Vectors and Kinematics Notes

1 – Review

Velocity is defined as the change in displacement with respect to time.

$$\boxed{\vec{V} = \frac{\Delta \vec{d}}{\Delta t}}$$

remember
"$\rightharpoonup$" means vector
"Δ" means change in

Note that this formula is only valid for finding **constant** velocity or **average** velocity. Also, if acceleration is constant:

$$\boxed{V_{avg} = \frac{V + V_0}{2}}$$

Ex: A sprinter runs from the 50.0 m mark to the 100.0 m mark in 4.50 s, what is his velocity?

$$V = \frac{\Delta d}{t} = \frac{d_f - d_i}{t} = \frac{100.0\,m - 50.0\,m}{4.50\,s}$$
$$= 11.1\ m/s$$

Ex: A car traveling at 22 m/s slows down to 14 m/s in 3.00 s. What is its average velocity during this time?

$$V_{avg} = \frac{V + V_0}{2} = \frac{22\ m/s + 14\ m/s}{2} = 18\ m/s$$

Whenever an object undergoes acceleration, we need to rely on our 3 kinematics equations. The variables for these are:

v : final velocity
v_o : initial velocity
a : acceleration
d : displacement
t : time

There are three kinematics equations that use these variables.

1) $\boxed{V = V_0 + at}$

2) $\boxed{d = V_0 t + \frac{1}{2}at^2}$

3) $\boxed{V^2 = V_0^2 + 2ad}$

Ex: A jet traveling at 65 m/s accelerates at 25 m/s^2 for 8.00 s. What is its final velocity?

$V = ?$
$V_0 = 65\ m/s$
$a = 25\ m/s^2$
$d =$
$t = 8.00\ s$

$$V = V_0 + at$$
$$V = 65\ m/s + (25\ m/s^2)(8.00\ s)$$
$$= 265\ m/s$$

Ex: A textbook is dropped from a high cliff and hits the ground 3.5 s later. What is the book's displacement?

$V =$
$V_0 = 0\ m/s$
$a = -9.80\ m/s^2$
$d = ?$
$t = 3.5\ s$

$$d = \cancel{V_0 t}^{\,0} + \frac{1}{2}at^2$$
$$= \frac{1}{2}(9.80)(3.5)^2$$
$$= -60.\ m$$

Remember: acceleration due to gravity *near the Earth's surface* is the same for all objects regardless of mass!!!

g = $-9.80\ m/s^2$

Note:
We generally assign up and right as "+" down and left as "–"

Ex: A student throws a ball straight up in the air at 14.2 m/s. What is its velocity when it is 6.0 m above its point of release?

$V = ?$
$V_0 = 14.2\ m/s$
$a = -9.80\ m/s^2$
$d = 6.0\ m$
$t =$

$$V^2 = V_0^2 + 2ad$$
$$V = \sqrt{V_0^2 + 2ad}$$
$$= \sqrt{(14.2\ m/s)^2 + 2(-9.80)(6.0)}$$
$$= \pm 9.2\ m/s$$

wait... what? Why is that?

Note: Displacements, velocities and accelerations can all be negative because they are *vectors*, which have both a magnitude and direction.

<u>Vector and Kinematics Notes</u>

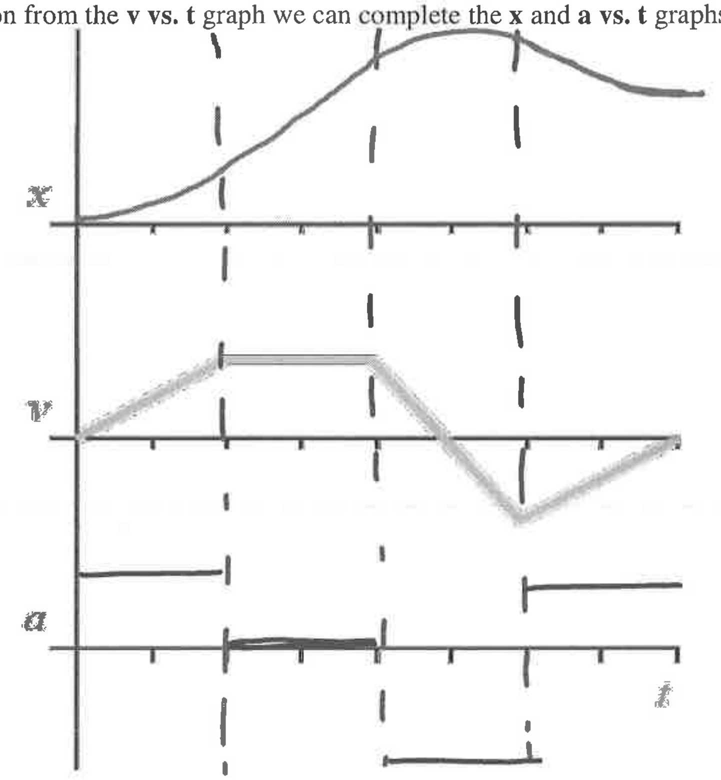
3 - Graphs

There is certain information that can be taken from position vs. time (d vs. t) and velocity vs. time (v vs. t) graphs.

