F Math 12

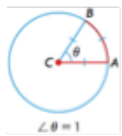
8.1 Understanding Angles p. 514

Name ____________________

Date ____________________

Goal: Estimate and determine benchmarks for angle measure.

1. **radian**: The measure of the central angle of a circle subtended by an arc that is the same length as the radius of the circle.

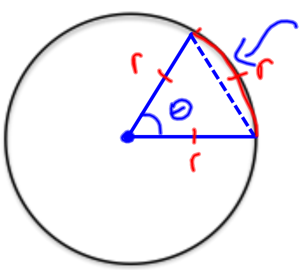


Key Ideas:

- Angles can be measured using different units. These include degrees, radians, gradients and minutes and seconds.
- Any angle measures presented a real numbers without units are considered to be in radians.

Units of Measurement for Angles

- Degrees: devised in ancient Babylon; 1 complete rotation = 360°
- Gradients: devised in 18th century; 1 complete rotation = 400 grad
- Radians: devised by mathematicians and scientists; 1 complete rotation = 2π



this side is reshaped to follow the arc of the circle

≈ approx. equal to

$$\theta = 1\ radian \approx 57.296°$$

$$2\pi\ radians \approx 6.28\ radians = 360°$$

$$\pi\ \cancel{raidians} \approx 3.14\ radians = 180°$$

radians

Example 1: Relating degrees to radians in a circle.

d → degrees
r → radians

$$\frac{d}{360^\circ} = \frac{r}{2\pi}$$

$$d = \frac{360^\circ r}{2\pi} \qquad r = \frac{2\pi d}{360^\circ}$$

$$d = \frac{180^\circ r}{\pi} \qquad r = \frac{\pi d}{180^\circ}$$

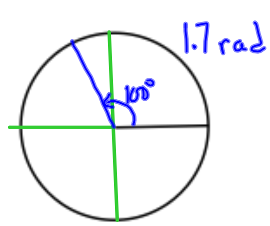
$$180^\circ = \pi \text{ rad} \Rightarrow \frac{180^\circ}{\pi} \text{ or } \frac{\pi}{180^\circ}$$

Example 2: Calculate the value of each angle in **radian** measure, to the nearest tenth, and then sketch each angle.

a. $\frac{100^\circ}{1} \times \frac{\pi}{180^\circ}$

$\frac{100\pi}{180}$

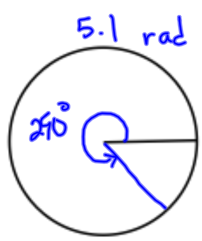
1.7 rad



b. $\frac{290^\circ}{1} \times \frac{\pi}{180^\circ}$

$\frac{290\pi}{180}$

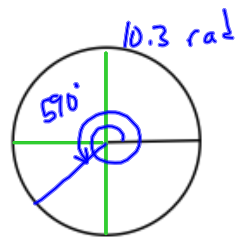
5.1 rad



c. $590^\circ \times \frac{\pi}{180}$

$\frac{590\pi}{180}$

10.3 rad



Example 3: Calculate the value of each angle in **degree** measure, to the nearest degree, and then sketch each angle.

a. $\frac{5.7696}{1} \times \frac{180°}{\pi}$

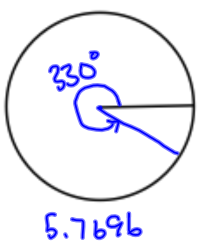
$331°$

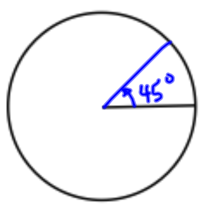
b. $\frac{0.7854}{1} \times \frac{180°}{\pi}$

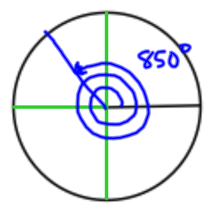
$45°$

c. $\frac{14.8353}{1} \times \frac{180°}{\pi}$

$850°$







Example 4: For each pair of angle measures, determine which measure is greater.

