***Pre-Calculus 11***

***Unit 7: Rational Expressions and Equations***

***Worksheet 7.1—Equivalent Rational Expressions***

1. State whether each expression is a rational expression. If the expression is not rational,

explain why.

1. $\frac{4x+2}{3}$ b) $x^{2}-x-6$ c) $\frac{4m-n}{m+2n}$

d) $\frac{x-\sqrt{x}}{2}$ e) $\frac{-4}{x^{3}+1}$ f) $\frac{2^{x}-1}{3x}$

2. Identify the non-permissible values for the variables in each rational expression.

1. $\frac{1}{6w}$ b) $\frac{5x}{x^{2}+3}$ c) $\frac{a-b}{b-1}$

d) $\frac{m+5}{m^{2}-1}$ e) $\frac{x^{2}-1}{x^{3}-8}$ f) $\frac{6x-5}{2x(x+4)}$

g) $\frac{2x+1}{5}$ h) $\frac{3}{(x+2)(x-1)}$ i) $\frac{4(x+1)}{(x+1)(x+2)}$

j) $\frac{3x}{3x^{2}+6x}$ k) $\frac{x(x+3)}{x^{2}+7x+12}$ l) $\frac{x+1}{3x^{2}+8x+5}$

m) $\frac{x+4}{x^{2}+16}$ n) $\frac{n+5}{x^{2}-25}$ o) $\frac{x^{2}-1}{x^{3}+1}$

3. Simplify each rational expression.

 a) $\frac{3m^{4}}{6m}$ b) $\frac{-12a^{3}b^{5}}{4a^{2}b^{7}}$ c) $\frac{15x^{2}y}{-18x^{3}y^{2}z}$

d) $\frac{4x+8}{2x+4}$ e) $\frac{2x-10}{3x-15}$ f) $\frac{3a+12}{6a+24}$

4. Simplify each rational expression.

1. $\frac{x-5}{10-2x}$ b) $\frac{9-3x}{x-3}$ c) $\frac{2x^{2}-10x}{4x-20}$

d) $\frac{3x^{2}-6x}{14-7x}$ e) $\frac{10xy-15x^{2}y}{6x^{2}-4x}$ f) $\frac{60a^{2}b^{2}-24ab}{16ab-40a^{2}b^{2}}$

5. Simplify each rational expression.

1. $\frac{x-3}{x^{2}+3x-18}$ b) $\frac{m^{2}-7m+10}{m-2}$ c) $\frac{x+4}{x^{2}-16}$

d) $\frac{a^{2}+5a-14}{a^{2}-6a+8}$ e) $\frac{4a^{2}-16b^{2}}{4a^{2}-8ab}$ f) $\frac{x^{2}+6x+8}{x^{2}+7x+12}$

6. Simplify each rational expression.

1. $\frac{m^{2}-9mn+20n^{2}}{3m^{2}-15mn}$ b) $\frac{x^{2}+9xy+18y^{2}}{2x^{2}+12xy}$ c) $\frac{8t^{2}-32}{2t^{2}+12t+16}$
2. $\frac{3m^{2}-15m}{3m^{2}-16m+5}$ e) $\frac{2x^{2}+3xy+y^{2}}{3x^{2}+2xy-y^{2}}$ f) $\frac{x^{2}-5x-6}{36-x^{2}}$

7. Simplify each rational expression.

1. $\frac{25-x^{2}}{2x^{2}-9x-5}$ b)$ \frac{16+x^{2}}{x^{4}+5x^{2}+4}$ c) $\frac{x^{4}-5x^{2}+4}{x^{2}+x+2}$

d) $\frac{x^{2}-4xy+4y^{2}}{x^{4}-16y^{4}}$ e) $\frac{x^{4}-y^{4}}{\left(x^{2}+y^{2}\right)\left(x^{2}-5xy+4y^{2}\right)}$ f) $\frac{16x^{4}-y^{4}}{\left(4x^{2}+y^{2}\right)^{2}\left(2x^{2}+3xy-2y^{2}\right)}$

8. Give and example of a rational expression that has the following characteristic:

 a) Has non-permissible values 3 and $-2$.

 b) Has non-permissible values $-\frac{1}{2}$ and $\frac{2}{3}$.

 c) Is defined for all real values of *x*.