For Example:

<u>d vs. t graphs</u>:	<u>v vs. t graphs</u>:
slope = velocity	slope = acceleration area under curve = displacement

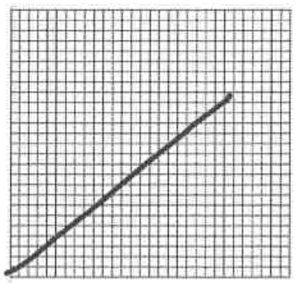
Given the information from the **v vs. t** graph we can complete the **x** and **a vs. t** graphs

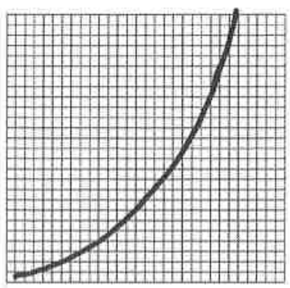


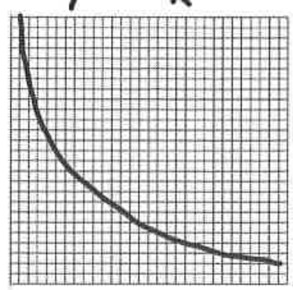
In Physics 12 you will be expected to perform more advanced graphical analysis on tests and in labs. EVERY time you make a graph you should follow the following rules.

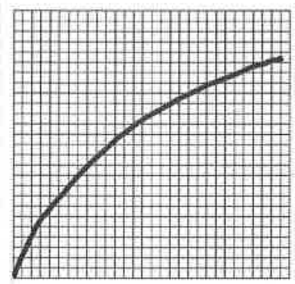
- Label the axis
  - Manipulated variable on the x-axis
  - Responding variable on the y-axis
- Give the graph an appropriate title.
- Scale each axis
  - Use… as much grid as possible
  - Choose a scale that is… easy to read
  -
- Plot the points and draw a best fit curve.
- Determine if the curve is linear or not

Direct:	Quadratic :	Inverse / Inverse Square	Square Root:
$y \propto x$	$y \propto x^2$	$y \propto \frac{1}{x}$ / $y \propto \frac{1}{x^2}$	$y \propto \sqrt{x}$









Finding Slope

To find the slope of a straight line:

- Choose…2 points
- Choose them as…as far apart as possible
- Use only…points on the line NO DATA POINTS!

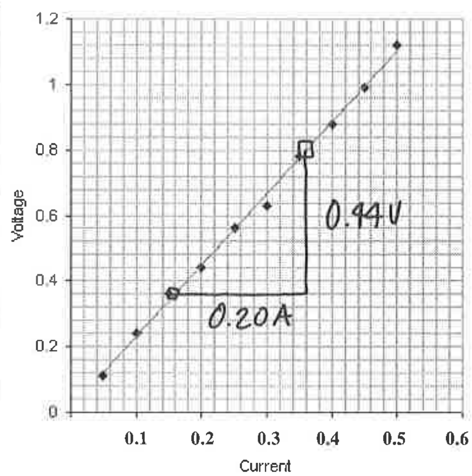
Remember the equation of a line is:

$$\boxed{y = mx + b}$$

Determine the slope and y-intercept of the graph shown and write the equation describing this line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0.44\,V}{0.20\,A} = 2.2\ V/A$$

y-int = 0



Curve Straightening

Ex 1: A car starts at a certain speed and accelerates uniformly. A student collects data of velocity at different displacements.

v	d
⋮	⋮

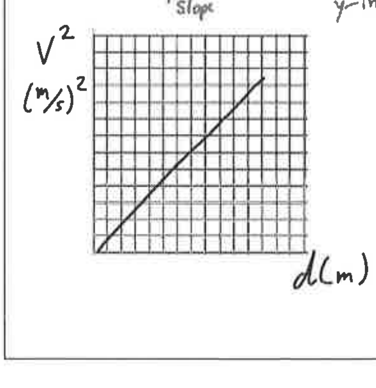
$v^2 = v_0^2 + 2ad$

so $v^2 \propto d$

$y = mx + b$

$v^2 = 2ad + v_0^2$

slope y-int



Ex 2: An astronaut standing on an asteroid measures the force of gravity acting on a 10 kg mass at different distances from the center of the asteroid.

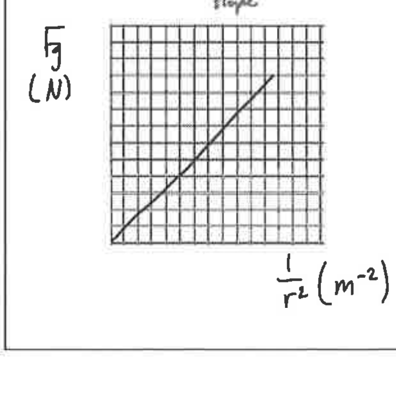
Fg	r
⋮	⋮

$F_g = \frac{Gm_1m_2}{r^2}$ so $F_g \propto \frac{1}{r^2}$

$y = mx + b$

$F_g = Gm_1m_2 \cdot \frac{1}{r^2}$

slope



Ex 3: A student pushes a wooden block over a rough surface with different amounts of force and measures the acceleration each time.

