

# Pre-Calculus II - Year In Review

## Unit 1

1.  $2.6, -1.8, \dots$

a)  $d = -1.8 - 2.6$   
 $= -4.4$

b)  $t_n = t_1 + (n-1)d$   
 $t_5 = 2.6 + (5-1)(-4.4)$   
 $= 2.6 + 4(-4.4)$   
 $= 2.6 + -17.6$   
 $= -15$

c)  $S_n = \frac{n(2t_1 + (n-1)d)}{2}$   
 $S_8 = \frac{8(2(2.6) + (8-1)(-4.4))}{2}$   
 $= \frac{8(5.2 + 7(-4.4))}{2}$   
 $= 4(5.2 - 30.8)$   
 $= 4(-25.6)$   
 $= -102.4$

2.  $3x, 7x, 11x, \dots$

a)  $d = 7x - 3x$   
 $= 4x$

b)  $t_n = t_1 + (n-1)d$   
 $t_8 = 3x + (8-1)(4x)$   
 $= 3x + 7(4x)$   
 $= 3x + 28x$   
 $= 31x$

c)  $S_n = \frac{n(2t_1 + (n-1)d)}{2}$   
 $S_5 = \frac{5(2(3x) + (5-1)(4x))}{2}$   
 $= \frac{5(6x + 4(4x))}{2}$   
 $= \frac{5(6x + 16x)}{2}$   
 $= \frac{5(22x)}{2}$   
 $= 55x$

3.  $t_1 = -3$   
 $d = 4$   
 $t_8 = ?$   
 $n = 8$

$t_n = t_1 + (n-1)d$   
 $t_8 = -3 + (8-1)(4)$   
 $= -3 + 7(4)$   
 $= -3 + 28$   
 $= 25$

PC 11 - Review con 4... 2 Unit 1

4.  $5 + 2 - 1 - \dots$

$$\begin{aligned} S_{14} &= ? \\ n &= 14 \\ t_1 &= 5 \\ d &= -3 \end{aligned}$$

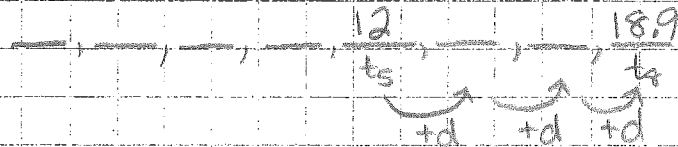
$$\begin{aligned} S_n &= \frac{n(2t_1 + (n-1)d)}{2} \\ S_{14} &= \frac{14(2(5) + (14-1)(-3))}{2} \\ &= \frac{14(10 - 39)}{2} \\ &= 7(-29) \\ &= -203 \end{aligned}$$

5.  $S_9 = 57.6$   
 $t_1 = 2$   
 $d = ?$   
 $n = 9$

$$\begin{aligned} S_n &= \frac{n(2t_1 + (n-1)d)}{2} \\ 2. \quad 57.6 &= \frac{9(2(2) + (9-1)d)}{2} \\ \frac{57.6(2)}{9} &= \frac{9(4 + 8d)}{9} \end{aligned}$$

$$\begin{aligned} 12.8 &= 4 + 8d \\ -4 & \quad -4 \\ 8.8 &= 8d \\ \frac{8.8}{8} &= \frac{8d}{8} \\ 1.1 &= d \end{aligned}$$

6.  $t_5 = 12$   
 $t_8 = 18.9$   
 $d = ?$



$$\begin{aligned} 12 + 3d &= 18.9 \\ -12 & \quad -12 \\ 3d &= 6.9 \\ \frac{3d}{3} &= \frac{6.9}{3} \rightarrow d = 2.3 \end{aligned}$$

7.  $-3, -1.1, 0.8, \dots, 19.8$   
 $t_n$   
 $d = -1.1 - (-3)$   
 $= -1.1 + 3 = 1.9$

$$\begin{aligned} t_n &= t_1 + (n-1)d \\ 19.8 &= -3 + (n-1)(1.9) \\ +3 & \quad +3 \\ 22.8 &= (n-1)(1.9) \\ \frac{22.8}{1.9} &= \frac{(n-1)(1.9)}{1.9} \\ 12 &= n-1 \\ +1 & \quad +1 \\ 13 &= n \end{aligned}$$

# PC11 - Review con't... 3 Unit 1

$$8. \begin{array}{l} t_2 = 12 \\ S_2 = 20 \\ d = ? \end{array} \quad \begin{array}{l} t_1 + t_2 = S_2 \\ t_1 + 12 = 20 \\ \quad -12 \quad -12 \\ \hline t_1 = 8 \end{array} \quad \begin{array}{l} 8, 12, \dots \\ d = 12 - 8 \\ = 4 \end{array}$$

$$9. \sum_{j=2}^9 2j \quad \text{Number of terms} = \begin{array}{l} \text{upper} \\ \text{bound} \end{array} - \begin{array}{l} \text{lower} \\ \text{bound} \end{array} + 1 \\ = 9 - 2 + 1 \\ = 8$$

$$10. 3 + 1 + \frac{1}{3} + \frac{1}{9} + \dots \quad r = -\frac{1}{3}$$

$$11. 8000, 4000, 2000$$

$$r = \frac{4000}{8000} = \frac{1}{2}$$

$$t_7 = ?$$

$$t_n = t_1 r^{n-1}$$

$$\begin{aligned} t_7 &= 8000 \left(\frac{1}{2}\right)^{7-1} \\ &= 8000 \left(\frac{1}{2}\right)^6 \\ &= 8000 \left(\frac{1}{64}\right) \end{aligned}$$

$$= 125$$

$$12. S_5 = -328$$

$$r = -4$$

$$t_1 = ?$$

$$n = 5$$

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

$$-328 = \frac{t_1(1-(-4)^5)}{1-(-4)}$$

$$-328 = \frac{t_1(1-(-1024))}{5}$$

$$-328(5) = t_1(1025)$$

$$\frac{-1640}{1025} = \frac{t_1(1025)}{1025}$$

$$-1.6 = t_1$$

# PC11 - Reviews con't. 4 Unit 1

13.  $\sum_{k=1}^{\infty} 50 \left(\frac{1}{4}\right)^{k-1}$

$$= 50 \left(\frac{1}{4}\right)^{1-1} + 50 \left(\frac{1}{4}\right)^{2-1} + 50 \left(\frac{1}{4}\right)^{3-1} + 50 \left(\frac{1}{4}\right)^{4-1} + \dots$$

$$= 50 \left(\frac{1}{4}\right)^0 + 50 \left(\frac{1}{4}\right)^1 + 50 \left(\frac{1}{4}\right)^2 + 50 \left(\frac{1}{4}\right)^3 + \dots$$

$$= 50 + \frac{50}{4} + \frac{50}{16} + \frac{50}{64} + \dots$$

$$t_1 = 50$$

$$r = \frac{1}{4}$$

$$S_{\infty} = \frac{t_1}{1-r}$$

$$= \frac{50}{1-\frac{1}{4}}$$

$$= \frac{50}{\frac{3}{4}}$$

$$= 50 \cdot \frac{4}{3}$$

$$= 50 \cdot \frac{4}{3}$$

$$= \frac{200}{3}$$

14.  $160 - 40 + 10 - \dots$

$$r = \frac{-40}{160} = -\frac{1}{4}$$

$$t_1 = 160$$

$$S_{\infty} = \frac{t_1}{1-r}$$

$$= \frac{160}{1-\left(-\frac{1}{4}\right)}$$

$$= \frac{160}{\frac{4}{4} + \frac{1}{4}}$$

$$= \frac{160}{\frac{5}{4}}$$

$$= 160 \div \frac{5}{4}$$

$$= 160 \cdot \frac{4}{5} = 128$$

15.  $2, k, 18$

$2, 6, 18$  or  $2, -6, 18$

$$\frac{18}{k} = \frac{k}{2} \quad \leftarrow \text{both} = r$$

$$\therefore r = 3 \text{ or } r = -3$$

$$k^2 = 2(18) \quad \text{cross multiply}$$

$$k^2 = 36$$

$$k = \pm \sqrt{36}$$

$$k = \pm 6$$

16.  $\sum_{k=2}^{\infty} 3(2)^k = 3(2)^2 + 3(2)^3 + \dots$

$$= 3(4) + 3(8) + \dots$$

$$= 12 + 24 + \dots$$

$$\uparrow t_2 = 24$$

# PC II - Review cont... 5 Unit 1

17.  $8x, 4x^2, \dots$

$$r = \frac{4x^2}{8x} = \frac{x}{2}$$

$$t_1 = 8x$$

$$t_4 = ?$$

$$n = 4$$

$$t_n = t_1 r^{n-1}$$

$$t_4 = 8x \left(\frac{x}{2}\right)^{4-1}$$

$$= 8x \left(\frac{x}{2}\right)^3$$

$$\begin{aligned} &\rightarrow = 8x \left(\frac{x^3}{8}\right) \\ &= \frac{8x^4}{8} \\ &= x^4 \end{aligned}$$

18.  $S_\infty = 27$

$$t_1 = 18$$

$$r = ?$$

$$S_\infty = \frac{t_1}{1-r}$$

$$27 = \frac{18}{1-r}$$

$$27(1-r) = 18$$

$$27 - 27r = 18$$

$$27 - 27r = 18$$

$$-27r = -9$$

$$-27r = -9$$

$$\frac{-27r}{-27} = \frac{-9}{-27}$$

$$r = \frac{1}{3}$$

19.  $t_1 = -5.8$

$$t_{12} = 17.7$$

$$S_{12} = ?$$

$$n = 12$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{12} = \frac{12(-5.8 + 17.7)}{2}$$

$$= \frac{12(11.9)}{2}$$

$$= 71.4$$

20.  $x, 3x+3, 5x+6, \dots$

a)  $d = (3x+3) - x$   
 $= 2x+3$

$$t_1 = x$$

$$t_6 = ?$$

b)  $t_n = t_1 + (n-1)d$

$$29x+42 = x + (n-1)(2x+3)$$

$$\frac{28x+42}{2x+3} = \frac{(n-1)(2x+3)}{2x+3}$$

$$28x+42 = n-1$$

$$\frac{14(2x+3)}{(2x+3)} = n-1$$

$$14 = n-1$$

$$+1 \quad +1$$

$$15 = n$$

$$t_n = t_1 + (n-1)d$$

$$t_6 = x + (6-1)(2x+3)$$

$$= x + 5(2x+3)$$

$$= x + 10x + 15$$

$$t_6 = 11x + 15$$

# PC11 - Review con't... 6 Unit 1

20. con't...