9. Two points on a coordinate grid are represented by $A\left(p, 3\right)$ and $B\left(2p+1,p-5\right)$.

 a) Write a rational expression for the slope of a line passing through A and B.

 Write your answer in simplest form.

 b) Determine a value for p such that the line passing though A and B has a negative slope.

 c) Describe the line through A and B for any non-permissible value of p.

10. Write in simplest form

 a) $\frac{\left(x+2\right)^{2}-\left(x+2\right)-20}{x^{2}-9}$ b) $\frac{4\left(x^{2}-9\right)^{2}-\left(x-3\right)^{2}}{x^{2}-6x+9}$

 c) $\frac{\left(x^{2}-x\right)^{2}-8\left(x^{2}-x\right)+12}{\left(x^{2}-4\right)^{2}-\left(x-2\right)^{2}}$ d) $\frac{\left(x^{2}+4x+4\right)^{2}-10\left(x^{2}+4x+4\right)+9}{\left(2x+1\right)^{2}-\left(x+2\right)^{2}}$

***Solutions***

1. a) Rational

 b) Rational

 c) Rational

 d) Not Rational $\rightarrow "\sqrt{x}"$ not a polynomial

 e) Rational

 f) Not Rational $\rightarrow "2^{x}"$ not a polynomial

2. a) $w=0$ b) $x\in R$ c) $b=0$ d) $m=-1, 1$

 e) $x=2$ f) $x=0, -4$ g) $x\in R$ h) $x=-2 , 1$

 i) $x=-1, -2$ j) $x=0, -6 $ k) $x=-3, -4$ l) $x=\frac{-5}{3}$, $-1$

 m) $x\in R$ n) $x=\pm 5$ o) $x=-1$

3. a) $\frac{m^{3}}{2}$ b) $\frac{-3a}{b^{2}}$ c) $ \frac{5}{-6xyz}$ d) $2$ e) $\frac{2}{3}$ f) $\frac{1}{2}$

4. a) $\frac{-1}{2}$ b) $-3$ c) $\frac{x}{2}$ d) $\frac{-3x}{7}$ e) $\frac{-5y}{2}$ f) $\frac{-3}{2}$

5. a) $\frac{1}{x+6}$ b) $m-5$ c) $\frac{1}{x-4}$ d) $\frac{a+7}{a-4}$ e) $\frac{a+2b}{a}$ f) $\frac{x+2}{x+3}$

6. a) $\frac{m-4n}{3m}$ b) $\frac{x+3y}{2x}$ c) $\frac{4\left(t-2\right)}{t+4}$ d) $\frac{3m}{3m-1}$ e) $\frac{2x+y}{3x-y}$ f) $-\left(\frac{x+1}{x+6}\right)=\frac{-x-1}{x+6}$

7. a) $-\frac{\left(x+5\right)}{\left(2x+1\right)}=\frac{-x-5}{2x+1}$ b) $\frac{16+x^{2}}{\left(x^{2}+4\right)\left(x^{2}+1\right)}$ c) $\left(x+1\right)\left(x-2\right)$

 d) $\frac{1}{x^{2}+4y^{2}}$ e) $\frac{x+y}{x-4y}$ f) $\frac{2x+y}{x+2y}$

8. a) $\frac{1}{x^{2}-x-6}$ b) $\frac{1}{6x^{2}-x-2}$ c) Any denominator that can never equal zero. $\left(ie. \frac{1}{x^{2}+1}\right)$

9. a) $m=\frac{p-8}{p+1}$

1. Any value $-1<p<8$ will give a negative slope. Example if $p=0$, $m=-\frac{8}{1}$
2. If $p=-1$, then the expression is undefined, and the line is vertical.

10. a) $\frac{x+6}{x+3}, x\ne \pm 3$ b) $\left(2x-7\right)\left(2x-5\right), x\ne -3$

 c) $\frac{\left(x-3\right)\left(x+2\right)}{\left(x+3\right)\left(x-2\right)}, x\ne -3,-1, 2$ d) $\frac{\left(x+5\right)\left(x+3\right)}{3}, x\ne \pm 1$