F_{app}	a
⋮	⋮

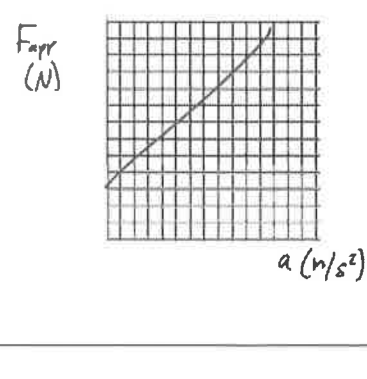
$F_{app} - F_f = ma$

$F_{app} \propto a$

$y = mx + b$

$F_{app} = ma + F_f$

slope y-int



Vectors and Kinematics Notes

4 – Vector Addition and Subtraction

SCALAR	VECTOR
speed	velocity
time	displacement
distance	acceleration
mass	force
temperature	momentum
energy	

When we draw vectors we represent them as arrows.

Vector Addition

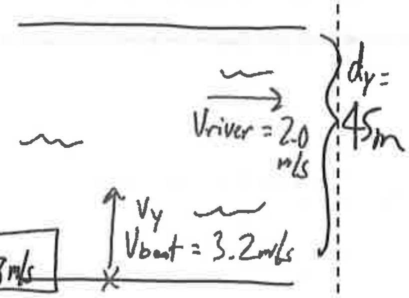
Whenever we add vectors we use...
tip to tail method

To find the total or resultant vector, simply draw...
an arrow from the start to the finish

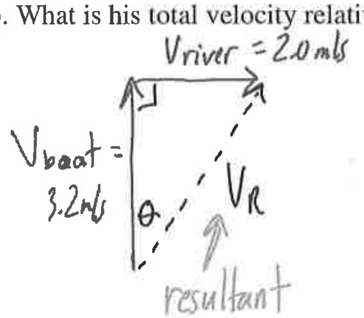
Ex: A student in a canoe is trying to cross a 45 m wide river that flows due East at 2.0 m/s. The student can paddle at 3.2 m/s

a. If he points due North and paddles how long will it take him to cross the river?

$V_y = \frac{d_y}{t}$ $\quad t = \frac{d_y}{V_y} = \frac{45m}{3.2m/s} = \boxed{14s}$



b. What is his total velocity relative to his starting point in part a?



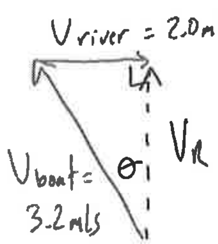
$V_R^2 = V_{boat}^2 + V_{river}^2$

$V_R = \sqrt{(3.2)^2 + (2.0)^2} = 3.77 m/s = \boxed{3.8 m/s}$

$\tan\theta = \frac{2.0}{3.2}$ $\quad \theta = \tan^{-1}\left(\frac{2.0}{3.2}\right) = 32°$ E of N

$\boxed{3.8 m/s\ 32°\ E of N}$

c. If he needs to end up directly North across the river from his starting point, what heading should he take?



$\sin\theta = \frac{2.0}{3.2}$ $\quad \sin^{-1}\left(\frac{2.0}{3.2}\right) = 39°$ W of N

d. How long will it take him to cross the river at this heading?

$V_{boat}^2 = V_R^2 + V_{river}^2$ $\quad V_R = \sqrt{V_{boat}^2 - V_{river}^2} = \sqrt{(3.2)^2 - (2.0)^2} = 2.50 m/s$

$V_y = \frac{d_y}{t}$ $\quad t = \frac{d_y}{V_y} = \frac{45m}{2.50m/s} = \boxed{18 s}$

Vector Addition – Trig Method

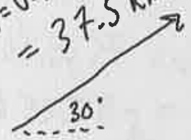
In the previous example we added perpendicular vectors which gave us a nice simple right triangle. In reality it's not always going to be that easy.

Ex. A zeppelin flies at 15 km/h 30° N of E for 2.5 hr and then changes heading and flies at 20 km/h 70° W of N for 1.5 hr. What was its final displacement?

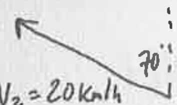
$v_1 = 15\ \text{Km/h}$

$d_1 = v \cdot t = (15)(2.5)$

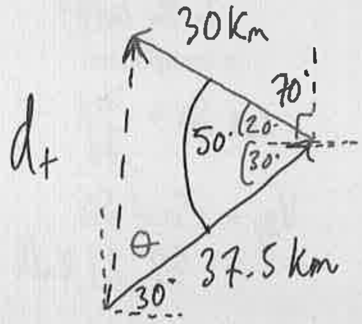
$= 37.5\ \text{Km}$



$V_2 = 20\ \text{Km/h}$



$d_2 = (20)(1.5) = 30\ \text{Km}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$= (37.5)^2 + (30)^2 - 2(37.5)(30)\cos 50°$

$\sqrt{c^2} = \sqrt{859.98}$

$c = 29.3\ \text{Km}$

$\frac{\sin\theta}{30} = \frac{\sin 50}{29.3}$ $\quad \theta = \sin^{-1}\left(\frac{30\sin 50}{29.3}\right)$

$= 52°$

$d_t = 29.3\ \text{Km}\ 82°\ \text{N of E}$

or

$8°\ \text{E of N}$

In order to solve non-right angle triangles, we will need to be familiar with the **Sine Law** and the **Cosine Law.**

Sine Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Law:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Vector Addition – The Component Method

There is another method that we can use when adding vectors. This method is a very precise, stepwise approach, however it is the only way we can add 3 or more vectors.

