Example 2.
Add the displacements \( D_1 + D_2 + D_3 = R \) where \( D_1 = 6 \text{ km north}, \ D_2 = 3 \text{ km east}, \) and \( D_3 = 4 \text{ km (45° S of E)} \).

Assume north is ‘up’. Use the scale 1 grid length = 1 km and:
- draw the vector-sum tip-to-tail:
- draw the resultant \( R \)
- measure the length \( R \) and convert to kilometers
- measure the angle using a protractor
- report \( R \).

\[\begin{align*}
D_1 & \rightarrow \\
D_2 & \rightarrow 45° \\
D_3 & \rightarrow \text{becomes} \\
\theta & \text{becomes} \\
R & \rightarrow
\end{align*}\]

\( R \) should measure to be 6.7 km; \( \theta = 63° \)

\( \rightarrow R = 6.7 \text{ km at 63° E of N (or 27° N of E)} \)
Example 3.
Given the vectors shown:

\[ \vec{A} \text{ (6.3 cm)} \]
\[ 18^\circ \]
\[ \vec{B} \text{ (5 cm)} \]
\[ 53^\circ \]

Draw diagrams and find \( R \) for:
(a) \( \vec{A} + \vec{B} = \vec{R} \)
(b) \( \vec{A} - \vec{B} = \vec{R} \)
(c) \( \vec{B} - \vec{A} = \vec{R} \)

- Scale: 1 grid length = 1 cm

(a) \( \vec{A} + \vec{B} \)

\[ \vec{R} = 6.6 \text{ cm @ } 26^\circ \text{ E of N} \]
(b) \( A - B \)

![Diagram of A - B relationship with vector \( R = 9.1 \text{ cm } \) @ 77º E of S]

(c) \( B - A \)

![Diagram of B - A relationship with vector \( R = 9.1 \text{ cm } \) @ 77º E of S]
Example 4.
A plane with an air speed of 105 m/s heads west when a 25 m/s north wind is blowing. What is the velocity of the plane relative to the ground?

To find resultant velocity, vector add plane velocity and wind velocity:

\[ R = \sqrt{105^2 + 25^2} = 108 \text{ m/s} \]
\[ \theta = \tan^{-1} \left( \frac{25}{105} \right) = 13^\circ \]

Answer: 108 m/s @ 13° N of W
Example 5.
A plane is capable of 120 m/s is still air. Where must the pilot head the plane in order to end up going due north when there is a 35 m/s west wind?

\[
\theta = \sin^{-1} \left[ \frac{35}{120} \right]
\]

\[ \theta = 17^\circ \text{ E of N} \]
Example 6.
A boat is capable of 12 m/s in still water. If a river flows at 7.0 m/s due east and is 500 m wide:

(a) What is the velocity of the boat relative to the shore if the boat heads south, perpendicular to the current?

\[ R = \sqrt{12^2 + 7^2} = 14 \text{ m/s} \]
\[ \theta = \tan^{-1} \left( \frac{7}{12} \right) = 30^\circ \]

\[ R = 14 \text{ m/s @ } 30^\circ \text{E of S} \]

(b) How long would it take to cross the river?

\[ d = v_{aw} t \]
\[ t = \frac{d}{v_{aw}} = \frac{500}{12} \]
\[ t = 42 \text{ s} \]
(c) Where would the boat have to aim in order to end up directly across from its starting point?

\[ \text{→ boat should aim into the current} \]

\[ \sqrt{12 \text{ m/s}} \]

\[ \text{→ vector-add boat velocity + current velocity so that the resultant is directed due south (across the river)} \]

\[ \theta = \sin^{-1} \left( \frac{7}{12} \right) = 36^\circ \]

Answer: \[ 36^\circ \text{ W of S} \]
Example 7.
A cannon is shot at a muzzle velocity of 1500 m/s at an angle of 60° to the horizontal. What are the vertical and horizontal components of the velocity?

\[ v = 1500 \text{ m/s} \]

vertical component: \[ \frac{v_{\text{Vert}}}{1500} = \sin 60° \]

\[ v_{\text{Vert}} = 1500 \sin 60° \]

\[ v_{\text{Vert}} = 1.3 \times 10^3 \text{ m/s} \]

horizontal component: \[ \frac{v_{\text{Hor}}}{1500} = \cos 60° \]

\[ v_{\text{Hor}} = 1500 \cos 60° \]

\[ v_{\text{Hor}} = 7.5 \times 10^2 \text{ m/s} \]
Example 8.
A boy pulls a wagon with a force of 100 N at 40 degrees to the horizontal. Find the pulling force ($F_x$) and the lifting force ($F_y$).

Pulling Force: \[
\frac{F_x}{100} = \cos 40^\circ
\]

\[
F_x = 100 \cos 40^\circ
\]

\[
F_x = 76.6 \text{ N}
\]

Lifting Force: \[
\frac{F_y}{100} = \sin 40^\circ
\]

\[
F_y = 100 \sin 40^\circ
\]

\[
F_y = 64.3 \text{ N}
\]