$$c) S_8 = 7$$

$$n = 8$$

$$t_1 = x$$

$$d = 2x + 3$$

$$S_n = \frac{n(2t_1 + (n-1)d)}{2}$$

$$S_8 = \frac{8(2(x) + (8-1)(2x+3))}{2}$$

$$= \frac{8(2x + 7(2x+3))}{2}$$

$$= 4(2x + 14x + 21)$$

$$= 4(16x + 21)$$

$$= 64x + 84$$

$$d) t_4 = -12$$

$$d = ?$$

$$x, 3x+3, 5x+6, -12$$

$$d = -12 - (5x+6) \quad (t_4 - t_3)$$

And from part (a) we know  $d = 2x + 3$

$$2x + 3 = -12 - (5x + 6)$$

$$2x + 3 = -12 - 5x - 6$$

$$2x + 3 = -18 - 5x$$

$$+5x - 3 \quad -3 + 5x$$

$$7x = -21$$

$$\frac{7x}{7} = \frac{-21}{7}$$

$$x = -3$$

$$d = 2x + 3$$

$$= 2(-3) + 3$$

$$= -6 + 3$$

$$d = -3$$

# PC II - Reviews cont... 7

## Unit II

$$1. a) 2|-2-3|$$

$$= 2|-5|$$

$$= 2(5)$$

$$= 10$$

$$b.) |-7| + 3|5|$$

$$= 7 + 3(5)$$

$$= 7 + 15$$

$$= 22$$

$$c.) 5|-6+4| + 2|-3|$$

$$= 5|-2| + 2(3)$$

$$= 5(2) + 6$$

$$= 10 + 6 = 16$$

$$2. |x^2 + 6x - 2| \quad x = -3 \rightarrow = |-9 - 2|$$

$$= |(-3)^2 + 6(-3) - 2|$$

$$= |9 - 18 - 2|$$

$$= |-11|$$

$$= 11$$

$$3. a) 5\sqrt{2}$$

$$= \sqrt{5^2} \sqrt{2}$$

$$= \sqrt{25} \sqrt{2}$$

$$= \sqrt{50}$$

$$b) 3\sqrt[3]{5}$$

$$= \sqrt[3]{3^3} \sqrt[3]{5}$$

$$= \sqrt[3]{27} \sqrt[3]{5}$$

$$= \sqrt[3]{135}$$

$$4. \sqrt{12}$$

$$\boxed{3\sqrt{3}}$$

$$-5\sqrt{3}$$

$$\sqrt{3}$$

$$= \sqrt{9} \sqrt{3}$$

$$= -\sqrt{25} \sqrt{3}$$

$$= \sqrt{27}$$

$$= -\sqrt{75}$$

↑  
largest

↑ keep the negative outside

$$5. \sqrt[4]{7x^3}$$

→ you can only take the fourth root of a positive number.

$$\therefore x \geq 0$$

$$6. \sqrt{32a^5b^4}$$

→ You can only take the square root of a positive number

note: "b<sup>4</sup>" will always be positive.

$$\therefore a \geq 0 \quad b \in \mathbb{R}$$

# PC 11 - Review con't... 8

$$\begin{aligned}
 7. a) \quad & \sqrt{\frac{48}{50}} \\
 &= \frac{\sqrt{48}}{\sqrt{50}} \\
 &= \frac{\sqrt{16 \cdot 3}}{\sqrt{25 \cdot 2}} \\
 &= \frac{4\sqrt{3}}{5\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{or} \quad &= \frac{4\sqrt{3} \cdot \sqrt{2}}{5\sqrt{2} \cdot \sqrt{2}} \\
 &= \frac{4\sqrt{6}}{5(2)} \\
 &= \frac{4\sqrt{6}}{10} \\
 &= \frac{2\sqrt{6}}{5}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \sqrt[3]{\frac{-24}{27}} \\
 &= \frac{\sqrt[3]{-24}}{\sqrt[3]{27}} \\
 &= \frac{\sqrt[3]{-8 \cdot 3}}{\sqrt[3]{27}} \\
 &= \frac{-2\sqrt[3]{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 8.) a) \quad & \frac{2}{3} \sqrt{\frac{1}{2}} \\
 &= \frac{\sqrt{2^2}}{\sqrt{3^2}} \sqrt{\frac{1}{2}} \\
 &= \frac{\sqrt{8}}{\sqrt{27}} \sqrt{\frac{1}{2}} \\
 &= \frac{\sqrt{8}}{\sqrt{54}} \\
 &= \frac{\sqrt{4}}{\sqrt{27}}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & -3 \sqrt{\frac{2}{3}} \\
 &= -\sqrt{\frac{3^2}{1}} \sqrt{\frac{2}{3}} \\
 &= -\sqrt{\frac{9}{1}} \sqrt{\frac{2}{3}} \\
 &= -\sqrt{\frac{18}{3}} \\
 &= -\sqrt{6}
 \end{aligned}$$

note: the negative must stay outside the square root since you cannot take the square root of a negative.

$$\begin{aligned}
 9.) a) \quad & 5\sqrt{2m^2} + \sqrt{8m^2} - \sqrt{50m^2} \\
 &= 5\sqrt{2}\sqrt{m^2} + \sqrt{4}\sqrt{2}\sqrt{m^2} - \sqrt{25}\sqrt{2}\sqrt{m^2} \\
 &= 5m\sqrt{2} + 2m\sqrt{2} - 5m\sqrt{2} \\
 &= 2m\sqrt{2}
 \end{aligned}$$

$\uparrow m \geq 0$  because when you take the square root of something, you get a positive number.

$$\therefore = 2|m|\sqrt{2}$$

$$\begin{aligned}
 b) \quad & \sqrt{3b} + \sqrt{12b} - \sqrt[3]{3b} - \sqrt{75b} \\
 &= \sqrt{3b} + \sqrt{4 \cdot 3b} - \sqrt[3]{3b} - \sqrt{25 \cdot 3b} \\
 &= \sqrt{3b} + 2\sqrt{3b} - \sqrt[3]{3b} - 5\sqrt{3b} \\
 &= -2\sqrt{3b} - \sqrt[3]{3b}
 \end{aligned}$$



# PC 11 - Review con 4... 8.

## Unit II con't...

$$10. a) \sqrt{3}(\sqrt{5}+3) \\ = \sqrt{15} + 3\sqrt{3}$$

$$b) \sqrt{2x}(\sqrt{5x}+\sqrt{7}) \\ = \sqrt{10x^2} + \sqrt{14x} \\ = x\sqrt{10} + \sqrt{14x}$$

$$c) (2\sqrt{a}-2\sqrt{b})(2\sqrt{a}+\sqrt{b}) \\ = 4\sqrt{a^2} + 2\sqrt{ab} - 4\sqrt{ab} - 2\sqrt{b^2} \\ = 4a - 2\sqrt{ab} - 2b$$

$$11. a) \frac{6+\sqrt{2}}{\sqrt{3}+5} \\ = \frac{(6+\sqrt{2})(\sqrt{3}-5)}{(\sqrt{3}+5)(\sqrt{3}-5)} \\ = \frac{6\sqrt{3}-30+\sqrt{6}-5\sqrt{2}}{3-5\sqrt{3}+5\sqrt{3}-25} \\ = \frac{6\sqrt{3}-30+\sqrt{6}+5\sqrt{2}}{-22} \\ = -\frac{6\sqrt{3}-30+\sqrt{6}+5\sqrt{2}}{22}$$

$$c) \frac{7}{\sqrt[3]{3}} \\ = \frac{7\sqrt[3]{3}\sqrt[3]{3}}{\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3}} \\ = \frac{7\sqrt[3]{3^2}}{3} \\ = \frac{7\sqrt[3]{9}}{3}$$

$$b) \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{6}} + \frac{\sqrt{2}}{\sqrt{3}} \quad \text{lcd} = \sqrt{6} \\ = \frac{\sqrt{3}\sqrt{3}}{\sqrt{2}\sqrt{3}} - \frac{1}{\sqrt{6}} + \frac{\sqrt{2}\sqrt{2}}{\sqrt{3}\sqrt{2}} \\ = \frac{3}{\sqrt{6}} - \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} \\ = \frac{4}{\sqrt{6}}$$

$$= \frac{4\sqrt{6}}{\sqrt{6}\sqrt{6}} \\ = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

$$\text{OR} // \frac{\sqrt{3}\sqrt{2}}{\sqrt{2}\sqrt{2}} - \frac{\sqrt{6}}{\sqrt{6}\sqrt{6}} + \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} \\ = \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{6} + \frac{\sqrt{6}}{3} \\ = \frac{3\sqrt{6}}{6} - \frac{\sqrt{6}}{6} + \frac{2\sqrt{6}}{6} \\ = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

$$d) \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}} \\ = \frac{2\sqrt{3}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} \\ = \frac{2\sqrt{6}-2(3)}{2-\sqrt{6}+\sqrt{6}-3} \\ = \frac{2\sqrt{6}-6}{-1} \\ = -2\sqrt{6}+6$$

$$e) \frac{3\sqrt{3}-\sqrt{5}}{\sqrt{7}+2\sqrt{2}} \\ = \frac{(3\sqrt{3}-\sqrt{5})(\sqrt{7}-2\sqrt{2})}{(\sqrt{7}+2\sqrt{2})(\sqrt{7}-2\sqrt{2})} \\ = \frac{3\sqrt{21}-6\sqrt{6}-\sqrt{35}+2\sqrt{10}}{7-2\sqrt{14}+2\sqrt{14}-4(2)} \\ = \frac{3\sqrt{21}-6\sqrt{6}-\sqrt{35}+2\sqrt{10}}{7-8} \\ = \frac{3\sqrt{21}-6\sqrt{6}-\sqrt{35}+2\sqrt{10}}{-1} \\ = -3\sqrt{21}+6\sqrt{6}+\sqrt{35}-2\sqrt{10}$$