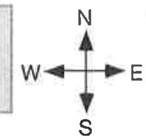
- **Draw** each vector
- **Resolve** each vector into x and y components
- Find the **total sum** of x and y vectors
- **Add** the x and y vectors
- **Solve** using trig

REMEMBER: When using x and y components…

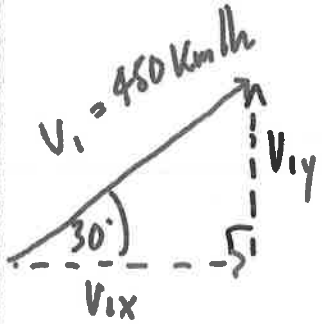
- up and right are "+"
- down and left are "−"

Ex. An airplane heading at 450 km/h, 30° ~~north~~ of east encounters a 75 km/h wind blowing towards a direction 50° west of north. What is the resultant velocity of the airplane relative to the ground?

total



Airplane vector:



x-component:

$\cos 30° = \frac{V_{1x}}{450}$

$V_{1x} = 450 \cos 30°$

$= 389.71 \text{ Km/h}$

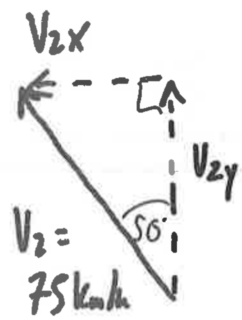
y-component:

$\sin 30° = \frac{V_{1y}}{450}$

$V_{1y} = 450 \sin 30°$

$= 225 \text{ Km/h}$

Wind vector:



x-component:

$\sin 50° = \frac{V_{2x}}{75}$

$V_{2x} = 75 \sin 50°$

$= -57.45 \text{ Km/h}$

to the left!

y-component:

$\cos 50° = \frac{V_{2y}}{75}$

$V_{2y} = 75 \cos 50°$

$= 48.21 \text{ Km/h}$

Adding the two vectors:

x-components of resultant:

$\sum V_x = V_{1x} + V_{2x}$

$= 389.71 + (-57.45)$

$= 332.26 \text{ km/h}$

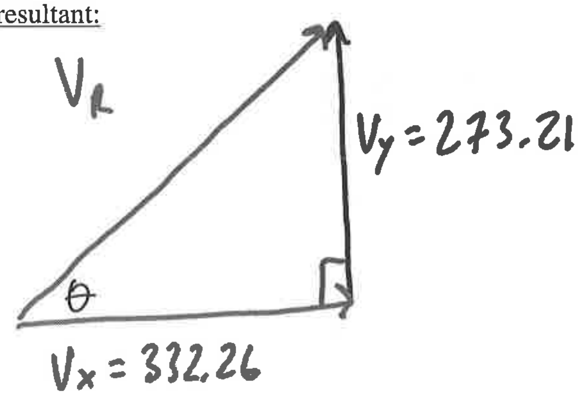
y-components of resultant:

$\sum V_y = V_{1y} + V_{2y}$

$= 225 + 48.21$

$= 273.21 \text{ Km/h}$

Total resultant:



$V_R = \sqrt{V_x^2 + V_y^2} = 430 \text{ Km/h}$

$\theta = \tan^{-1}\left(\frac{273.21}{332.26}\right) = 39° \text{ N of E}$

Vector Subtraction

With vectors a negative sign indicates that…it points in the exact opposite direction

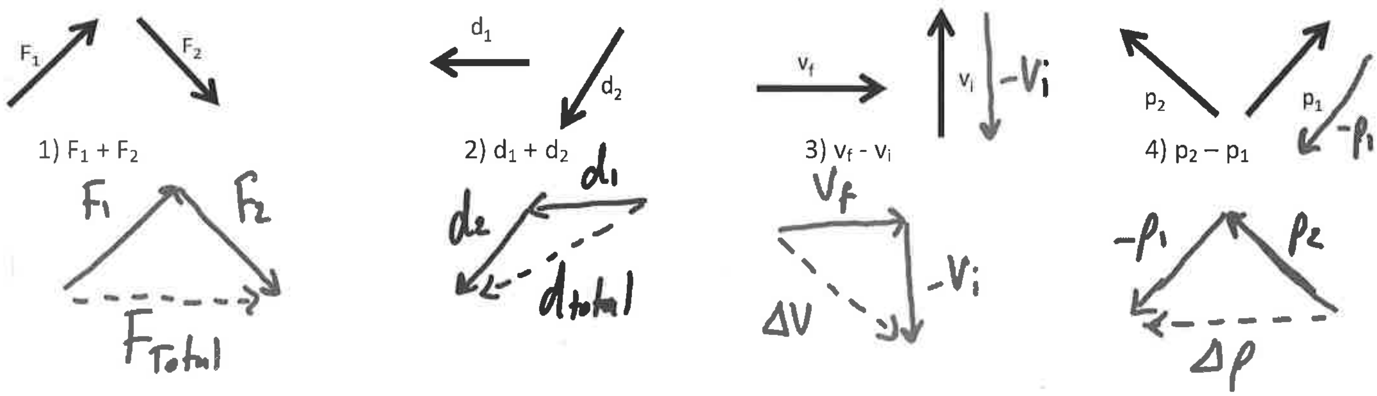
When subtracting vectors we still draw them *tip to tail*, except…we reverse the negative vector

We generally subtract vectors when dealing with a change in a vector quantity.