a. 3π radians or 8 radians

$3\pi \approx 9.42$ rad

$3\pi > 8$

b. 400° or 6.5 radians

$6.5 \times \frac{180°}{\pi} = 372°$

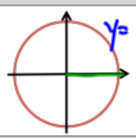
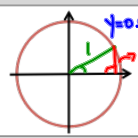
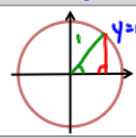
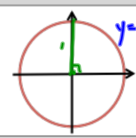
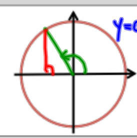
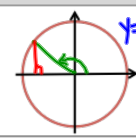
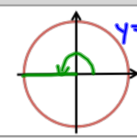
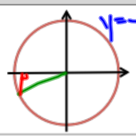
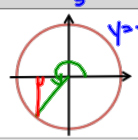
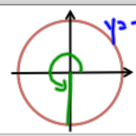
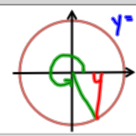
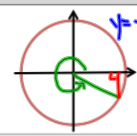
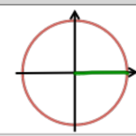
$400° > 6.5$

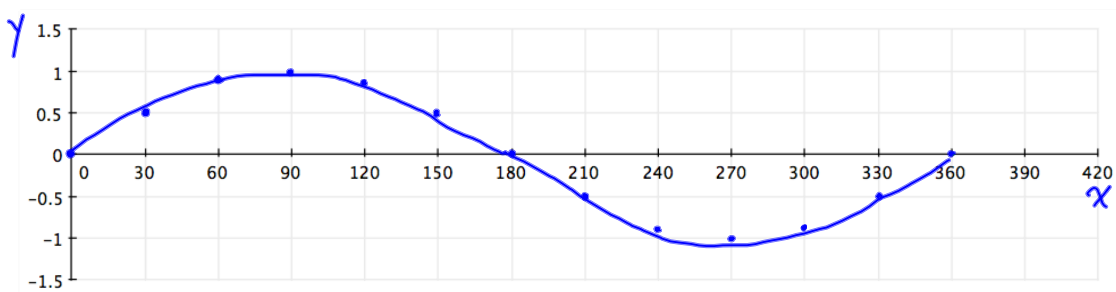
HW: 8.1 pp. 519-520 #1, 2, 4-6, 8 & 11

8.2 Exploring the Graphs of Periodic Functions (p.521)

1. Graph $y = \sin\theta$, $0° \le \theta \le 360°$

$$30° \times \frac{\pi}{180°} = \frac{\pi}{6}$$

0° or 0	30° or $\frac{\pi}{6}$	60° or $\frac{\pi}{3}$	90° or $\frac{\pi}{2}$	120° or $\frac{2\pi}{3}$	150° or $\frac{5\pi}{6}$	180° or π
 $y = 0$	 $y = 0.5$	 $y = 0.87$	 $y = 1$	 $y = 0.87$	 $y = 0.5$	 $y = 0$
210° or $\frac{7\pi}{6}$	240° or $\frac{4\pi}{3}$	270° or $\frac{3\pi}{2}$	300° or $\frac{5\pi}{3}$	330° or $\frac{11\pi}{6}$	360° or 2π	
 $y = -0.5$	 $y = -0.87$	 $y = -1$	 $y = -0.87$	 $y = -0.5$		



SOH CAH TOA

$$\sin\theta = \frac{opp}{hyp} \quad ; \quad \text{radius} = \text{hypotenuse} = 1$$

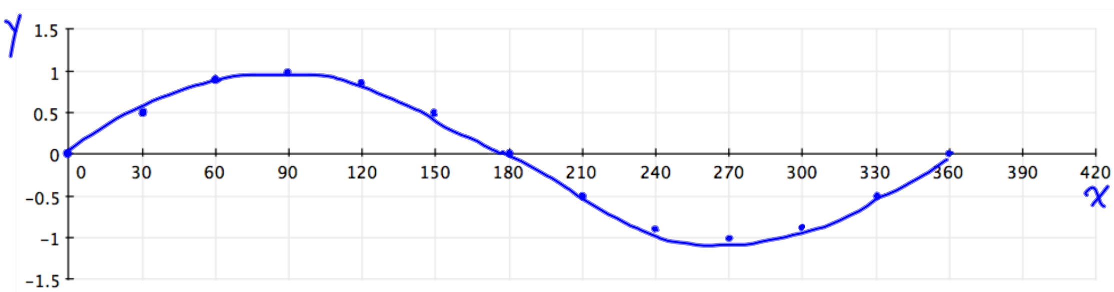
$$\sin\theta = \frac{y}{1} \rightarrow \boxed{y = \sin\theta}$$