# PC11 - Review con't...9

## Unit II con't...

$$\begin{aligned}
 11. f.) \quad & \frac{2}{\sqrt[4]{2}} \\
 & = \frac{2}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} \\
 & = \frac{2\sqrt[4]{2^3}}{\sqrt[4]{2^4}} = \frac{2\sqrt[4]{8}}{2} = \sqrt[4]{8}
 \end{aligned}$$

$$\begin{aligned}
 12. a.) \quad & \frac{2\sqrt{-x}}{2} = \frac{10}{2} \\
 & \sqrt{-x} = 5 \\
 & (\sqrt{-x})^2 = 5^2 \\
 & -x = 25 \\
 & x = -25
 \end{aligned}$$

$$\begin{aligned}
 d.) \quad & \sqrt[4]{x-1} + 2 = 5 \\
 & \quad \quad \quad -2 \quad -2 \\
 & \sqrt[4]{x-1} = 3 \\
 & (\sqrt[4]{x-1})^4 = (3)^4 \\
 & x-1 = 81 \\
 & \quad \quad \quad +1 \quad +1 \\
 & x = 82
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad & \sqrt{3x+2} + 4 = 8 \\
 & \quad \quad \quad -4 \quad -4 \\
 & \sqrt{3x+2} = 4 \\
 & (\sqrt{3x+2})^2 = (4)^2 \\
 & 3x+2 = 16 \\
 & \quad \quad \quad -2 \quad -2 \\
 & 3x = 14 \\
 & \quad \quad \quad \frac{3}{3} \quad \frac{3}{3} \\
 & x = \frac{14}{3}
 \end{aligned}$$

$$\begin{aligned}
 e.) \quad & 2\sqrt[3]{x-3} + 4 = 8 \\
 & \quad \quad \quad -4 \quad -4 \\
 & 2\sqrt[3]{x-3} = 4 \\
 & \quad \quad \quad \frac{2}{2} \quad \frac{2}{2} \\
 & \sqrt[3]{x-3} = 2 \\
 & (\sqrt[3]{x-3})^3 = (2)^3 \\
 & x-3 = 8 \\
 & \quad \quad \quad +3 \quad +3 \\
 & x = 11
 \end{aligned}$$

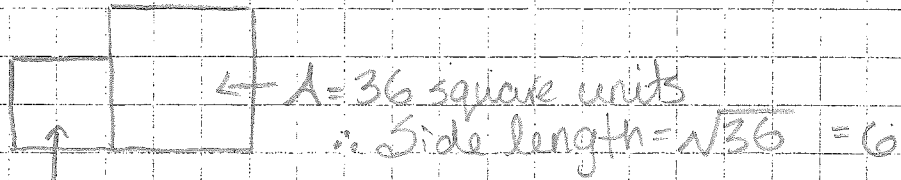
$$\begin{aligned}
 c.) \quad & 2\sqrt{x-3} = \sqrt{7x-5} \\
 & (2\sqrt{x-3})^2 = (\sqrt{7x-5})^2 \\
 & 4(x-3) = 7x-5 \\
 & 4x-12 = 7x-5 \\
 & -7x+12 \quad -7x+12 \\
 & -3x = 7 \\
 & x = -\frac{7}{3}
 \end{aligned}$$

$$\begin{aligned}
 f.) \quad & 3\sqrt{2x+3} = \sqrt{3x-5} \\
 & (3\sqrt{2x+3})^2 = (\sqrt{3x-5})^2 \\
 & 9(2x+3) = 3x-5 \\
 & 18x+27 = 3x-5 \\
 & -3x-27 \quad -3x-27 \\
 & 15x = -32 \\
 & x = -\frac{32}{15}
 \end{aligned}$$

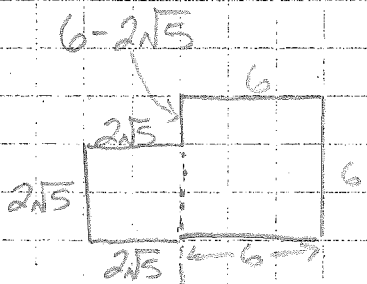
RC II - Review cont., 10

Unit II cont.,

13.



$A = 20$  square units  
 $\therefore$  Side length =  $\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$



Perimeter =  $6 + 6 + 6 + 2\sqrt{5} + 2\sqrt{5} + 2\sqrt{5} + 6 - 2\sqrt{5}$   
 $= 18 + 6\sqrt{5} + 6 - 2\sqrt{5}$   
 $= 24 + 4\sqrt{5}$

# PCII - Review cont. 11

## Unit III

$$1. a) x^2 + 14x + 24 \\ = (x+12)(x+2)$$

$$b) m^2 + 11m - 26 \\ = (m+13)(m-2)$$

$$c) r^4 - 8r^2 + 7 \\ = (r^2 - 7)(r^2 - 1) \\ = (r^2 - 7)(r+1)(r-1)$$

$$d) m^2 - 16 \\ = m^2 - 4^2 \\ = (m+4)(m-4)$$

$$e) 9x^2 - 81 \\ = (3x)^2 - (9)^2 \\ = (3x-9)(3x+9) \\ = 3(x-3)(3)(x+3) \\ = 9(x-3)(x+3)$$

$$\text{or} // 9x^2 - 81 \\ = 9(x^2 - 9) \\ = 9(x+3)(x-3)$$

$$f) m^2x^2 - 4y^2 \\ = (mx)^2 - (2y)^2 \\ = (mx+2y)(mx-2y)$$

$$g) (x+2y)^2 - 9 \\ = (x+2y)^2 - (3)^2 \\ = (x+2y+3)(x+2y-3)$$

$$h) 4m^2 + 7m - 2 \quad mn = 4(-2) \\ = -8 \\ \begin{array}{l} 4 \quad -1 \\ 8 \quad -1 \end{array} \\ = 4m^2 + 8m - 1m - 2 \\ = 4m(m+2) - 1(m+2) \\ = (m+2)(4m-1)$$

$$i) 10x^2 - 19x + 6 \quad mn = 10(6) \\ = 60 \\ \begin{array}{l} 5 \quad 2 \\ -15 \quad -4 \end{array} \\ = 10x^2 - 15x + 4x + 6 \\ = 5x(2x-3) - 2(2x-3) \\ = (2x-3)(5x-2)$$

# PC II - Review cont... 12

## Unit III cont...

$$2. a) \begin{aligned} a^2 - 18 &= -7a \\ &+7a \quad +7a \\ a^2 + 7a - 18 &= 0 \\ (a+9)(a-2) &= 0 \\ a+9=0 & \quad a-2=0 \\ a=-9 & \quad a=2 \end{aligned}$$

$$b) \begin{aligned} x(x-4) &= 12 \\ x^2 - 4x - 12 &= 0 \\ (x-6)(x+2) &= 0 \\ x-6=0 & \quad x+2=0 \\ x=6 & \quad x=-2 \end{aligned}$$

$$c) \begin{aligned} -x^2 - 49 - 2x &= 12x \\ 0 &= 12x + x^2 + 49 + 2x \\ 0 &= x^2 + 14x + 49 \\ 0 &= (x+7)(x+7) \\ x+7=0 & \quad x+7=0 \\ x &= -7 \end{aligned}$$

$$d) \begin{aligned} 6m^2 - m &= 35 & mn=6(-35) \\ 6m^2 - m - 35 &= 0 & = 210 \\ 6m^2 - 15m + 4m - 35 &= 0 \\ 3m(2m-5) + 7(2m-5) &= 0 \\ (2m-5)(3m+7) &= 0 \\ 2m-5=0 & \quad 3m+7=0 \\ m=\frac{5}{2} & \quad m=-\frac{7}{3} \end{aligned}$$

$$e) \begin{aligned} -10 + 3x^2 &= 5x & mn=3(-10) \\ 3x^2 - 5x - 10 &= 0 & = -30 \end{aligned}$$

Note: This doesn't factor  
so we use the quadratic formula

$$e) \text{cont...}$$

$$3x^2 - 5x - 10 = 0$$

$$a=3 \quad b=-5 \quad c=-10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-10)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 + 120}}{6}$$

$$= \frac{5 \pm \sqrt{145}}{6}$$

$$f) \begin{aligned} x(6x+1) &= 15 & mn=6(-15) \\ 6x^2 + x - 15 &= 0 & = -90 \\ 6x^2 + 10x - 9x - 15 &= 0 \\ 2x(3x+5) - 3(3x+5) &= 0 \\ (3x+5)(2x-3) &= 0 \\ 3x+5=0 & \quad 2x-3=0 \\ x=-\frac{5}{3} & \quad x=\frac{3}{2} \end{aligned}$$

$$g) \begin{aligned} 14a^2 + 3a &= 2 & mn=14(-2) \\ 14a^2 + 3a - 2 &= 0 & = -28 \\ 14a^2 + 7a - 4a - 2 &= 0 \\ 7a(2a+1) - 2(2a+1) &= 0 \\ (2a+1)(7a-2) &= 0 \\ 2a+1=0 & \quad 7a-2=0 \\ a=-\frac{1}{2} & \quad a=\frac{2}{7} \end{aligned}$$

# PC II - Reviews con. 13

## Unit III con. 13

2. h)  $3m^2 = -6(m-1) + 5$

$$3m^2 = -6m + 6 + 5$$

$$3m^2 + 6m - 6 - 5 = 0$$

$$3m^2 + 6m - 11 = 0$$

$$a = 3 \quad b = 6 \quad c = -11$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(3)(-11)}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{36 + 132}}{6}$$

$$= \frac{-6 \pm \sqrt{168}}{6}$$

$$= \frac{-6 \pm \sqrt{168}}{6}$$

$$= \frac{-6 \pm \sqrt{168}}{6}$$

$$= \frac{-6 \pm \sqrt{168}}{6}$$

$$= \frac{-6 \pm \sqrt{168}}{6}$$

i)  $(2x-1)^2 - 4 = 7x$

$$(2x-1)(2x-1) - 4 = 7x$$

$$4x^2 - 2x - 2x + 1 - 4 - 7x = 0$$

$$4x^2 - 11x - 3 = 0 \quad mn = 4(-3)$$

$$= -12$$

$$4x^2 - 12x + 1x - 3 = 0$$

$$4x(x-3) + 1(x-3) = 0$$

$$(4x+1)(x-3) = 0$$

$$4x+1=0 \quad x-3=0$$

$$x = -\frac{1}{4}$$

$$x = 3$$

3. a)  $x^2 - 11 + 6x = 0$

$$x^2 + 6x - 11 = 0$$

$$\frac{1}{2}(6) = 3$$

$$\hookrightarrow (3)^2 = 9$$

$$x^2 + 6x + 9 - 9 - 11 = 0$$

$$(x^2 + 6x + 9) - 9 - 11 = 0$$

$$(x+3)^2 - 20 = 0$$

$$(x+3)^2 = 20$$

$$x+3 = \pm \sqrt{20}$$

$$x = -3 \pm \sqrt{4 \cdot 5}$$

$$x = -3 \pm 2\sqrt{5}$$

b)  $a^2 + 3 = 9a$

$$a^2 - 9a + 3 = 0$$

$$\frac{1}{2}(-9) = -\frac{9}{2}$$

$$\hookrightarrow \left(-\frac{9}{2}\right)^2 = \frac{81}{4}$$

$$a^2 - 9a + \frac{81}{4} - \frac{81}{4} + 3 = 0$$

$$(a^2 - 9a + \frac{81}{4}) - \frac{81}{4} + \frac{12}{4} = 0$$

$$(a - \frac{9}{2})^2 - \frac{69}{4} = 0$$

$$(a - \frac{9}{2})^2 = \frac{69}{4} = 17.25$$

$$a - \frac{9}{2} = \pm \sqrt{17.25}$$

$$a = \frac{9}{2} \pm \sqrt{17.25}$$

$$a = 4.5 \pm \sqrt{17.25}$$

c)  $2x^2 - 2x - 9 = -6x$

$$+6x \quad - \quad +6x$$

$$2x^2 + 4x - 9 = 0$$

$$2(x^2 + 2x) - 9 = 0$$

$$\text{leading coefficient } \frac{1}{2}(2) = 1$$

$$\text{must be } +1 \quad \hookrightarrow 1^2 = 1$$

$$2(x^2 + 2x + 1 - 1) - 9 = 0$$

$$2(x^2 + 2x + 1) - 2 - 9 = 0$$

$$2(x+1)^2 - 11 = 0$$

$$+11 \quad +11$$

$$2(x+1)^2 = 11$$

$$\frac{2}{2} \quad \frac{2}{2}$$

$$(x+1)^2 = 5.5$$

$$x+1 = \pm \sqrt{5.5}$$

$$-1 \quad -1$$

$$x = -1 \pm \sqrt{5.5}$$

# PC11 - Review cont... 14

## Unit III cont...

4.  $h = -4.9t^2 + 36.4t$        $h = 20$

$$20 = -4.9t^2 + 36.4t$$

$$-20 = -4.9t^2 + 36.4t - 20$$

$$0 = -4.9t^2 + 36.4t - 20$$

$$a = -4.9 \quad b = 36.4 \quad c = -20$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(36.4) \pm \sqrt{(36.4)^2 - 4(-4.9)(-20)}}{2(-4.9)}$$