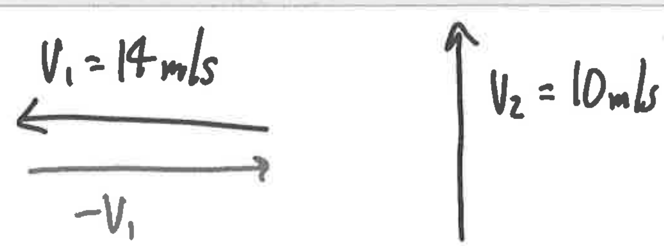
Recall:

Change = final − initial $\qquad \vec{\Delta V} = \vec{V_f} - \vec{V_i}$

Draw the Following

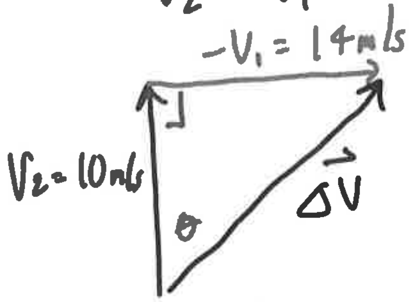


Ex: A cyclist is traveling at 14 m/s west when he turns due north and continues at 10 m/s. If it takes him 4.0 s to complete the turn what is the magnitude and direction of his acceleration?



$$a = \frac{\Delta V}{t} = \frac{V_f - V_i}{t}$$

$$\vec{\Delta V} = \vec{V_2} - \vec{V_1}$$



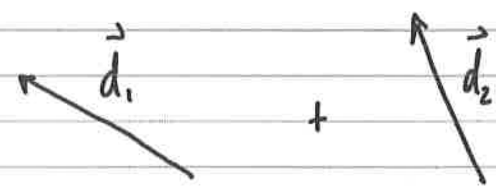
$$\Delta V = \sqrt{V_2^2 + V_1^2} = 17.2 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{14}{10}\right) = 54° \text{ E of N}$$

$$a = \frac{\Delta V}{t} = \frac{17.2 \text{ m/s}}{4.0 \text{ s}} = 4.3 \text{ m/s}^2 \; 54° \text{ E of N}$$

Summary of Methods

Say we want to add two vectors $\vec{d}_1 + \vec{d}_2$

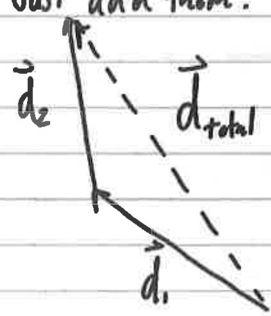


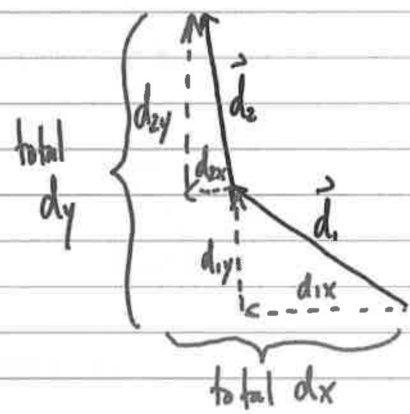
You have 2 choices

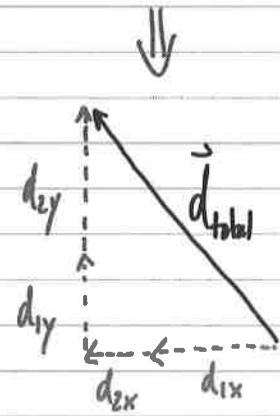
1) <u>Trig Method</u>

2.) <u>Component Method</u>

Just add them!



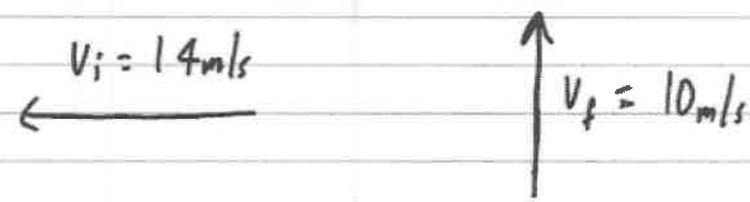




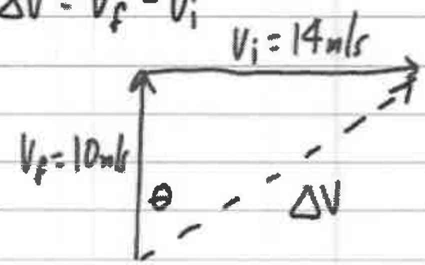
Relative Velocity and Navigation

Ex: A cyclist is traveling at 14 m/s west when he turns due north and continues at 10 m/s. If it takes him 4.0 s to complete the turn what is his average acceleration?