8.2 Exploring the Graphs of Periodic Functions (p.521)

1. Graph $y = sin\theta$, $0° \le \theta \le 360°$

$30° \times \frac{\pi}{180°} = \frac{\pi}{6}$

0°or 0	30°or $\frac{\pi}{6}$	60°or $\frac{\pi}{3}$	90°or $\frac{\pi}{2}$	120°or $\frac{2\pi}{3}$	150°or $\frac{5\pi}{6}$	180°or π
$y=0$	$y=0.5$	$y=0.87$	$y=1$	$y=0.87$	$y=0.5$	$y=0$
210°or $\frac{7\pi}{6}$	240°or $\frac{4\pi}{3}$	270°or $\frac{3\pi}{2}$	300°or $\frac{5\pi}{3}$	330°or $\frac{11\pi}{6}$	360°or 2π	
$y=-0.5$	$y=-0.87$	$y=-1$	$y=-0.87$	$y=-0.5$		



SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$; radius = hypotenuse = 1

$\sin\theta = \frac{y}{1} \rightarrow \boxed{y = \sin\theta}$

F Math 12

8.2 Exploring the Graphs of Periodic Functions p. 521

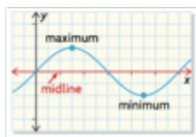
Name ____________

Date ____________

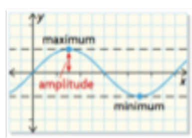
Goal: Investigate the characteristics of the graphs of sine and cosine functions.

1. **periodic function:** A function whose graph repeats in regular intervals or cycles.

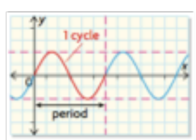
2. **midline**: The horizontal line halfway between the maximum and minimum values of a periodic function.



3. **amplitude:** The distance from the midline to either the maximum or minimum value of a periodic function; the amplitude is always expressed as a positive number.

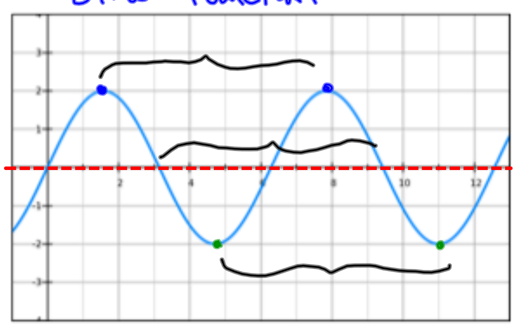


4. **period**: The length of the interval of the domain to complete one cycle.



Example 1: Correctly label the **midline**, **maximum** and **minimum** points, **amplitude** and **period** for the graphs below. State which graph is a **sine** function and which graph is a **cosine** function.

Sine function



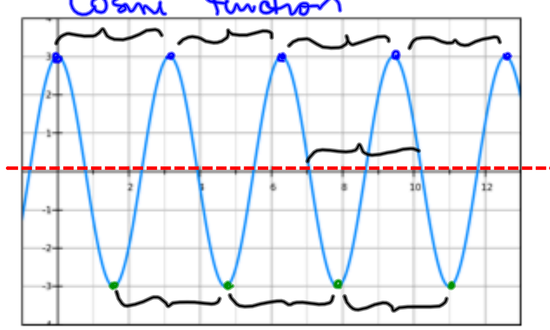
• maximum = +2

• minimum = -2

--- midline, y = 0

~ period

Cosine function



• maximum = +3

• minimum = -3

---- midline, y = 0

~ period

HW: 8.2 pp. 524-525 #1-6

F Math 12

8.3 The Graphs of Sinusoidal Functions p. 527

Name ______________

Date ______________

Goal: Identify characteristics of the graphs of sinusoidal functions.