$$= \frac{-36.4 \pm \sqrt{1324.96 - 392}}{-9.8}$$

$$= \frac{-36.4 \pm \sqrt{932.96}}{-9.8}$$

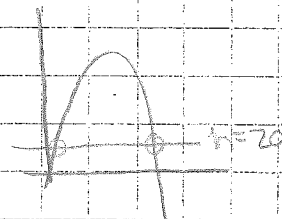
$$t = \frac{-36.4 + 30.544}{-9.8}$$

$$= 0.598 \text{ seconds}$$

$$t = \frac{-36.4 - 30.544}{-9.8}$$

$$= 6.831 \text{ seconds}$$

Sketch:



5. Let  $n$  be the number

$$n^2 - 2n = 35$$

$$n^2 - 2n - 35 = 0$$

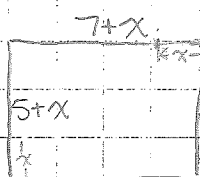
$$(n-7)(n+5) = 0$$

$$n = 7, \quad n = -5$$

b.



$$\text{Area} = 5(7) = 35 \text{m}^2$$



$$\text{Area} = 35 + 45 = 80 \text{m}^2$$

$$(7+x)(5+x) = 80$$

$$35 + 7x + 5x + x^2 = 80$$

$$x^2 + 12x + 35 - 80 = 0$$

$$x^2 + 12x + 45 = 0$$

$$(x+15)(x-3) = 0$$

$$x = -15, \quad x = 3$$

can't be negative

$$\therefore x = 3$$

So dimensions are  $7+3=10\text{m}$

by  $5+3=8\text{m}$

# PC II - Reviews con't. 15

## Unit IV

1a.)  $y = x^2 + 4x$

$$\frac{1}{2}(4) = 2$$

$$\hookrightarrow 2^2 = 4$$

Vertex  $(-2, -4)$

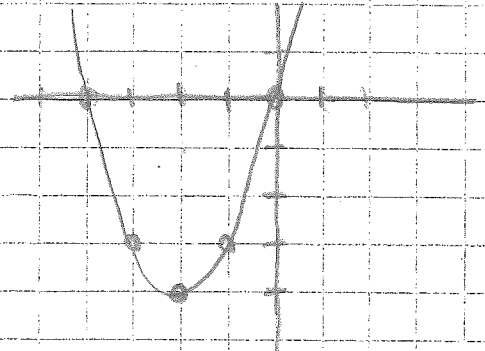
AoS:  $x = -2$

R:  $\{y \mid y \geq -4\}$

$$y = x^2 + 4x + 4 - 4$$

$$y = (x^2 + 4x + 4) - 4$$

$$y = (x + 2)^2 - 4$$



b.)  $y = \frac{1}{2}x^2 - x + 2$

$$= \frac{1}{2}(x^2 - 2x) + 2$$

$$\frac{1}{2}(-2) = -1$$

$$\hookrightarrow (-1)^2 = 1$$

Vertex  $(1, -2.5)$

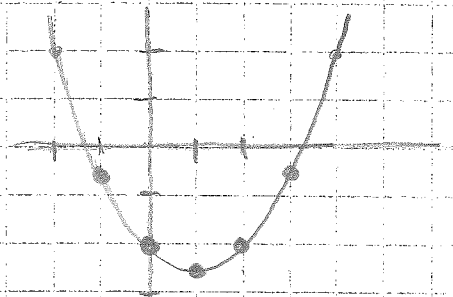
AoS:  $x = 1$

R:  $\{y \mid y \geq -2.5\}$

$$y = \frac{1}{2}(x^2 - 2x + 1 - 1) + 2$$

$$y = \frac{1}{2}(x^2 - 2x + 1) - \frac{1}{2} + 2$$

$$y = \frac{1}{2}(x - 1)^2 - \frac{5}{2}$$



c.)  $y = -3x^2 + 12x + 2$

$$y = -3(x^2 - 4x) + 2$$

$$\frac{1}{2}(-4) = -2$$

$$\hookrightarrow (-2)^2 = 4$$

Vertex  $(2, 14)$

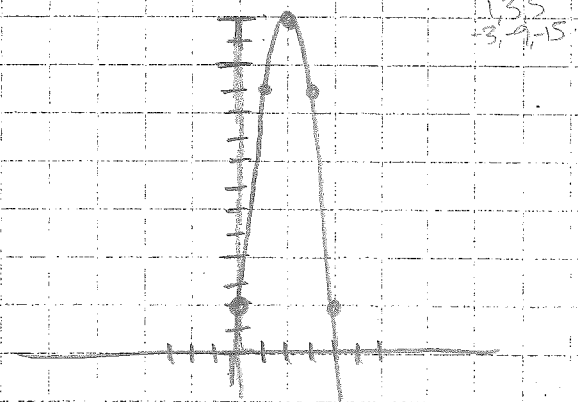
AoS:  $x = 2$

R:  $\{y \mid y \leq 14\}$

$$y = -3(x^2 - 4x + 4 - 4) + 2$$

$$y = -3(x^2 - 4x + 4) + 12 + 2$$

$$y = -3(x - 2)^2 + 14$$



2. a.)  $y = x^2 + 2x - 8 \rightarrow y\text{-int: } (0, -8)$

$$y = (x + 4)(x - 2)$$

$$x + 4 = 0 \quad x - 2 = 0$$

$$x = -4 \quad x = 2 \leftarrow x\text{-intercepts}$$

Axis of Symmetry

$$x = \frac{-4 + 2}{2}$$

$$= \frac{-2}{2}$$

$$= -1$$

$$x = -1$$

Vertex:  $x = -1$

$$y = (-1)^2 + 2(-1) - 8$$

$$= 1 - 2 - 8$$

$$= -9$$

Vertex  $(-1, -9)$



# PC11 - Review cont... 16

## Unit IV cont...

2. b.)  $y = 2x^2 + 16x + 14$   $mn = 2(14) = 28$   $\rightarrow$  y-intercept = 14

$$y = 2x^2 + 2x + 14x + 14$$

$$= 2x(x+1) + 14(x+1)$$

$$= (x+1)(2x+14)$$

$$x+1=0 \quad 2x+14=0$$

$$x = -1 \quad x = \frac{-14}{2} = -7 \quad \leftarrow \text{x-intercepts}$$

Axis of Symmetry

$$x = \frac{-1 + (-7)}{2} = \frac{-8}{2}$$

$$x = -4$$

Vertex:  $x = -4$

$$y = 2(-4)^2 + 16(-4) + 14$$

$$= 2(16) - 64 + 14$$

$$= 32 - 64 + 14$$

$$= -18$$

Vertex:  $(-4, -18)$

3. a.)  $x = -3, -1$  through  $(2, 12)$

$$y = a(x-x_1)(x-x_2)$$

$$y = a(x-(-3))(x-(-1))$$

$$y = a(x+3)(x+1)$$

$$12 = a(2+3)(2+1)$$

$$12 = a(5)(3)$$

$$12 = a(15)$$

$$\frac{12}{15} = a = \frac{4}{5}$$

$$y = \frac{4}{5}(x+3)(x+1)$$

$$y = \frac{4}{5}(x^2 + x + 3x + 3)$$

$$y = \frac{4}{5}(x^2 + 4x + 3) \rightarrow y = \frac{4}{5}x^2 + \frac{16}{5}x + \frac{12}{5}$$

b.) Vertex  $(-2, 5)$  y-int = -1

$$y = a(x-p)^2 + q$$

$$y = a(x-(-2))^2 + 5$$

$$y = a(x+2)^2 + 5$$

$(0, -1)$   
 $(x, y)$

$$-1 = a(0+2)^2 + 5$$

$$-1 = a(4) + 5$$

$$-5 = 4a$$

$$-6 = a(4)$$

$$\frac{-6}{4} = a = \frac{-3}{2}$$

$$y = \frac{-3}{2}(x+2)^2 + 5$$

$$y = \frac{-3}{2}(x+2)(x+2) + 5$$

$$= \frac{-3}{2}(x^2 + 2x + 2x + 4) + 5$$

$$= \frac{-3}{2}(x^2 + 4x + 4) + 5$$

$$= \frac{-3}{2}x^2 - \frac{12}{2}x - \frac{12}{2} + \frac{10}{2} \rightarrow y = \frac{-3}{2}x^2 - 6x - 1$$