Ex 4



$a = \frac{\Delta V}{t} \longrightarrow \Delta V = V_f - V_i$



$\Delta V = \sqrt{V_f^2 + V_i^2} = 17.2\, m/s$

$\theta = \tan^{-1}\left(\frac{14}{10}\right) = 54^\circ$ E of N

$a = \frac{\Delta V}{t} = \frac{17.2\, m/s}{4.0\, s} = 4.3\, m/s^2$ 54° E of N

Whenever we talk about displacement or velocity, we must specify the reference point or reference frame from which it is measured, to avoid confusion about the values of the d or v.

For example, suppose a person is traveling on a train at 50 km/h. The person walks in the direction of the train's motion with a speed of 5 km/h. Then a person outside of the train will see the person walking at a speed of 55 km/h.

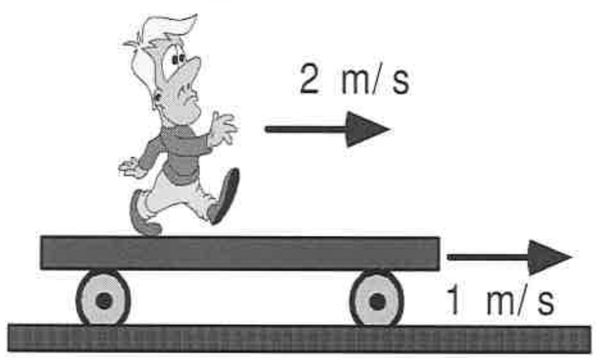
How can two people see different values for the same person?

When inside the train, our measurement of the speed is “relative to” the train

While outside, our measurement of the speed is “relative to” Earth.

Calculating Relative Velocity with Reference Frames

A man walks to the right with a velocity of 2 m/s on a platform that moves with a velocity of 1 m/s to the right.



a) What is the person's velocity relative to the platform?

Reference frame: platform

$v_{person\text{-}platform}$ = 2 m/s

b) What is the person's velocity relative to the ground?

Reference frame: ground

$v_{person\text{-}ground} = v_{platform} + v_{person\text{-}platform}$

$v_{person\text{-}ground}$ = 1 m/s + 2 m/s

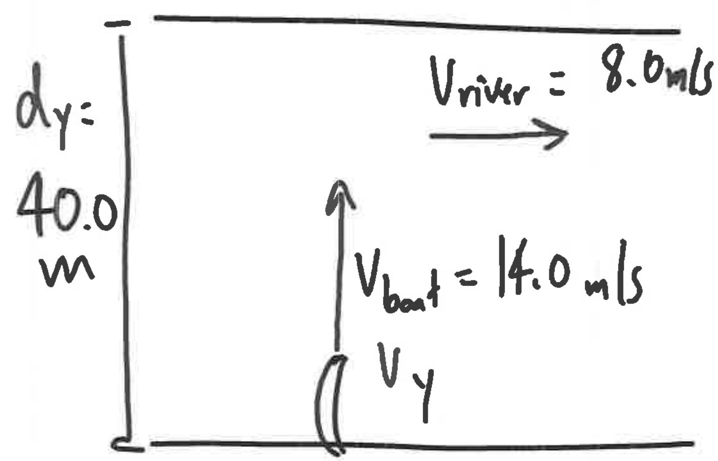
$v_{person\text{-}ground}$ = 3 m/s

In 1-D this we can simply add the vectors, however in 2-D we need to construct vector diagrams.

Ex: A student in a canoe is trying to cross a 40.0 m wide river that flows to the east at 8.0 m/s. The student can paddle at 14.0 m/s.

a) If he points due north and paddles, how long will it take him to cross?

NOTE: Perpendicular components don't affect each other. We only consider the width of the river and his velocity in that direction.



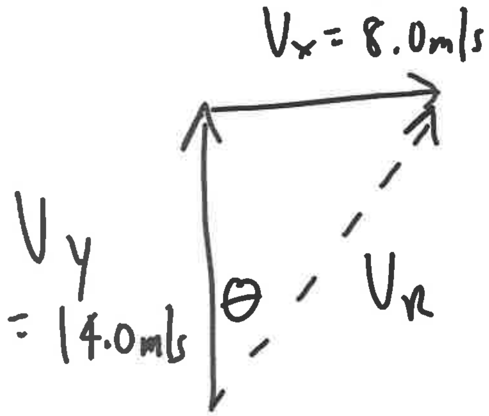
$$V_y = \frac{d_y}{t}$$

$$t = \frac{d_y}{V_y} = \frac{40.0\text{ m}}{14.0\text{ m/s}} = 2.86\text{ s}$$

In 1-D this we can simply add the vectors, however in 2-D we need to construct vector diagrams.

b) What will be his velocity (relative to his starting point) in part a?