1. **sinusoidal function:** Any periodic function whose graph has the same shape as that of $y = \sin x$.

Key Ideas:

- Range = Maximum value − Minimum value

  $R = \{y \mid \text{min.} \leq y \leq \text{max.}, y \in \mathbb{R}\}$

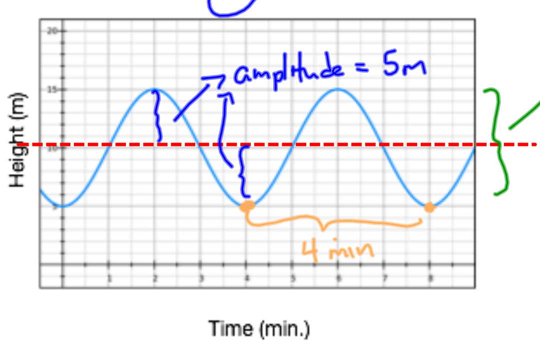
- Amplitude = $\dfrac{\text{maximum value} - \text{minimum value}}{2}$

- Equation of Midline =

  $y = \dfrac{\text{maximum value} + \text{minimum value}}{2}$

- Period: horizontal distance between consecutive maximum values or minimum values

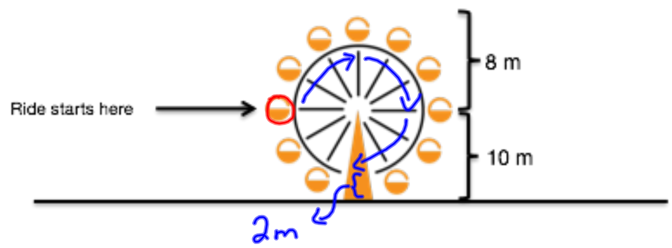
Example 1: The sine curve below shows a person's height above the ground as the person rides a ferris wheel. Label the **range**, **amplitude**, **midline** and **period**.



Range: $15 - 5 = 10\text{m} \Rightarrow \{y \mid 5 \leq y \leq 15, y \in \mathbb{R}\}$

midline, $y = 10\text{m}$

Example 2: The diagram below displays some of the key information about a particular Ferris wheel. One ride last 600 s and completes 10 rotations.



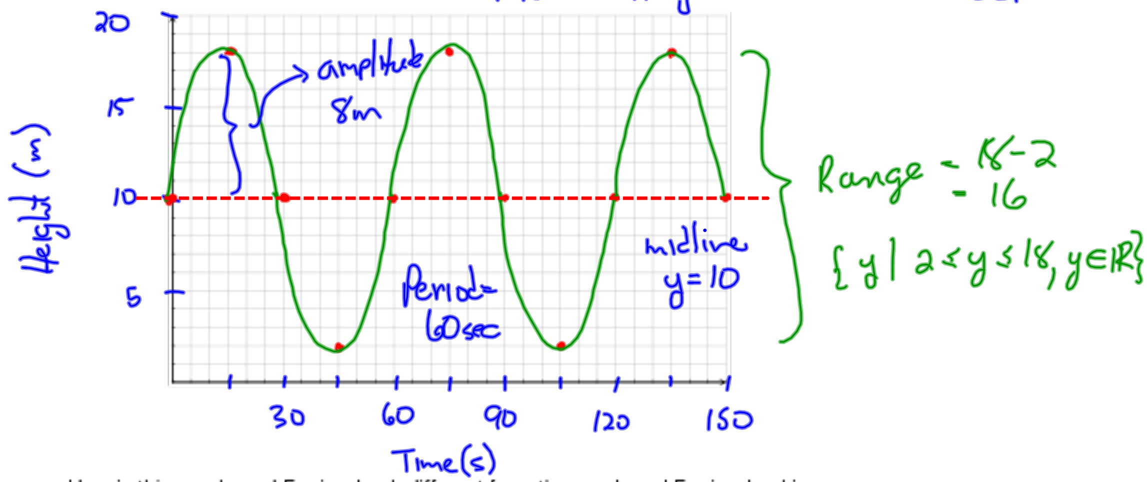
$$= \frac{600 \text{ sec}}{10 \text{ rotations}} = \frac{60 \text{ sec}}{1 \text{ rotation}}$$

a. Complete the table below to show a rider's height above the ground.