# PC II - Review cont. 17

## Unit IV cont.

3.c) Zero:  $x=5 \rightarrow (5,0)$  Vertex  $(3,2)$

$$y = a(x-p)^2 + q$$

$$y = a(x-3)^2 + 2$$

$$0 = a(5-3)^2 + 2$$

$$0 = a(2)^2 + 2$$

$$-2 = a(4)$$

$$\frac{-2}{4} = \frac{a(4)}{4}$$

$$a = -\frac{1}{2}$$

$$y = -\frac{1}{2}(x-3)^2 + 2$$

$$y = -\frac{1}{2}(x-3)^2 + 2$$

$$y = -\frac{1}{2}(x-3)(x-3) + 2$$

$$y = -\frac{1}{2}(x^2 - 3x - 3x + 9) + 2$$

$$y = -\frac{1}{2}(x^2 - 6x + 9) + 2$$

$$= -\frac{1}{2}x^2 + \frac{6}{2}x - \frac{9}{2} + \frac{4}{2}$$

$$y = -\frac{1}{2}x^2 + 3x - \frac{5}{2}$$

4.a) Vertex  $(2,3)$  Point  $(1,2)$

$$y = a(x-p)^2 + q$$

$$y = a(x-2)^2 + 3$$

$$\rightarrow 2 = a(1-2)^2 + 3$$

$$-3 = a(-1)^2 - 3$$

$$-1 = a(1)$$

$$-1 = a(1)$$

$$a = -1$$

$$y = -(x-2)^2 + 3$$

b) Vertex  $(-3,-4)$  Point  $(-2,-2)$

$$y = a(x-p)^2 + q$$

$$y = a(x-(-3))^2 - 4$$

$$y = a(x+3)^2 - 4$$

$$\rightarrow -2 = a(-2+3)^2 - 4$$

$$+4 = a(1)^2 - 4$$

$$2 = a(1)^2$$

$$2 = a$$

$$y = 2(x+3)^2 - 4$$

c) Vertex  $(1,-2)$  Point  $(-2,4)$

$$y = a(x-p)^2 + q$$

$$y = a(x-1)^2 - 2$$

$$\rightarrow 4 = a(-2-1)^2 - 2$$

$$+2 = a(9) - 2$$

$$6 = a(9)$$

$$6 = a(9)$$

$$a = \frac{6}{9} = \frac{2}{3}$$

$$y = \frac{2}{3}(x-1)^2 - 2$$

$$y = \frac{2}{3}(x-1)^2 - 2$$

# PC11 - Review cont... 18

## Unit IV cont...

5. Let  $x$  be the number of \$2 increases.

$$\begin{aligned}\text{Revenue} &= (\text{Cost of Eggs}) (\# \text{ of eggs sold}) \\ &= (20 + 2x)(120 - 5x) \\ &= 2400 - 100x + 240x - 10x^2 \\ &= -10x^2 + 140x + 2400 \quad \text{Find Maximum}\end{aligned}$$

$$\begin{aligned}&= -10(x^2 - 14x) + 2400 && \frac{1}{2}(-14) = -7 \\ &= -10(x^2 - 14x + 49 - 49) + 2400 && \hookrightarrow (-7)^2 = 49 \\ &= -10(x^2 - 14x + 49) + 490 + 2400 \\ &= -10(x - 7)^2 + 2890\end{aligned}$$

Maximum (Vertex)

$$x = 7, \quad y = 2890$$

$\therefore$  With 7 increases of \$2, max revenue is \$2890

$$\begin{aligned}\hookrightarrow & 20 + 2(7) \\ &= 20 + 14 = \$34\end{aligned}$$

Lenny should charge \$34 per egg to maximize revenue

6. Let  $2x+1$  be one odd integer  
then  $2x+3$  is the next consecutive odd integer.

$$\begin{aligned}\text{Sum of Squares} &= (2x+1)^2 + (2x+3)^2 \\ &= (2x+1)(2x+1) + (2x+3)(2x+3) \\ &= 4x^2 + 2x + 2x + 1 + 4x^2 + 6x + 6x + 9 \\ &= 8x^2 + 16x + 10 \quad \text{Find Minimum}\end{aligned}$$

$$\begin{aligned}&= 8(x^2 + 2x) + 10 && \frac{1}{2}(2) = 1 \\ &= 8(x^2 + 2x + 1 - 1) + 10 && \hookrightarrow 1^2 = 1 \\ &= 8(x^2 + 2x + 1) - 8 + 10 \\ &= 8(x+1)^2 + 2\end{aligned}$$

Minimum (Vertex)

$$x = -1, \quad \text{Sum} = 2.$$

$$\therefore 2x+1 \Rightarrow 2(-1)+1 = -2+1 = -1$$

$$2x+3 \Rightarrow 2(-1)+3 = -2+3 = 1$$

The two integers are -1 and 1

# PC II - Review con't... 19

## Unit IV con't...

7. Let  $x$  be the number of \$2 increases.

$$\begin{aligned}
 \text{Revenue} &= (\text{Cost of T-shirt})(\# \text{ of T-shirts sold}) \\
 &= (20 + 2x)(1200 - 60x) \\
 &= 24000 - 1200x + 2400x - 120x^2 \\
 &= -120x^2 + 1200x + 24000 \quad \text{Find Maximum} \\
 &= -120(x^2 - 10x) + 24000 \quad \left| \begin{array}{l} \frac{1}{2}(-10) = -5 \\ \hookrightarrow (-5)^2 = 25 \end{array} \right. \\
 &= -120(x^2 - 10x + 25 - 25) + 24000 \\
 &= -120(x^2 - 10x + 25) + 3000 + 24000 \\
 &= -120(x - 5)^2 + 27000
 \end{aligned}$$

Maximum (Vertex)

$$x = 5 \quad y = 27000$$

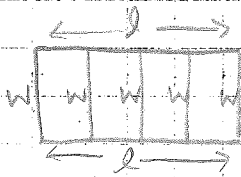
∴ With 5 increases of \$2, Max Revenue is \$27000

$$\hookrightarrow 20 + 2(5)$$

$$= 20 + 10 = \$30$$

The manager should sell shirts for \$30 each.

8.



$$2l + 5w = 1200$$

$$\frac{2l}{2} = \frac{1200 - 5w}{2}$$

$$l = 600 - \frac{5}{2}w$$

$$\begin{aligned}
 \text{Area} &= lw \\
 &= w \left( 600 - \frac{5}{2}w \right) \\
 &= 600w - \frac{5}{2}w^2 \\
 &= -\frac{5}{2}w^2 + 600w
 \end{aligned}$$

Maximum

$$\text{width} = 120$$

$$\text{Area} = 36000$$

$$\begin{aligned}
 \therefore \text{Maximum Area} \\
 &= 36000 \text{ m}^2
 \end{aligned}$$

$$\frac{1}{2}(-240)$$

$$= -120$$

$$\hookrightarrow (-120)^2$$

$$= 14400$$

$$= -\frac{5}{2}(w^2 - 240w)$$

$$= -\frac{5}{2}(w^2 - 240w + 14400 - 14400)$$

$$= -\frac{5}{2}(w^2 - 240w + 14400) + 36000$$

$$= -\frac{5}{2}(w - 120)^2 + 36000$$

# PC11 - Review con't... 20

## Unit V

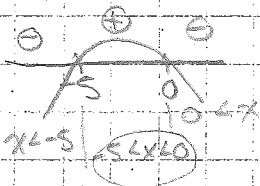
1. a)  $-x^2 - 5x > 0$  ← positive

Find zeros

$$-x^2 - 5x = 0$$

$$-x(x+5) = 0$$

$$x = 0 \quad x = -5$$



Solution

$$-5 < x < 0$$

b)  $3x^2 - 11x + 4 \geq 0$  ← positive

Find zeros

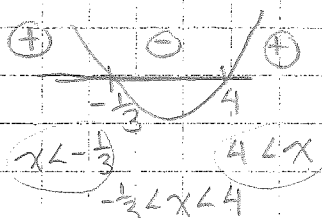
$$3x^2 - 11x + 4 = 0$$

$$3x^2 - 12x + 11x - 4 = 0$$

$$3x(x-4) + 1(x-4) = 0$$

$$(3x+1)(x-4) = 0$$

$$x = -\frac{1}{3} \quad x = 4$$



Solution

$$x < -\frac{1}{3}$$

and

$$4 < x$$

c)  $4x^2 - 20x + 10 < 2x^2 + 3x - 11$

$$+2x^2 - 3x + 11 \quad +2x^2 - 3x + 11$$

$$6x^2 - 23x + 21 < 0 \quad \leftarrow \text{negative}$$

Find zeros

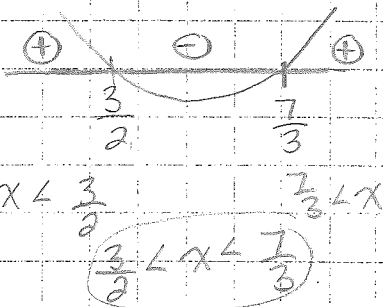
$$6x^2 - 23x + 21 = 0$$

$$6x^2 - 9x - 14x + 21 = 0$$

$$3x(2x-3) - 7(2x-3) = 0$$

$$(2x-3)(3x-7) = 0$$

$$x = \frac{3}{2} \quad x = \frac{7}{3}$$



Solution

$$\frac{3}{2} < x < \frac{7}{3}$$

2. a) Point:  $(-3, 3)$  and  $(2, 5)$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{5}$$

$$y = mx + b$$

$$y = \frac{2}{5}x + b \quad \text{point } (2, 5)$$

$$5 = \frac{2}{5}(2) + b$$

$$5 = \frac{4}{5} + b$$

$$5 - \frac{4}{5} = b$$

$$\frac{25}{5} - \frac{4}{5} + b = \frac{21}{5}$$

$$y = \frac{2}{5}x + \frac{21}{5}$$

shaded above

$$y \geq \frac{2}{5}x + \frac{21}{5}$$

# PC II - Review cont. ... 21

## Unit IV cont. ...

2. b)  $y = a(x-p)^2 + q$

$y = a(x+1)^2 + 4$

point  $(-3, 2)$   
 $\begin{matrix} x \\ y \end{matrix}$

$2 = a(-3+1)^2 + 4$

$2 = a(-2)^2 + 4$

$-4 = 4a$

$\frac{-4}{4} = \frac{a \cdot 4}{4}$

$-1 = a$

$y = -\frac{1}{2}(x+1)^2 + 4$

Shaded below

$y \leq -\frac{1}{2}(x+1)^2 + 4$

c)  $y = a(x-p)^2 + q$

$y = a(x-4)^2 - 4$

point  $(2, -5)$   
 $\begin{matrix} x \\ y \end{matrix}$

$-5 = a(2-4)^2 - 4$

$-1 = a(-2)^2$

$\frac{-1}{4} = \frac{a \cdot 4}{4}$

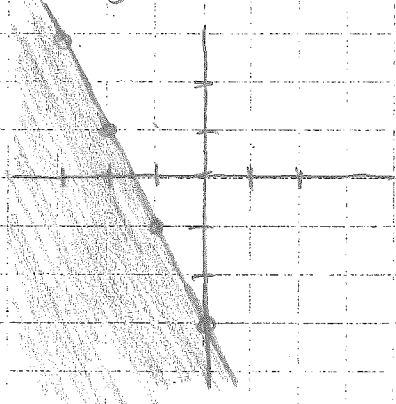
$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x-4)^2 - 4$