NOTE: Add the velocities with vector addition



$$V_R = \sqrt{V_x^2 + V_y^2} = \sqrt{(14.0)^2 + (8.0)^2}$$

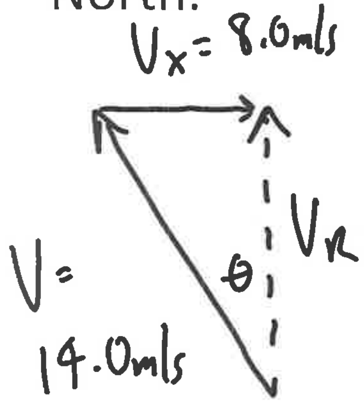
$$= \underline{16.1\ m/s}$$

$$\theta = \tan^{-1}\left(\frac{8.0}{14.0}\right) = \underline{29.7^\circ\ E\ of\ N}$$

In 1-D this we can simply add the vectors, however in 2-D we need to construct vector diagrams.

c) If he needs to end up directly across from his starting point, what direction should he head?

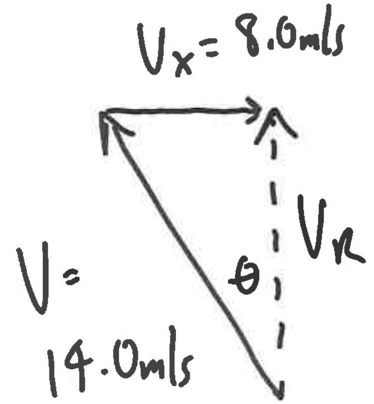
NOTE: Add the velocity vectors so that the resultant points due North.



$$\theta = \sin^{-1}\left(\frac{8.0}{14.0}\right) = 34.8^\circ \text{ W of N}$$

In 1-D this we can simply add the vectors, however in 2-D we need to construct vector diagrams.

d) In part c, how long will it take to cross the river?

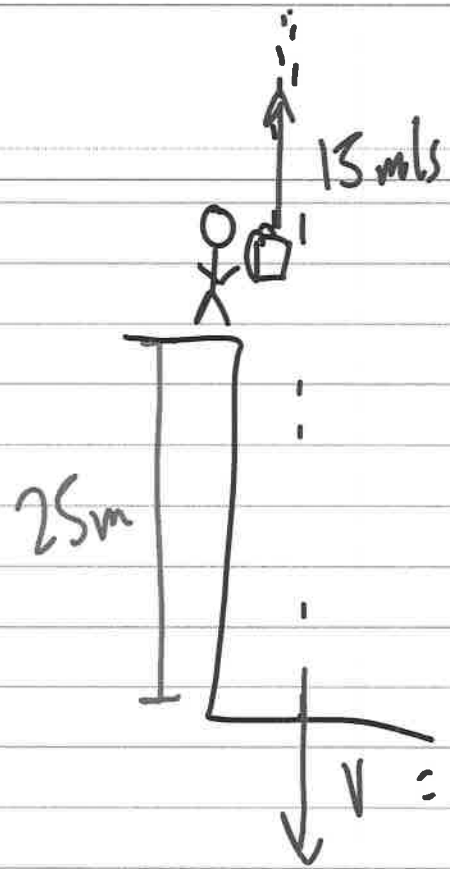


$$V_R = \sqrt{V^2 - V_x^2} = 11.5 m/s$$

$$V = \frac{d}{t} \qquad t = \frac{d}{V} = \frac{40.0 m}{11.5 m/s}$$

$$= \underline{3.48 s}$$

Note Title 24/09/2010



$V = \bigcirc$

$V_0 = 15\,m/s$

$a = -9.80\,m/s^2$

$d = -25\,m$

$t =$

$V^2 = V_0^2 + 2ad$

$V = \pm\sqrt{V_0^2 + 2ad}$

$= \pm\sqrt{(15)^2 + 2(-9.80)(-25)}$

$\boxed{= -27\,m/s}$

5 – Projectile Motion

Projectiles in 2-D – x and y components are perpendicular and therefore totally independent.

Nothing crosses this line or there will be trouble…

X-components

- No net force in the x direction
- Acceleration is always zero
- The only equation you can ever use is:

$$V = \frac{d}{t}$$

Y-components

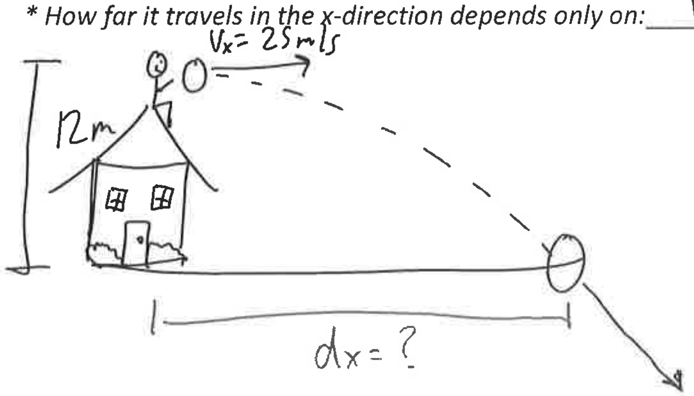
- Always a constant acceleration of 9.8 m/s² downwards ($-9.80\ m/s^2$)
- Need to use the Big 3 equations

The only value that can ever be used on both sides is time because it is scalar

Ex 1: A student sits on the roof of their house which is 12 m high. She can launch water-balloons from a slingshot at 25 m/s. If she fires a water-balloon directly horizontally:
a. How long will it be airborne?
b. How far forward will it travel?

