{2}}\right)$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 3

Simplify each of the following expressions. Any variables in your final answer should be written with positive exponents.

a) $\dfrac{10a^7b^9c^6}{5a^6b^{10}c^8}$

b) $\dfrac{-3a^{-7}b^{-11}}{12a^4b^{-3}}$

c) $\left(\dfrac{2}{5}a^{-3}b^{-1}\right)^{-3}$

d) $\left(\dfrac{4a^2b^3}{8ab^5}\right)^{-2}$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ OR } \left(\sqrt[n]{a}\right)^m$$

Example 4

Simplify. Any variables in your final answer should be written with positive exponents. Fractional exponents should be converted to a radical.

a) $\left(a^5\right)\left(a^{-\frac{1}{2}}\right)$

b) $\left(27a^{\frac{1}{2}}\right)^{\frac{2}{3}}$

c) $\left(\dfrac{9a^{-2}}{16b^{-4}}\right)^{-\frac{3}{2}}$

d) $\left(2^{-\frac{5}{4}}\right)\left(2^{-\frac{4}{3}}\right)$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 5

Simplify. Any variables in your final answer should be written with positive exponents. Fractional exponents should be converted to a radical.

a) $\dfrac{-20a^{-\frac{2}{3}}b}{4ab^{-\frac{1}{2}}}$

b) $\dfrac{2^{-3} + 2^{-4}}{2^{-5}}$

c) $\dfrac{\left(\frac{1}{16}\right)^{\frac{5}{4}}\left(\frac{1}{16}\right)^{-\frac{3}{4}}}{\left(\frac{1}{16}\right)^{-5}\left(\frac{1}{16}\right)^{4}}$

d) $9^{\frac{1}{2}}\left(\dfrac{a^{\frac{3}{4}}}{2b^{-\frac{1}{7}}}\right)^0$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ OR } \left(\sqrt[n]{a}\right)^m$$

Example 6

Write each of the following radical expressions with rational exponents and simplify.

a) $-\sqrt{a^3}$

b) $\sqrt{\sqrt{a}}$

c) $\sqrt{\sqrt[3]{a}}$

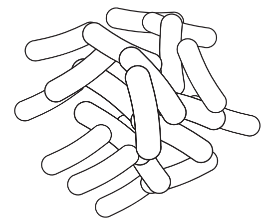
d) $\sqrt{\sqrt[3]{64a^6b^{12}}}$

Numbers, Radicals, and Exponents
LESSON SIX - *Exponents II*
Lesson Notes

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 7



A culture of bacteria contains 5000 bacterium cells. This particular type of bacteria doubles every 8 hours. If the amount of bacteria is represented by the letter A, and the elapsed time (*in hours*) is represented by the letter t, the formula used to find the amount of bacteria as time passes is:

$$A = 5000(2)^{\frac{t}{8}}$$

a) How many bacteria will be in the culture in 8 hours?

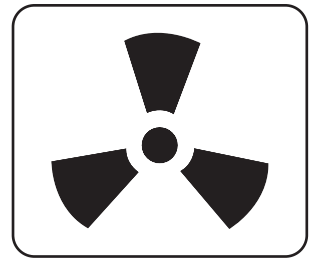
b) How many bacteria will be in the culture in 16 hours?

c) How many bacteria were in the sample 8 hours ago?

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{OR} \quad \left(\sqrt[n]{a}\right)^m$$

Example 8



Over time, a sample of a radioactive isotope will lose its mass. The length of time for the sample to lose half of its mass is called the *half-life* of the isotope. Carbon-14 is a radioactive isotope commonly used to date archaeological finds. It has a half-life of 5730 years.

If the initial mass of a Carbon-14 sample is 88 g, the formula used to find the mass remaining as time passes is given by:

$$A = 88\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

In this formula, A is the mass, and t is time *(in years)* since the mass of the sample was measured.

a) What will be the mass of the Carbon-14 sample in 2000 years?

b) What will be the mass of the Carbon-14 sample in 5730 years?

c) If the mass of the sample is measured 10000 years in the future, what percentage of the original mass remains?