Time on ride (s)	0	15	30	45	60	75	90
Height above the ground (m)	10	18	10	2	10	18	10

b. Sketch a graph to represent the rider's height above the ground during the ride. Label the **range, amplitude, midline** and **period**.

Rider's Height on Ferris Wheel

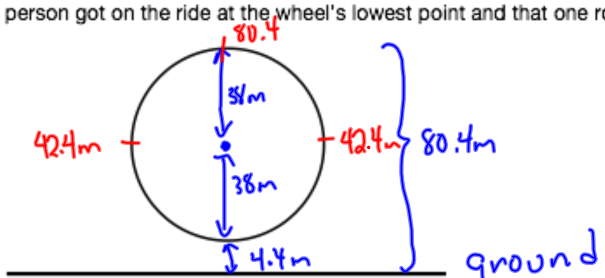


c. How is this graph, and Ferris wheel, different from the graph and Ferris wheel in Example 1?

Ferris wheel #2 has a bigger diameter (range)
Its radius is larger (radius = amplitude) and
it rotates faster (shorter period)

Example 3: The original Ferris wheel, designed by George Ferris in 1893, could carry 2 160 people at a time. It had a maximum height of 80.4 m and a radius of 38 m.

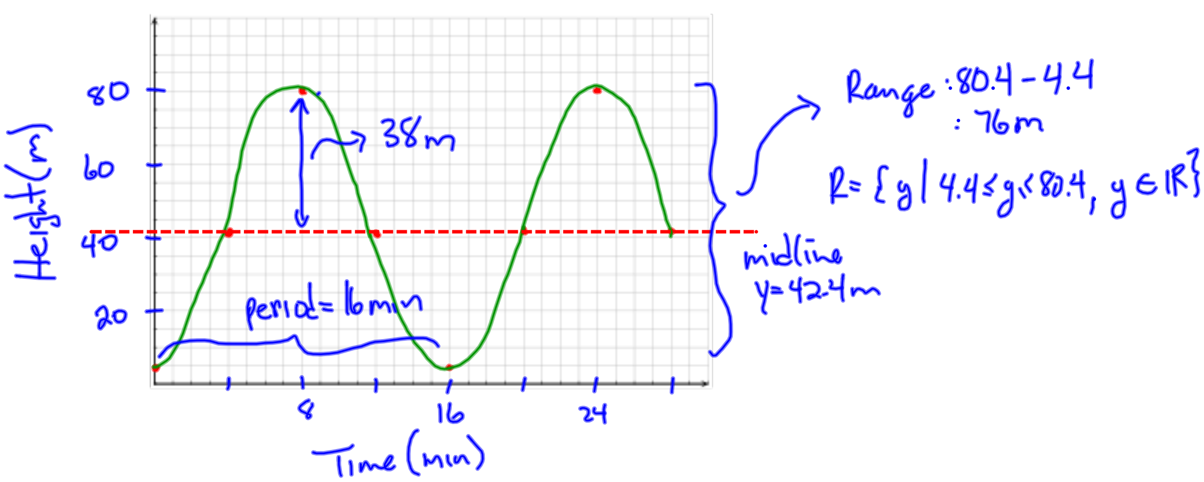
a. Fill in the table below for the height above the ground of a person on the Ferris wheel. Assume that the person got on the ride at the wheel's lowest point and that one rotation took 16 min.