** How long it is airborne only depends on:* vertical displacement

** How far it travels in the x-direction depends only on:* V_x *and* t



X	Y
$V_x = 25\ m/s$	$V_y =$
$d_x = ?$	$V_{y0} = 0$
$t = 1.565\ s$	$a_y = -9.8\ m/s^2$
	$d_y = -12\ m$
	$t = ? = 1.565\ s$

$$V_x = \frac{d_x}{t} \qquad d_x = V_x t$$

$$= (25\ m/s)(1.565\ s)$$

$$= \boxed{39\ m}$$

$$d = \cancel{V_0 t} + \frac{1}{2}at^2$$

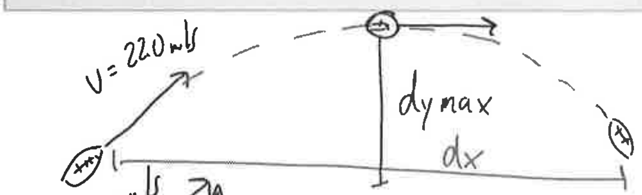
$$d = \frac{1}{2}at^2$$

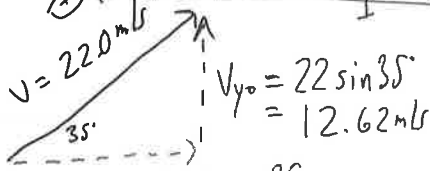
$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2(-12)}{-9.80}}$$

$$= 1.565\ s$$

$$= \boxed{1.6\ s}$$

Ex 2: A quarterback launches a ball to his wide receiver by throwing it at 22.0 m/s at 35° above horizontal.
a. How far downfield is the receiver?
b. How high does the ball go?
c. At what other angle could the quarterback have thrown the ball and reached the same displacement?





$V_{yo} = 22 \sin 35° = 12.62$ m/s

$V_x = 22 \cos 35 = 18.02$ m/s

X	y @ $t_{\frac{1}{2}}$
$V_x = 18.02$ m/s	$V_y = 0$
$d_x = ?$	$V_{yo} = 12.62$ m/s
$t = 2.575$ s	$a_y = -9.8$ m/s²
	$d_y =$
	$t_{\frac{1}{2}} = 1.288$

a) $d_x = V_x t = (18.02)(2.575) = \boxed{46.4\text{ m}}$

c) 55° (complimentary to 35°)

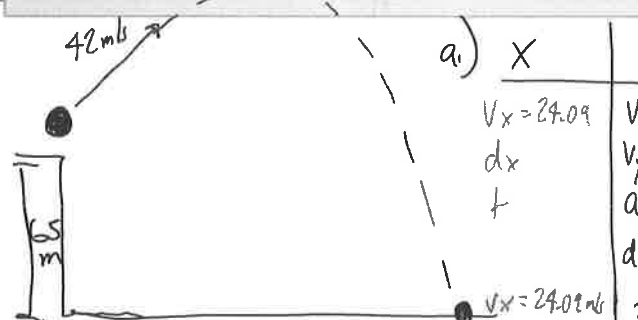
b) $v^2 = V_0^2 + 2ad$

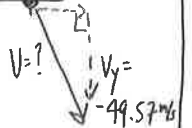
$d = \frac{V^2 - V_0^2}{2a} = \frac{0 - (12.62)^2}{2(-9.8)} = \boxed{8.13\text{m}}$

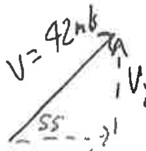
$V = V_0 + at$

$t = \frac{V - V_0}{a} = \frac{0 - 12.62}{-9.8} = 1.288$ s

Ex 3: A cannon sits on a 65 m high cliff (typical Trask...so typical...). A cannonball is fired at 42 m/s 55° above the horizontal.
a. How long is it airborne?
b. What is its final velocity?
c. What is its maximum height relative to the ground below?







$V_{yo} = 42 \sin 55 = 34.40$ m/s

$V_x = 42 \cos 55 = 24.09$ m/s

a)

X	Y
$V_x = 24.09$	$V_y = ? = -49.57$ m/s
d_x	$V_{yo} = 34.40$ m/s
t	$a_y = -9.80$ m/s²
	$d_y = -65$ m
	$t = ?$

$V^2 = V_0^2 + 2ad$

$V = \pm\sqrt{34.40^2 + 2(-9.8)(-65)}$

$= -49.57$ m/s

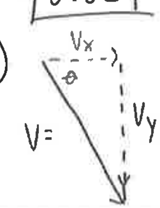
$V = V_0 + at$

$t = \frac{V - V_0}{a} = \frac{-49.57 - 34.40}{-9.8}$

$= 8.569$ s

$= \boxed{8.6\text{s}}$

b)

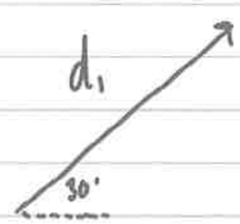


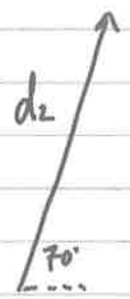
$V = \sqrt{V_y^2 + V_x^2}$

$= 55$ m/s

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 64°$ (below horiz)

Note Title 9/19/2011





$$\vec{d}_1 + \vec{d}_2 = \vec{d}_t$$