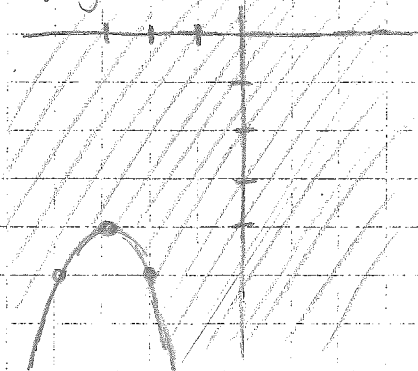
Shaded above

$y > -\frac{1}{4}(x-4)^2 - 4$

3. a)  $y \leq -2x + 3$



b)  $y \geq -(x+3)^2 - 4$



tp  $(0, 0)$

$0 \leq -2(0) + 3$

$0 \leq -3$  false

tp  $(0, 0)$

$0 \geq -(0+3)^2 - 4$

$0 \geq -9 - 4$

$0 \geq -13$  true

PC1- Review con't... 22. Unit V

3. c)  $y < -3x^2 - 30x - 71$

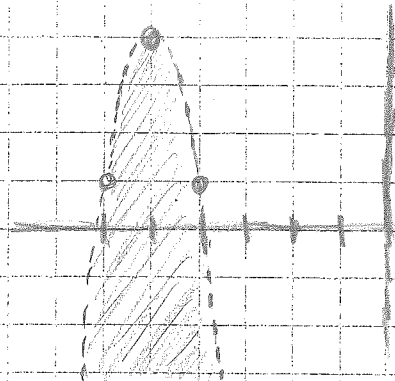
$$y < -3(x^2 + 10x) - 71 \quad \sqrt{\frac{1}{2}(10)} = 5$$

$$y < -3(x^2 + 10x + 25 - 25) - 71 \quad \rightarrow 5^2 = 25$$

$$y < -3(x^2 + 10x + 25) + 75 - 71$$

$$y < -3(x+5)^2 + 4$$

tp (0,0)  $\rightarrow$   $0 < -3(0)^2 - 30(0) - 71$   
 $0 < -71$  False



4. a)  $y = \frac{2}{3}x - 2$   
 $4x - 6y + 24 = 0$   
 Subs.

$$4x - 6\left(\frac{2}{3}x + 2\right) + 24 = 0$$

$$4x - 4x + 12 + 24 = 0$$

$$0 + 36 = 0$$

Uh oh... this isn't right...  
 $\therefore$  No Solution.

Check: Rearrange

$$4x - 6y + 24 = 0$$

$$4x + 24 = 6y$$

$$\frac{4x + 24}{6} = y$$

$\frac{2}{3}x + 4 = y$  This is parallel  
 to  $y = \frac{2}{3}x - 2$ .

b)  $y = \frac{1}{2}x + 2$   
 $y = 2(x-3)^2 + 1$   
 Subs.

$$\frac{1}{2}x + 2 = 2(x-3)^2 + 1$$

$$\frac{1}{2}x + 2 = 2(x-3)(x-3) + 1$$

$$\frac{1}{2}x + 2 = 2(x^2 - 6x + 9) + 1$$

$$\frac{1}{2}x + 2 = 2x^2 - 12x + 18 + 1$$

$$2\left(\frac{1}{2}x + 2\right) = 2(2x^2 - 12x + 19)$$

$$x + 4 = 4x^2 - 24x + 38$$

$$-x - 4 \quad -x - 4$$

$$0 = 4x^2 - 25x + 34$$

$$0 = 4x^2 - 8x - 17x + 34$$

$$0 = 4x(x-2) - 17(x-2)$$

$$0 = (x-2)(4x-17)$$

$$x = 2 \quad x = \frac{17}{4}$$

$x = 2: y = \frac{1}{2}x + 2$   
 $y = \frac{1}{2}(2) + 2$   
 $y = 1 + 2$   
 $y = 3$

$x = \frac{17}{4}: y = \frac{1}{2}x + 2$

$$y = \frac{1}{2}\left(\frac{17}{4}\right) + 2$$

$$= \frac{17}{8} + \frac{16}{8}$$

$$= \frac{33}{8}$$

$(2, 3)$  and  $\left(\frac{17}{4}, \frac{33}{8}\right)$

$$4. c) \quad y = x^2 + 2x - 3$$

$$y = -2(x-1)^2 + 6$$

Subs.

$$x^2 + 2x - 3 = -2(x-1)^2 + 6$$

$$x^2 + 2x - 3 = -2(x-1)(x-1) + 6$$

$$x^2 + 2x - 3 = -2(x^2 - 2x + 1) + 6$$

$$x^2 + 2x - 3 = -2x^2 + 4x - 2 + 6$$

$$x^2 + 2x - 3 + 2x^2 - 4x + 2 - 6 = 0$$

$$3x^2 - 2x - 7 = 0$$

$$a=3 \quad b=-2 \quad c=-7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-7)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 84}}{6}$$

$$x = \frac{2 \pm \sqrt{88}}{6}$$

$$x = \frac{2 + \sqrt{88}}{6}$$

$$x = 1.8968$$

$$x = \frac{2 - \sqrt{88}}{6}$$

$$= -1.2301$$

$$y = x^2 + 2x - 3$$

$$= (1.8968)^2 + 2(1.8968) - 3$$

$$= 4.3915$$

$$(1.8968, 4.3915)$$

$$y = x^2 + 2x - 3$$

$$= (-1.2301)^2 + 2(-1.2301) - 3$$

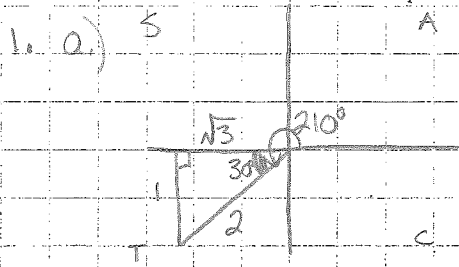
$$= -3.947$$

$$(-1.2301, -3.947)$$

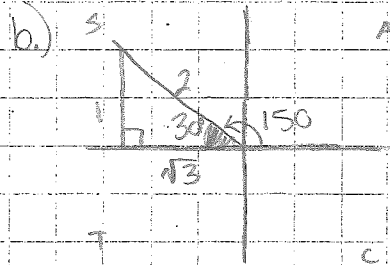


# PCII - Review con't. 24

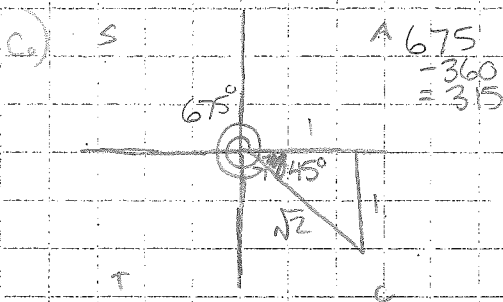
## Unit VI



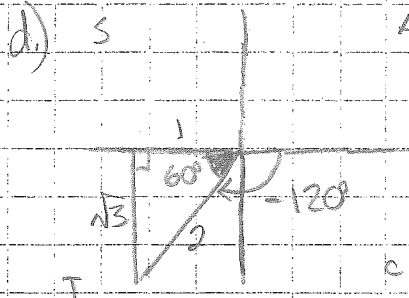
$$\sin 210^\circ = -\frac{\sqrt{3}}{2}$$



$$\tan 150^\circ = -\frac{1}{\sqrt{3}}$$

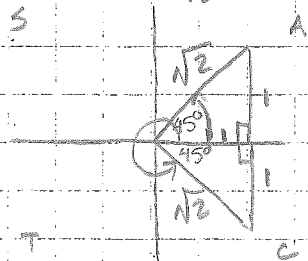


$$\tan(675^\circ) = -1$$



$$\cos(-120^\circ) = -\frac{1}{2}$$

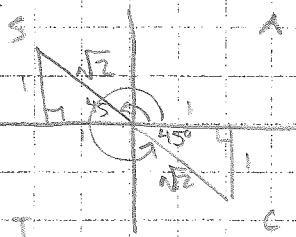
2. a.)  $\cos x = \frac{1}{\sqrt{2}}$  adj. hyp



$$x = 45^\circ$$

$$x_2 = 360 - 45 = 315^\circ$$

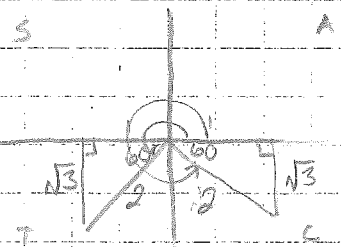
b.)  $\tan x = -1$



$$x_1 = 180 - 45 = 135^\circ$$

$$x_2 = 360 - 45 = 315^\circ$$

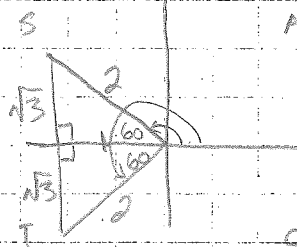
c.)  $\sin x = -\frac{\sqrt{3}}{2}$



$$x_1 = 180 + 60 = 240^\circ$$

$$x_2 = 360 - 60 = 300^\circ$$

d.)  $\cos x = -\frac{1}{2}$



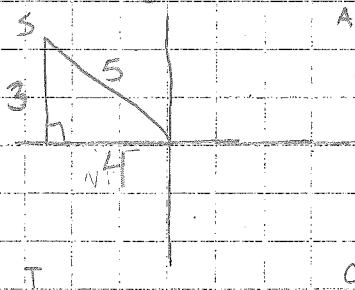
$$x_1 = 180 - 60 = 120^\circ$$

$$x_2 = 180 + 60 = 240^\circ$$

# PC II - Review cont. 25

## Unit VI

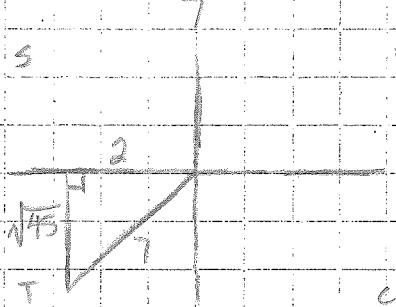
3.  $\sin x = \frac{3}{5}$  and  $\cos x < 0$  (negative)



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 3^2 &= 5^2 \\ a^2 + 9 &= 25 \\ a^2 &= 16 \\ a &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \cos x &= \frac{\text{adj}}{\text{hyp}} \\ \cos x &= -\frac{4}{5} \end{aligned}$$

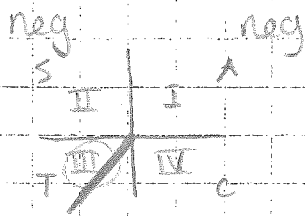
4.  $\cos x = -\frac{2}{7}$  and  $\sin x < 0$  (negative)



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 2^2 &= 7^2 \\ a^2 + 4 &= 49 \\ a^2 &= 45 \\ a &= \sqrt{45} \end{aligned}$$

