Vector Analysis: Relative Velocity

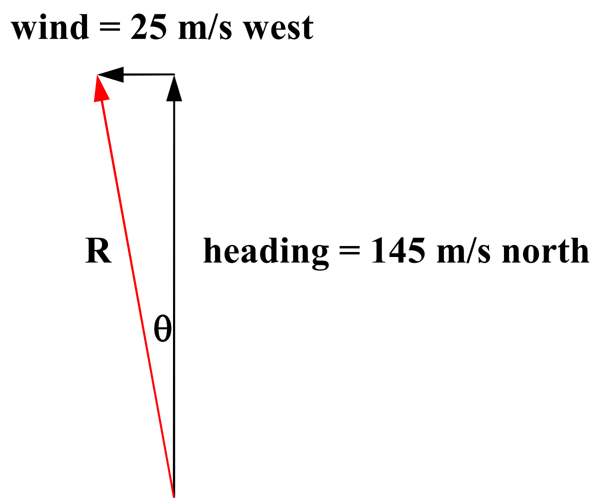
Suppose you are standing beside a highway watching car A move north at 20 m/s and car B moving south at 25 m/s. These are the observations you make as a stationary observer. However, the reality is quite different to each of the drivers in the two cars.

According to the driver in car A (i.e. relative to her), you are moving south at 20 m/s towards the car (the driver in car A thinks of herself as stationary and that you are moving). Car A also thinks that car B is moving at 45 m/s south! Similarly, the driver in Car B thinks that car A is moving at 45 m/s north.

These ideas are used to solve navigation problems for heading, displacement and time of travel.

Consider the following problem: A plane heads due north at 145 m/s. What is the plane's velocity relative to the ground if there is a 25 m/s wind blowing due west?

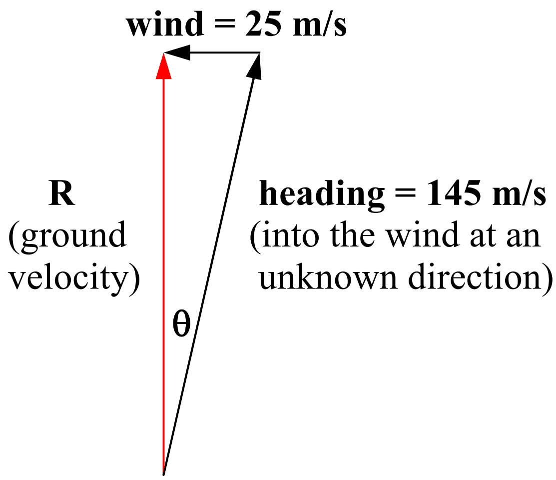
- The term "relative to the ground" describes how the plane appears to move according to an observer standing on the ground.
- To such an observer, the plane is being moved by:
  a) its engine, at 145 m/s due north (the heading);
  b) the wind, at 25 m/s due west.
- Therefore, the plane's velocity relative to the ground is simply the *resultant* of vector-adding engine heading and wind velocity. It represents the actual path of the aircraft over the ground.



- Using Pythagoras and simple trig, the plane's velocity relative to the ground is R = 147 m/s at 9.7° W of N.

Navigation involves directing a boat or plane in the correct direction to deal with wind or water currents. If the pilot in this same plane wanted to go straight north, what direction would she aim the plane (What is the heading?) Plane velocity is still 145 m/s and the wind is still 25 m/s to the west.

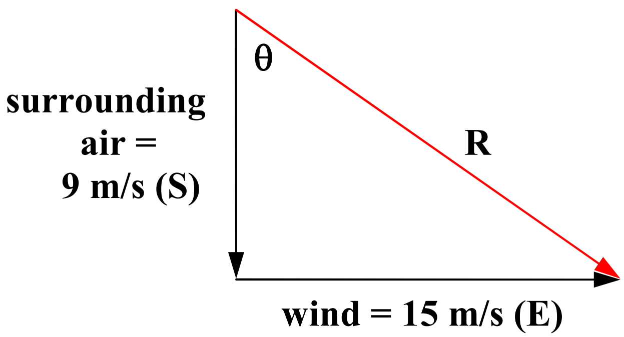
- Remember, **wind + heading = ground velocity (the resultant)**.
- In this case, we know the magnitude, but not the direction, of the heading.
- The vector-addition diagram looks like this:



- As you can see, vector-adding creates a resultant that points due north (where the pilot wants to head), and is <u>not</u> the hypotenuse.
- Using inverse-sine, the proper heading for the plane is 9.9° E of N. Note that the resultant speed in this case can also be determined, and is 143 m/s. This velocity is less than the engine's capable velocity, caused by the plane pointing into the wind.

Finally, think about this relative velocity problem: There is a 15.0 m/s wind blowing due east and you start riding your bike north at 9.0 m/s. What is the velocity of the wind in your face?

- This time, YOU are the observer. Relative to you, the bike is stationary.
- Even though you are travelling north, you observe that the ground and surrounding air is travelling *south*, at 9.0 m/s.
- You also observe the wind moving east at 15.0 m/s.
- Therefore, the apparent wind velocity that hits your face is simply the addition of these two vectors:



Remember, the 9 m/s points <u>south</u> because it represents the wind caused by your motion.

- The apparent velocity of the wind in your face is 17.5 m/s at 59° E of S.

Example 4.
A plane with an air speed of 105 m/s heads west when a 25 m/s north wind is blowing. What is the velocity of the plane relative to the ground?

(see Vectors Ex 4 for answer)

Example 5.
A plane is capable of 120 m/s is still air. Where must the pilot head the plane in order to end up going due north when there is a 35m/s west wind?

(see Vectors Ex 5 for answer)

Example 6.
A boat is capable of 12 m/s in still water. If a river flows at 7.0 m/s due east and is 500 m wide:

(a) What is the velocity of the boat relative to the shore if the boat heads south, perpendicular to the current?
(b) How long would it take to cross the river?
(c) Where would the boat have to aim in order to end up directly across from its starting point?

(see Vectors Ex 6 for answer)