Time on ride ~~(s)~~ (min)	0	4	8	12	16	20	24
Height above the ground (m)	4.4	42.4	80.4	42.4	4.4	42.4	80.4

b. Sketch a graph to represent the rider's height above the ground during the ride. Label the **range**, **amplitude**, **midline** and **period.**

Rider's Height on Ferris Wheel



HW: 8.3 pp. 536-542 #4, 6, 8 & 9

January 17, 2013

F Math 12

8.4 The Equations of Sinusoidal Functions p. 546

Name ______________

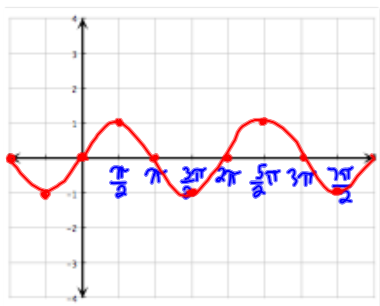
Date ______________

Goal: Identify characteristics of the equations of sinusoidal functions.

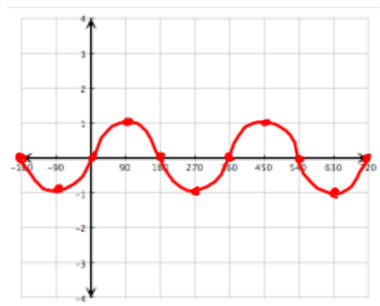
Investigating the characteristics of $y = a\sin b(\theta - c) + d$

$y = \sin\theta \rightarrow y = 1\sin 1(\theta - 0) + 0$

Radians



Degrees



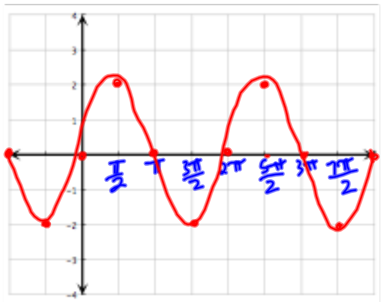
Amplitude = 1

Period = 2π

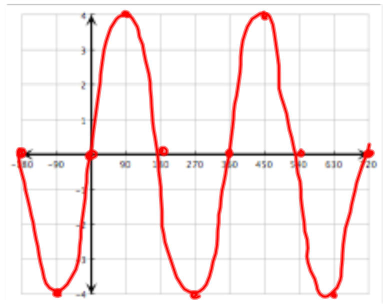
Amplitude = 1

Period = 360°

$y = 2\sin\theta$ $\quad (a = 2)$



$y = 4\sin\theta$ $\quad (a = 4)$



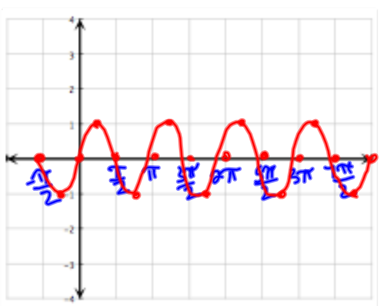
$y = \boldsymbol{a}\sin\theta$ What does the value of "$\boldsymbol{a}$" do to the **original** ($y = sin\theta$) sine function?

Changes the amplitude

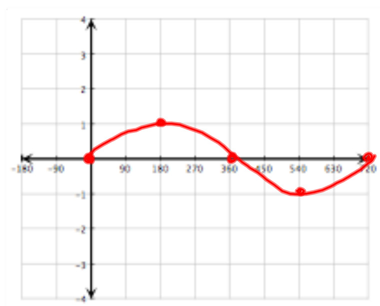
$y = 2\sin\theta$, amplitude $= 2$

$y = 4\sin\theta$, amplitude $= 4$

$y = \sin 2\theta$ $\quad (b = 2)$



$y = \sin 0.5\,\theta$ $\quad (b = 0.5)$



$y = \sin \boldsymbol{b}\theta$ What does the value of "$\boldsymbol{b}$" do to the **original** ($y = sin\theta$) sine function?