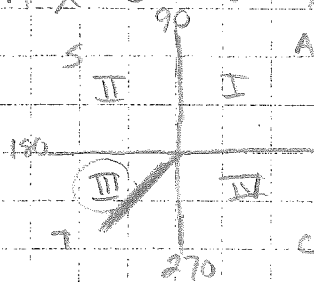
$$\begin{aligned} \tan x &= \frac{\text{opp}}{\text{adj}} \\ \tan x &= \frac{\sqrt{45}}{2} \end{aligned}$$

5. a)  $\sin x < 0$  &  $\cos x < 0$



Quadrant III

b)  $\sin x < 0$  and  $90^\circ < x < 270^\circ$

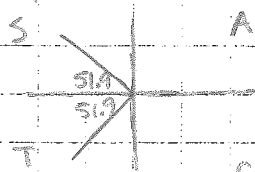


Quadrant III

6. a)  $\cos \theta = -0.6167$

$$\theta_r = \cos^{-1}(0.6167)$$

$$\theta_r = 51.9^\circ$$

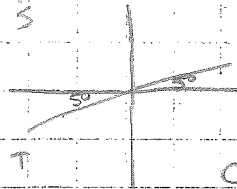


$$\begin{aligned} \theta &= 180 - 51.9^\circ \\ &= 128.1 \end{aligned}$$

$$\begin{aligned} \theta_2 &= 180 + 51.9^\circ \\ &= 231.9^\circ \end{aligned}$$

b)  $\tan \theta = 0.0875$

$$\theta_r = \tan^{-1}(0.0875) = 5^\circ$$



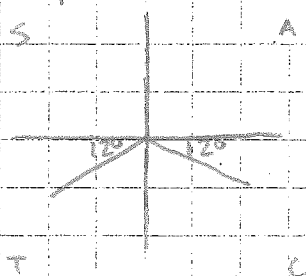
$$\theta_1 = 5^\circ$$

$$\theta_2 = 185^\circ$$

# PC11 - Review con't...26

## Unit VI

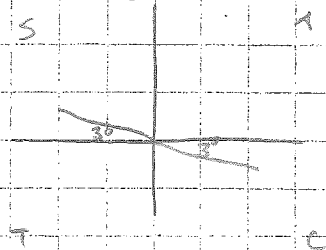
6. c)  $\sin \theta = -0.2079$   
 $\theta_p = \sin^{-1}(0.2079)$   
 $\theta_p = 12^\circ$



$$\theta_1 = 180^\circ + 12^\circ = 192^\circ$$

$$\theta_2 = 360^\circ - 12^\circ = 348^\circ$$

d)  $\tan \theta = -0.0524$   
 $\theta_p = \tan^{-1}(0.0524)$   
 $\theta_p = 3^\circ$



$$\theta_1 = 180^\circ - 3^\circ = 177^\circ$$

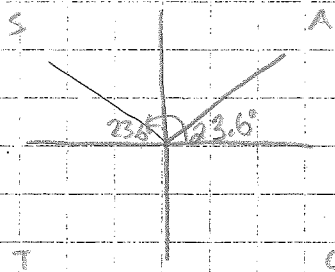
$$\theta_2 = 360^\circ - 3^\circ = 357^\circ$$

7. a)  $5 \sin^2 x - 2 = 0$

$$\sin^2 x = \frac{2}{5}$$

$$x_p = \sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{5}}\right)$$

$$x_p = 23.6^\circ$$



$$x_1 = 23.6^\circ$$

$$x_2 = 180^\circ - 23.6^\circ = 156.4^\circ$$

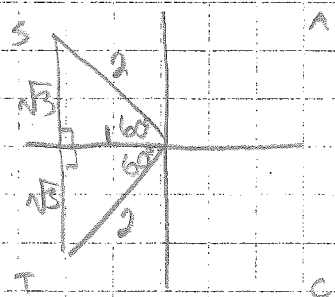
b)  $8 \cos^3 x = -1$

$$\cos^3 x = -\frac{1}{8}$$

$$(\cos x)^3 = -\frac{1}{8}$$

$$\cos x = \sqrt[3]{-\frac{1}{8}}$$

$$\cos x = -\frac{1}{2}$$



$$x_1 = 180^\circ - 60^\circ = 120^\circ$$

$$x_2 = 180^\circ + 60^\circ = 240^\circ$$

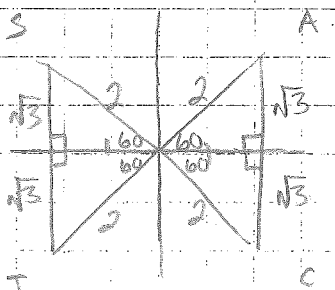
c)  $4 \sin^2 x - 3 = 0$

$$\sin^2 x = \frac{3}{4}$$

$$(\sin x)^2 = \frac{3}{4}$$

$$\sin x = \pm \sqrt{\frac{3}{4}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$



$$x_1 = 60^\circ$$

$$x_2 = 180^\circ - 60^\circ = 120^\circ$$

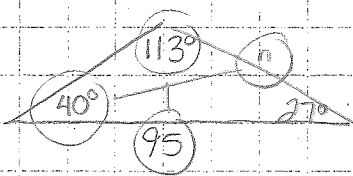
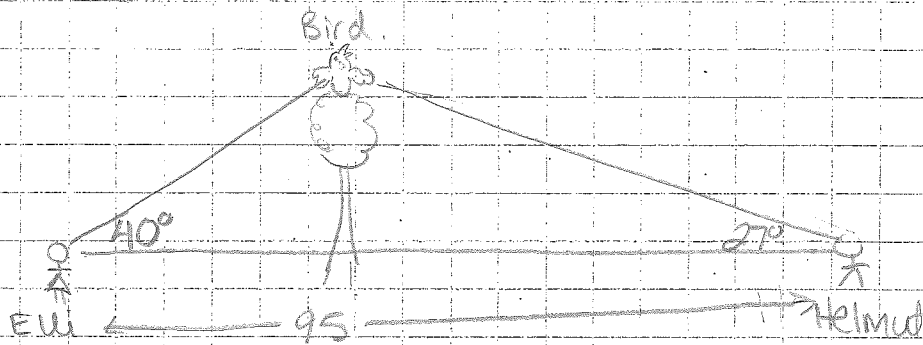
$$x_3 = 180^\circ + 60^\circ = 240^\circ$$

$$x_4 = 360^\circ - 60^\circ = 300^\circ$$

# PC11 - Reviews con't. sec 27.

## Unit VI

8.

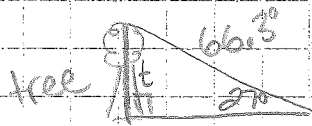


$$\frac{\sin 113^\circ}{95} = \frac{\sin 40^\circ}{n}$$

$$n \cdot \sin 113^\circ = 95 \sin 40^\circ$$

$$n = \frac{95 \sin 40^\circ}{\sin 113^\circ}$$

$$n = 66.3$$



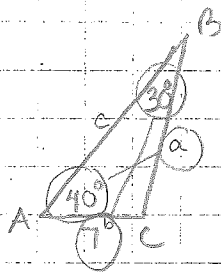
$$\sin 27^\circ = \frac{t}{66.3}$$

$$66.3 \sin 27^\circ = t$$

$$30.1 = t$$

8. tree = 30.1m

9. a.)

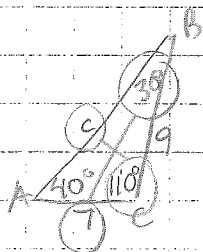


$$\frac{\sin 30^\circ}{7} = \frac{\sin 40^\circ}{a}$$

$$a \sin 30^\circ = 7 \sin 40^\circ$$

$$a = \frac{7 \sin 40^\circ}{\sin 30^\circ}$$

$$a = 9.0$$



$$\angle C = 180^\circ - 30^\circ - 40^\circ = 110^\circ$$

$$\frac{\sin 30^\circ}{7} = \frac{\sin 110^\circ}{c}$$

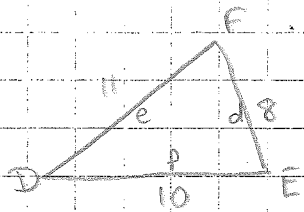
$$c \sin 30^\circ = \frac{7 \sin 110^\circ}{\sin 30^\circ}$$

$$c = 13.16 \approx 13.2$$

PC 11 - Review can't... 28

Unit VI

9. b.)



$$\angle D: d^2 = e^2 + f^2 - 2ef \cos D$$

$$d^2 - e^2 - f^2 = -2ef \cos D$$

$$\frac{8^2 - 11^2 - 10^2}{-2(11)(10)} = \cos D$$

$$\frac{-157}{-220} = \cos D$$

$$\frac{157}{220} = \cos D$$

$$\angle D = 44.47^\circ = 44.5^\circ$$

$\angle F:$

$$f^2 = d^2 + e^2 - 2de \cos F$$

$$f^2 - d^2 - e^2 = -2de \cos F$$

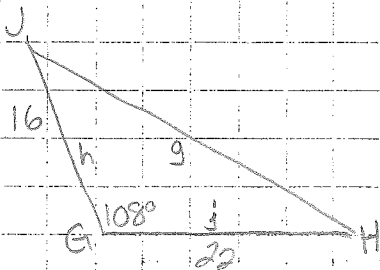
$$\frac{10^2 - 8^2 - 11^2}{-2(8)(11)} = \cos F$$

$$\frac{-85}{-176} = \cos F$$

$$\angle F = 61.1^\circ$$

$$\angle E = 180^\circ - 44.5^\circ - 61.1^\circ = 74.4^\circ$$

c.)



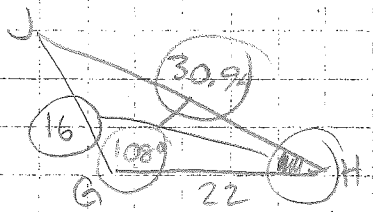
$$g^2 = j^2 + h^2 - 2jh \cos G$$

$$g^2 = 22^2 + 16^2 - 2(22)(16) \cos 108^\circ$$

$$g^2 = 957.547964$$

$$g = \sqrt{957.548}$$

$$g = 30.9$$



$$\frac{\sin 108}{30.94} = \frac{\sin H}{16}$$

$$\frac{16 \sin 108}{30.94} = \sin H$$

$$29.46 = \angle H$$

$$\angle H = 29.5^\circ$$

$$\angle J = 180^\circ - 108^\circ - 29.5^\circ = 42.5^\circ$$

# PC11 - Reviews con't...29

## Unit VII

$$\begin{aligned}
 1. a.) \quad & \frac{4}{x+4} - \frac{9}{3x^2+12x} \\
 & = \frac{4}{x+4} - \frac{9}{3x(x+4)} \quad \text{lcd} = 3x(x+4) \\
 & \quad \quad \quad x \neq 0 \quad x \neq -4 \\
 & = \frac{4(3x) - 9}{(x+4)(3x)} \\
 & = \frac{12x - 9}{3x(x+4)} \\
 & = \frac{3(4x-3)}{3x(x+4)} \\
 & = \frac{4x-3}{x(x+4)} \quad x \neq 0, -4
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad & \frac{5}{m+2} + \frac{2m}{m-5} \quad \text{lcd} = (m+2)(m-5) \\
 & \quad \quad \quad m \neq -2, 5 \\
 & = \frac{5(m-5)}{(m+2)(m-5)} + \frac{2m(m+2)}{(m-5)(m+2)} \\
 & = \frac{5m-25 + 2m^2+4m}{(m+2)(m-5)} \\
 & = \frac{2m^2+9m-25}{(m+2)(m-5)} \quad , \quad m \neq -2, 5
 \end{aligned}$$

$$\begin{aligned}
 c.) \quad & \frac{2}{x^2-x-12} - \frac{3}{9-x^2} \\
 & = \frac{2}{(x-4)(x+3)} - \frac{3}{(x^2-9)} \quad \text{lcd} = (x-4)(x+3)(x-3) \\
 & \quad \quad \quad x = 4, -3, 3 \\
 & = \frac{2}{(x-4)(x+3)} + \frac{3}{(x+3)(x-3)} \\
 & = \frac{2(x-3)}{(x-4)(x+3)(x-3)} + \frac{3(x-4)}{(x+3)(x-3)(x-4)} \\
 & = \frac{2x-6 + 3x-12}{(x-4)(x+3)(x-3)} \\
 & = \frac{5x-18}{(x-4)(x+3)(x-3)} \quad , \quad x = -3, 3, 4
 \end{aligned}$$

# PCII- Review con't. ... 30

## Unit VII

$$\begin{aligned}
 \text{1. d.) } & \frac{x}{x+7} + \frac{3x-1}{3x-1} \quad \text{lcd} = (x+7)(3x-1) \\
 & = \frac{x(3x-1) + 3x(x+7) - 1(x+7)(3x-1)}{(x+7)(3x-1)} \\
 & = \frac{3x^2 - x + 3x^2 + 21x - (3x^2 - x + 21x - 7)}{(x+7)(3x-1)} \\
 & = \frac{6x^2 + 20x - 3x^2 + x - 21x + 7}{(x+7)(3x-1)} \\
 & = \frac{3x^2 + 7}{(x+7)(3x-1)} \quad x \neq -7, \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{e.) } & \frac{3x+6}{x^2+7x+10} \\
 & = \frac{3(x+2)}{(x+5)(x+2)} \\
 & = \frac{3}{x+5} \quad x \neq -5, -2
 \end{aligned}$$

$$\begin{aligned}
 \text{f.) } & \frac{2x^2+4x}{16} \cdot \frac{3x}{x+2} \\
 & = \frac{2x(x+2)}{16} \cdot \frac{3x}{(x+2)} \\
 & = \frac{3x^2}{8}, \quad x \neq -2
 \end{aligned}$$

$$\begin{aligned}
 & 2x^2 + 15x + 7 \\
 & = 2x^2 + 14x + 1x + 7 \\
 & = 2x(x+7) + 1(x+7) \\
 & = (x+7)(2x+1)
 \end{aligned}$$

$$\begin{aligned}
 \text{g.) } & \frac{2x-10}{2x^2+15x+7} \cdot \frac{x+3}{x^2-2x-15} \div \frac{3x-6}{x^2+5x-14} \\
 & = \frac{2(x-5)}{(x+7)(2x+1)} \cdot \frac{(x+3)}{(x-5)(x+3)} \div \frac{3(x-2)}{(x+7)(x-2)} \quad x \neq -7, -\frac{1}{2}, 5, -3, -7, 2 \\
 & = \frac{2}{(x+7)(2x+1)} \cdot \frac{(x+7)(x-2)}{3(x-2)} \\
 & = \frac{2}{3(2x+1)} \quad x \neq -7, -3, -\frac{1}{2}, 2, 5
 \end{aligned}$$

# PCII - Review con't. sec 31

## Unit VII

$$2.a.) \frac{2}{3} = \frac{7}{5x} + 1 \quad \text{lcd} = 15x$$

$$\frac{2(5x)}{3(5x)} = \frac{7(3)}{(5x)(3)} + \frac{1(15x)}{15x}$$

$$10x = 21 + 15x$$

$$-15x \quad -15x$$

$$-5x = 21$$

$$x = \frac{-21}{5}$$

$$b.) \frac{3}{m+3} = \frac{5}{m}$$

$$3m = 5(m+3)$$

$$3m = 5m + 15$$

$$-5m \quad -5m$$

$$-2m = 15$$

$$m = \frac{-15}{2}$$

$$c.) \frac{3}{2x} - \frac{2x}{x+1} + 2 = 0$$

$$\frac{3(x+1)}{2x(x+1)} - \frac{2x(2x)}{(x+1)(2x)} + \frac{2(2x)(x+1)}{2x(x+1)} = \frac{0}{2x(x+1)}$$

$$3x+3 - 4x^2 + 4x(x+1) = 0$$

$$3x+3 - 4x^2 + 4x^2 + 4x = 0$$

$$7x+3 = 0$$

$$7x = -3$$

$$x = \frac{-3}{7}$$



# PC11 - Reviews cont. 32.

## Unit VII

$$2.d) \frac{p-5}{p^2-1} = 1 - \frac{2p}{p-1}$$

$$\text{lcd} = (p+1)(p-1)$$

$$\frac{p-5}{(p+1)(p-1)} = \frac{1 \cdot (p+1)(p-1) - 2p(p+1)}{(p+1)(p-1)} \quad p \neq 1, -1$$

$$p-5 = p^2-1 - 2p^2-2p$$

$$-p+5 = p^2-1 - 2p^2-2p$$

$$0 = -p^2 - 3p + 4$$

$$p^2 + 3p - 4 = 0$$

$$(p+4)(p-1) = 0$$

$$p = -4 \quad p = 1 \quad \uparrow \text{reject}$$

$$e) \frac{4m}{3m-2} + \frac{2m}{3m+2} = 2$$

$$\text{lcd} = (3m-2)(3m+2)$$

$$\frac{4m(3m+2)}{(3m-2)(3m+2)} + \frac{2m(3m-2)}{(3m+2)(3m-2)} = \frac{2(3m-2)(3m+2)}{(3m-2)(3m+2)}$$

$$12m^2 + 8m + 6m^2 - 4m = 2(9m^2 - 4)$$

$$18m^2 + 4m = 18m^2 - 8$$

$$-18m^2 \quad -18m^2$$

$$4m = -8$$

$$m = \frac{-8}{4} = -2$$

$$f) \frac{4}{x^2-8x+12} - \frac{1}{x-6} = \frac{x}{x-2}$$

$$\text{lcd} = (x-6)(x-2)$$

$$x \neq 6, 2$$

$$\frac{4}{(x-6)(x-2)} - \frac{1}{(x-6)(x-2)} = \frac{x}{(x-2)(x-6)}$$

$$4 - x + 2 = x^2 - 6x$$

$$0 = x^2 - 6x + x - 4 - 2$$

$$0 = x^2 - 5x - 6$$

$$0 = (x-6)(x+1)$$

$$x = 6 \quad x = -1$$

reject:  $\uparrow$

Unit VII

3. Let  $x$  be the time it takes for Carla to paint the barn on her own in hours

∴  $\frac{1}{x}$  = Amount of the door Carla would paint in 1 hour

$\frac{1}{13}$  = Amount of the door Lenny would paint in 1 hour

$\frac{1}{6}$  = Amount they would paint of the door together in 1 hour

$$\frac{1}{x} + \frac{1}{13} = \frac{1}{6} \quad \text{LCD} = 78x$$

$$\frac{1(78)}{x(78)} + \frac{1(6x)}{13(6x)} = \frac{1(13x)}{6(13x)}$$

$$\frac{78}{78x} + \frac{6x}{78x} = \frac{13x}{78x}$$

$$78 + 6x = 13x$$

$$\begin{array}{r} 78 \\ -6x \\ \hline 78 = 7x \end{array}$$

$$\frac{78}{7} = \frac{7x}{7}$$

$$11.14 = x$$

∴ Carla would paint the barn in 11.14 hours.

4.  $\text{Speed} = \frac{\text{distance}}{\text{time}} \rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$

Let  $x$  = the speed Lenny travels at  
then  $x-4$  = the speed of Carla

	distance	speed	time = $\frac{\text{dist}}{\text{speed}}$
Lenny	104	$x$	$\frac{104}{x}$
Carla	96	$x-4$	$\frac{96}{x-4}$

Time for each is equal:  $\frac{104}{x} = \frac{96}{x-4}$

PC 11 - Review cont... 34

Unit VII

4. cont...

$$\frac{104}{x} = \frac{96}{x-4}$$

$$\frac{104(x-4)}{x(x-4)} = \frac{96x}{(x-4)(x)}$$

$$104(x-4) = 96x$$

$$104x - 416 = 96x$$

$$-104x \quad -104x$$

$$\frac{-416}{-8} = \frac{-8x}{-8}$$

$$x = 52$$

∴ Lenny travels at 52 km/h

5.  $\text{time} = \frac{\text{distance}}{\text{speed}}$

Let  $x = \text{Speed of the boat in still water}$

	Distance	Speed	time
Upstream	85	$x-3$	$\frac{85}{x-3}$
Downstream	115	$x+3$	$\frac{115}{x+3}$

$$\frac{85}{x-3} = \frac{115}{x+3}$$

$$85(x+3) = 115(x-3)$$

$$85x + 255 = 115x - 345$$

$$-115x \quad -115x$$

$$-30x + 255 = -345$$

$$\frac{-255}{-30} \quad \frac{-255}{-30}$$

$$\frac{-30x}{-30} = \frac{-600}{-30}$$

$$x = 20$$

∴ The speed of the boat in still water is 20 km/h

Unit VII

6. Let  $x$  = amount of sand added.  
 then  $50+x$  will be the total volume of  
 topsoil and sand.

$$\frac{x}{50+x} = 0.23$$

$$\begin{aligned} x &= 0.23(50+x) \\ x &= 11.5 + 0.23x \\ -0.23x & \quad -0.23x \end{aligned}$$

$$\begin{aligned} 0.77x &= 11.5 \\ \frac{0.77x}{0.77} &= \frac{11.5}{0.77} \\ x &= 14.94 \end{aligned}$$

∴ 14.94 L of sand should be added.