Changes the period ; radians $\rightarrow$ period $= \frac{2\pi}{b}$

degrees $\rightarrow$ period $= \frac{360°}{b}$

$y = \sin 2\theta$; period $= \frac{2\pi}{2} = \pi$

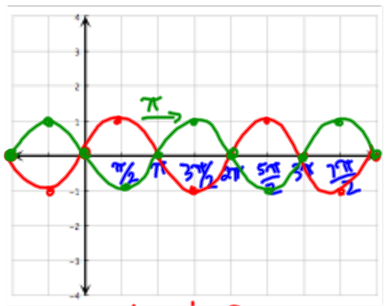
$y = \sin 0.5\theta$; period $= \frac{360°}{0.5} = 720°$

$y = \sin(\theta - (-90°))$ *

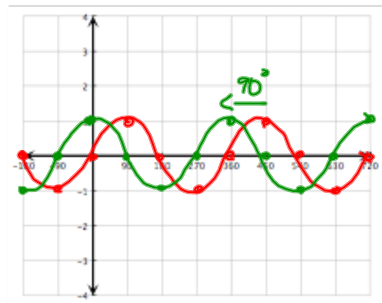
$y = \sin(\theta - \pi)$ ($c = \pi$)

$y = \sin(\theta + 90°)$ ($c = -90°$)

* always change a + sign into a double negative



$y = \sin\theta$



$y = \sin\theta$

$y = \sin(\theta - \boldsymbol{c})$ What does the value of "***c***" do to the **original** ($y = sin\theta$) sine function?

shifts the graph left or right (phase shift)

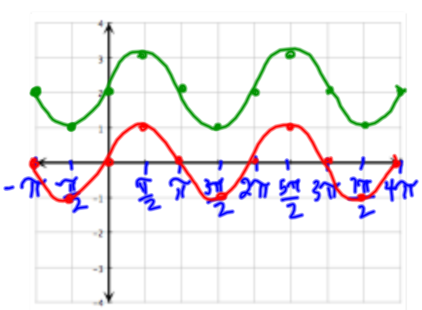
$y = \sin(\theta - \pi)$; shift right π units

$y = \sin(\theta + 90°)$; shift left 90°

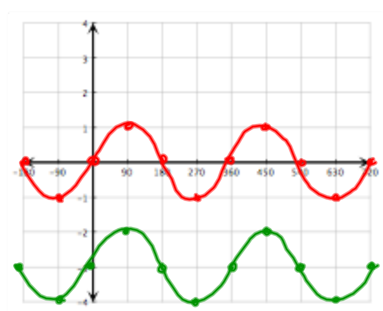
* shifts the opposite direction of the "sign" when written in brackets

$y = \sin\theta + 2$ ($d = 2$)

$y = \sin\theta - 3$ ($d = -3$)



$y = \sin\theta$



$y = \sin\theta$

$y = \sin\theta + \boldsymbol{d}$ What does the value of "***d***" do to the **original** ($y = sin\theta$) sine function?

shifts the graph up or down

$y = \sin\theta + 2$; shift up 2

$y = \sin\theta - 3$; shift down 3

* shifts same direction as "sign" when # is outside the brackets

$\cos\theta = \sin(90° - \theta)$

$\cos\theta = \sin(\pi - \theta)$

$$y = a \sin b(\theta - c) + d$$

amplitude

period

period $= \frac{2\pi}{b}$

period $= \frac{360°}{b}$

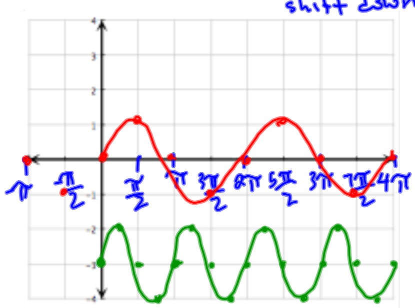
horizontal shift

vertical shift

$$y = a \cos b(\theta - c) + d$$

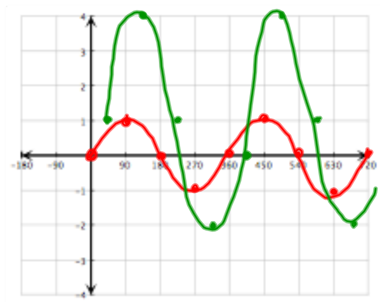
$y = \sin 2\theta - 3$

amp. $= 1$
period $= \pi$
shift down "3"



$y = 3\sin(\theta - 45°) + 1$

amp. $= 3$
period $= 360°$
shift right $= 45°$
shift up 1



$y = \sin\theta$

HW: 8.4 pp. 558-561 1-4, 5-9,12, 13 & 14